

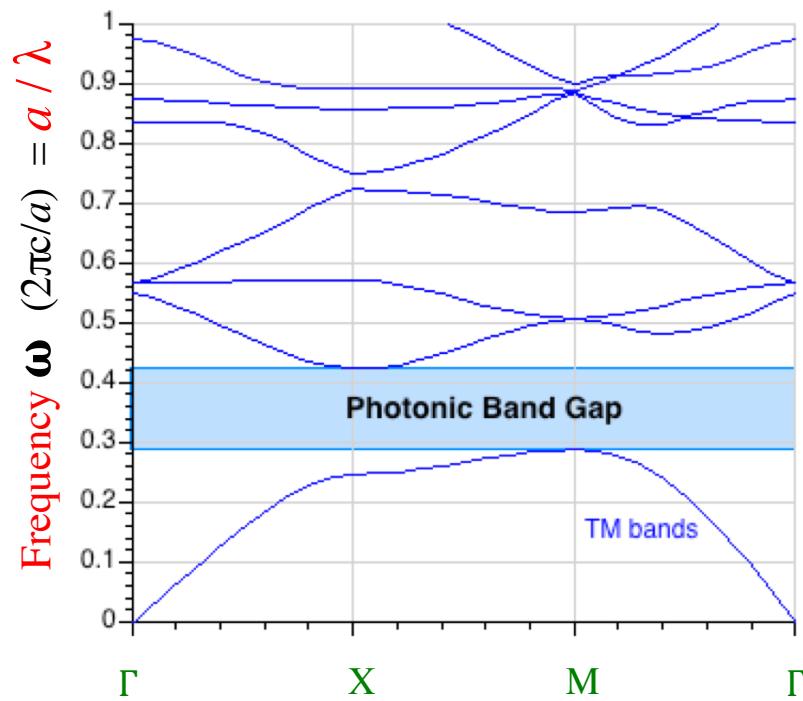
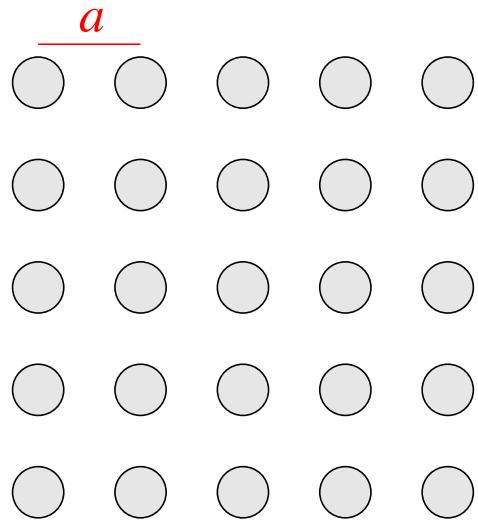
# 2d photonic crystals

Steven G. Johnson, MIT

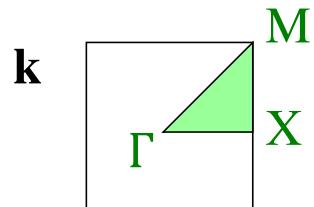
18.369/8.315, Spring 2018

See also chapters 5 and 10 of *Photonic Crystals: Molding the Flow of Light*

2d periodicity,  $\varepsilon=12:1$



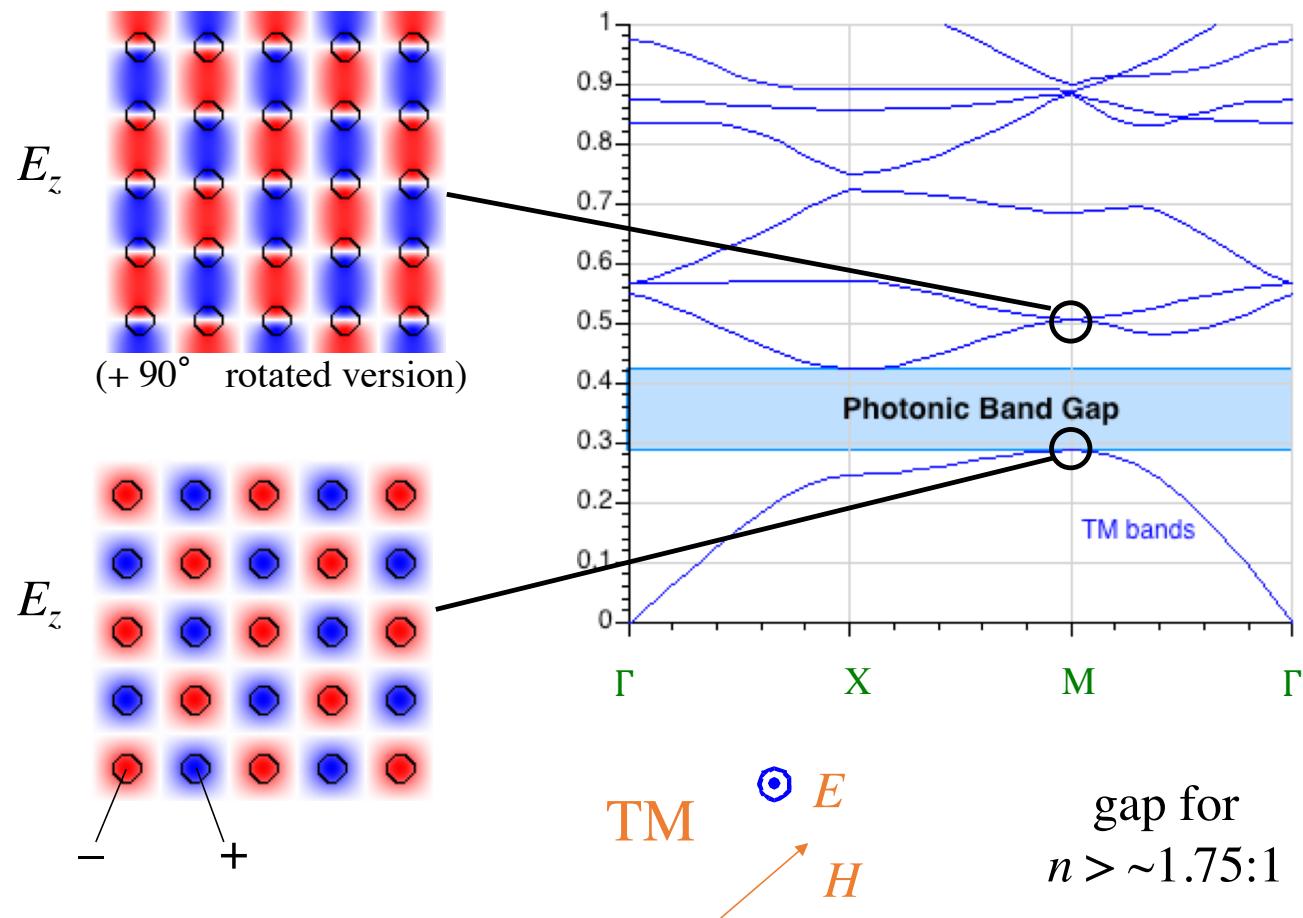
irreducible Brillouin zone



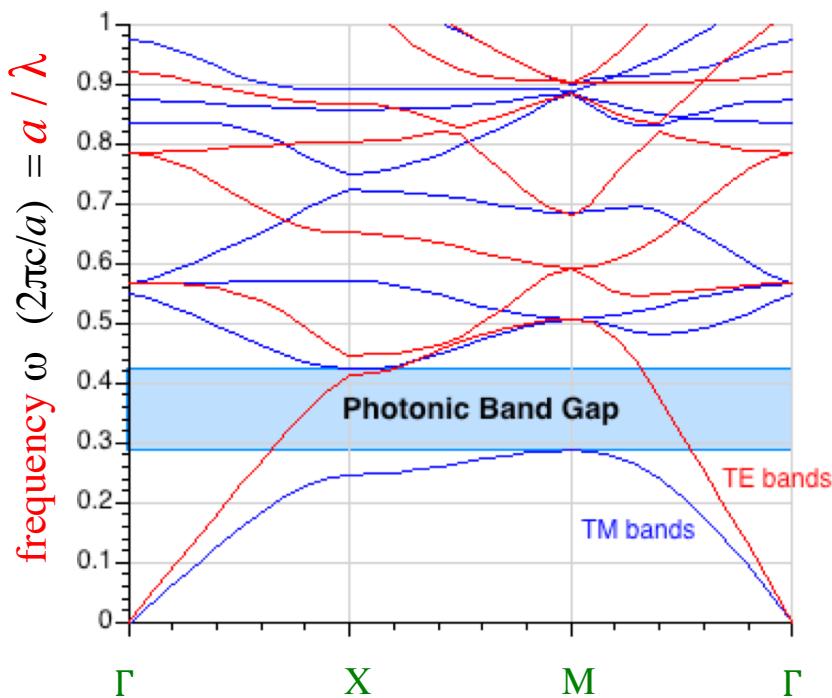
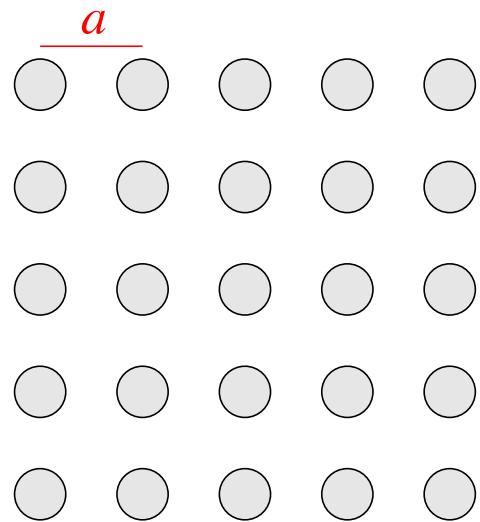
TM  $\circlearrowleft E$   
H

gap for  
 $n > \sim 1.75:1$

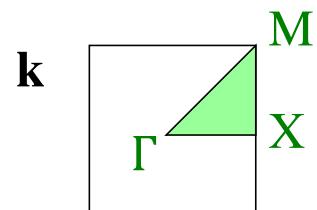
## 2d periodicity, $\epsilon=12:1$



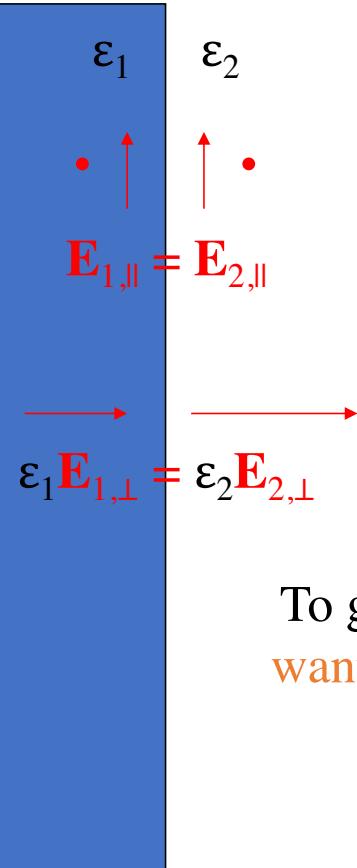
2d periodicity,  $\varepsilon=12:1$



irreducible Brillouin zone



# What a difference a boundary condition makes...



$E_{\parallel}$  is continuous:  
energy density  $\epsilon|E|^2$   
more in **larger**  $\epsilon$

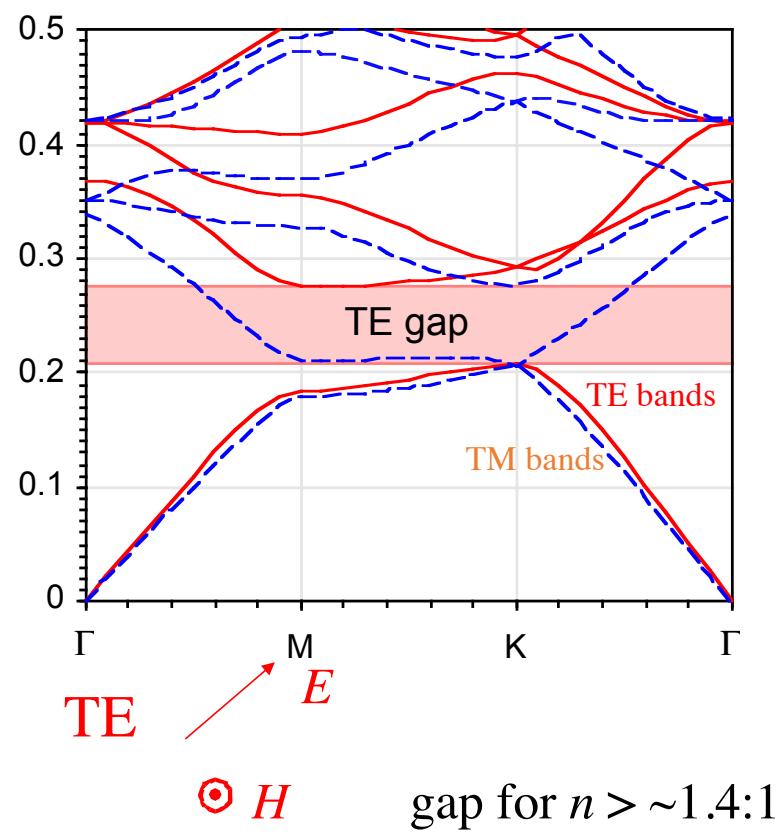
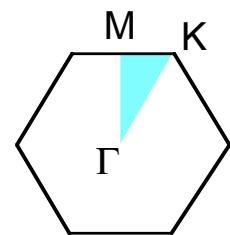
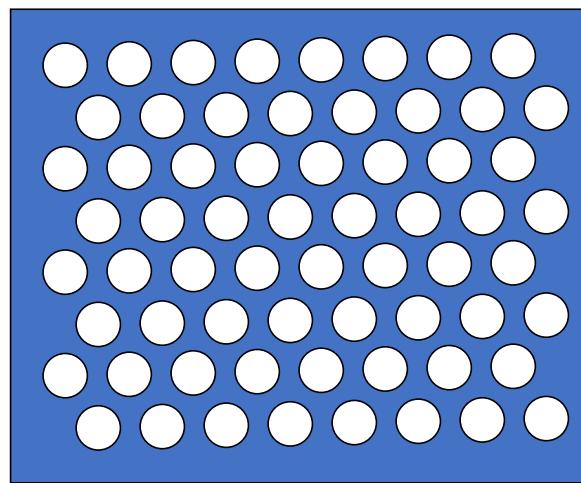
$\epsilon E_{\perp}$  is continuous:  
energy density  $|\epsilon E|^2/\epsilon$   
more in **smaller**  $\epsilon$

To get strong confinement & gaps,  
want **E mostly parallel to interfaces**

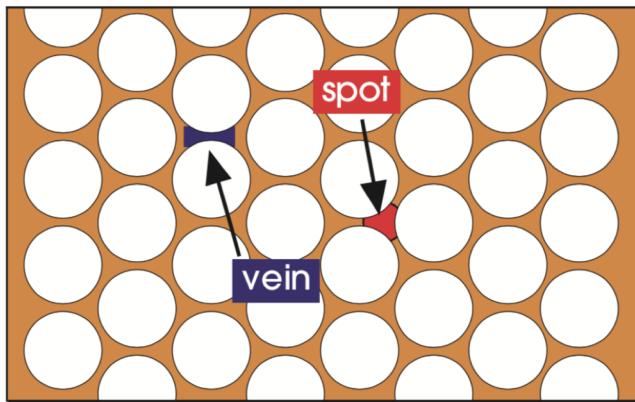
TM:  $\parallel$

TE:  $\perp$

## 2d photonic crystal: TE gap, $\epsilon=12:1$

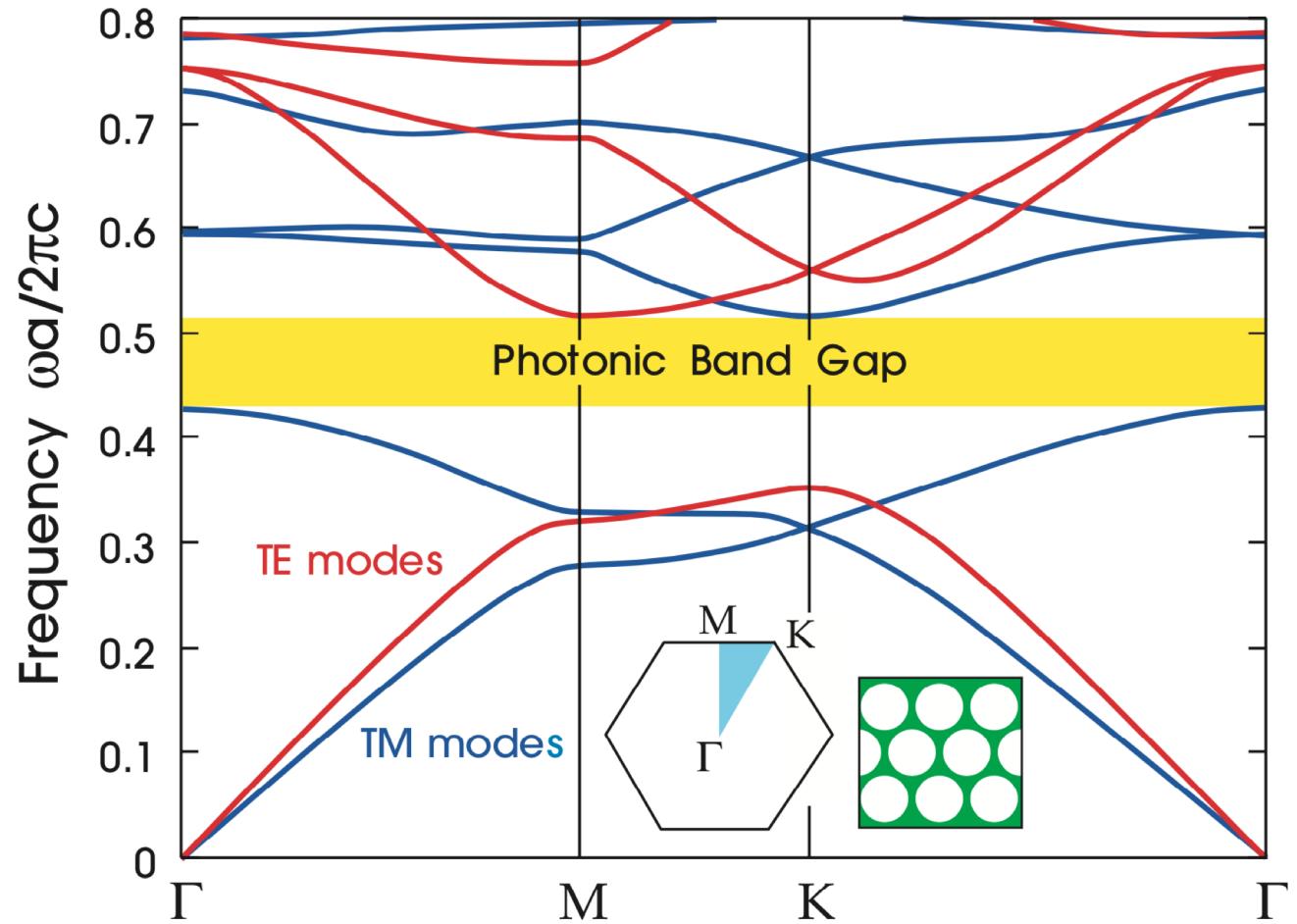


# “Complete” (TE+TM) gap in 2d

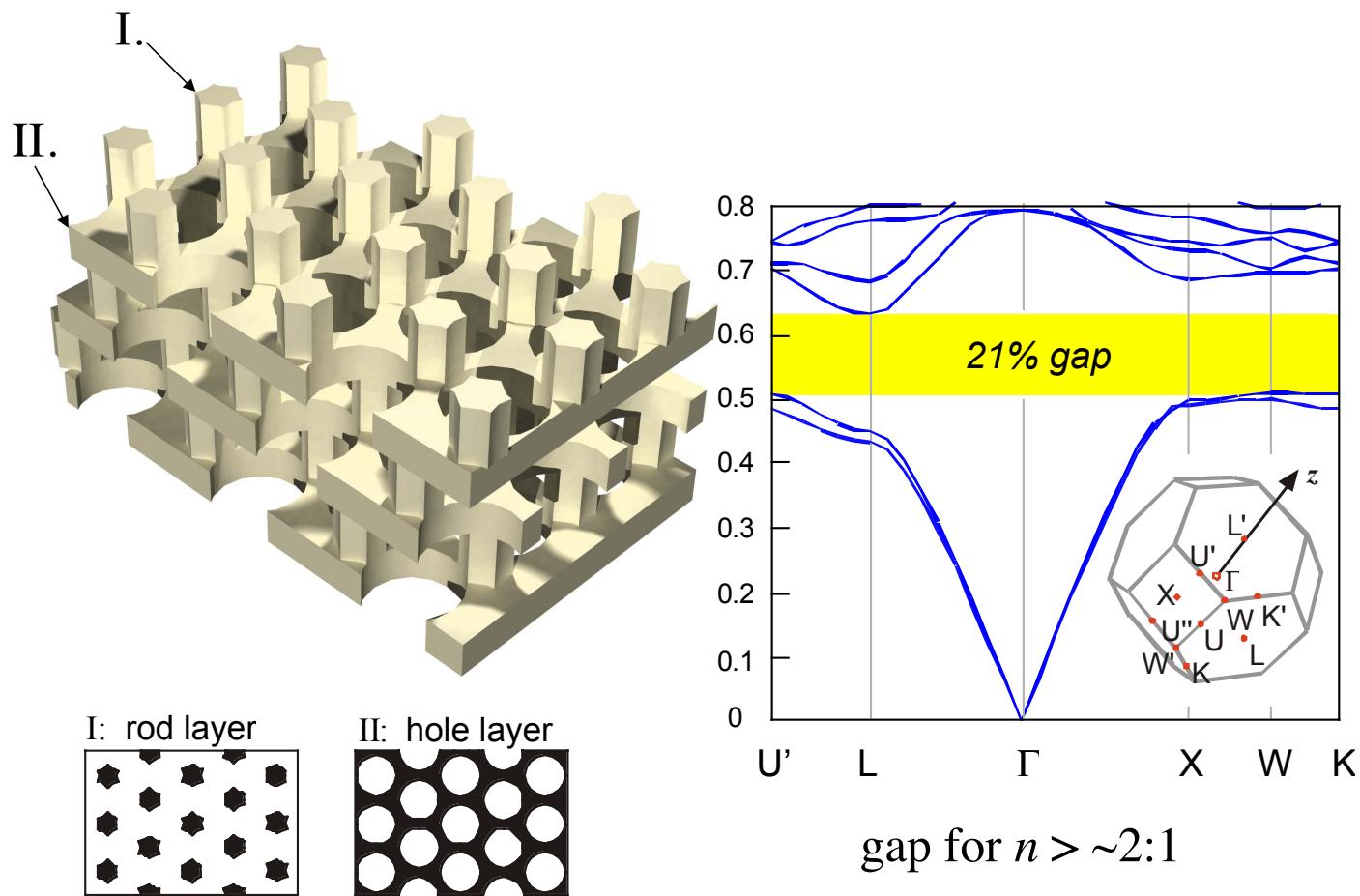


Spots: big enough for lowest TM bands  
to concentrate (gap with 3rd band)

Veins: lowest TE band circles around holes



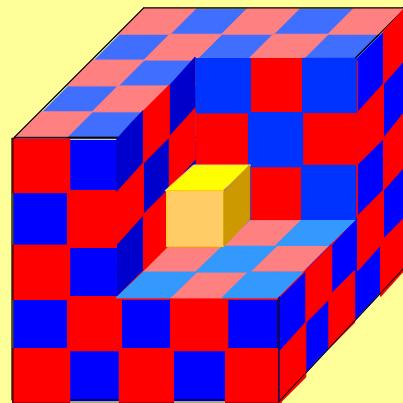
# 3d photonic crystal: complete gap , $\varepsilon=12:1$



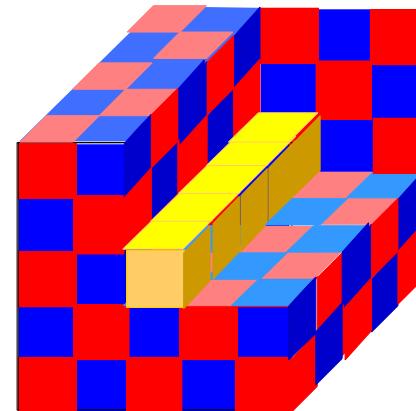
[ S. G. Johnson *et al.*, *Appl. Phys. Lett.* **77**, 3490 (2000) ]

Intentional “defects” are good

microcavities

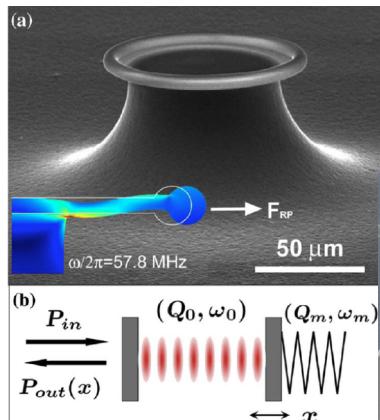


waveguides (“wires”)

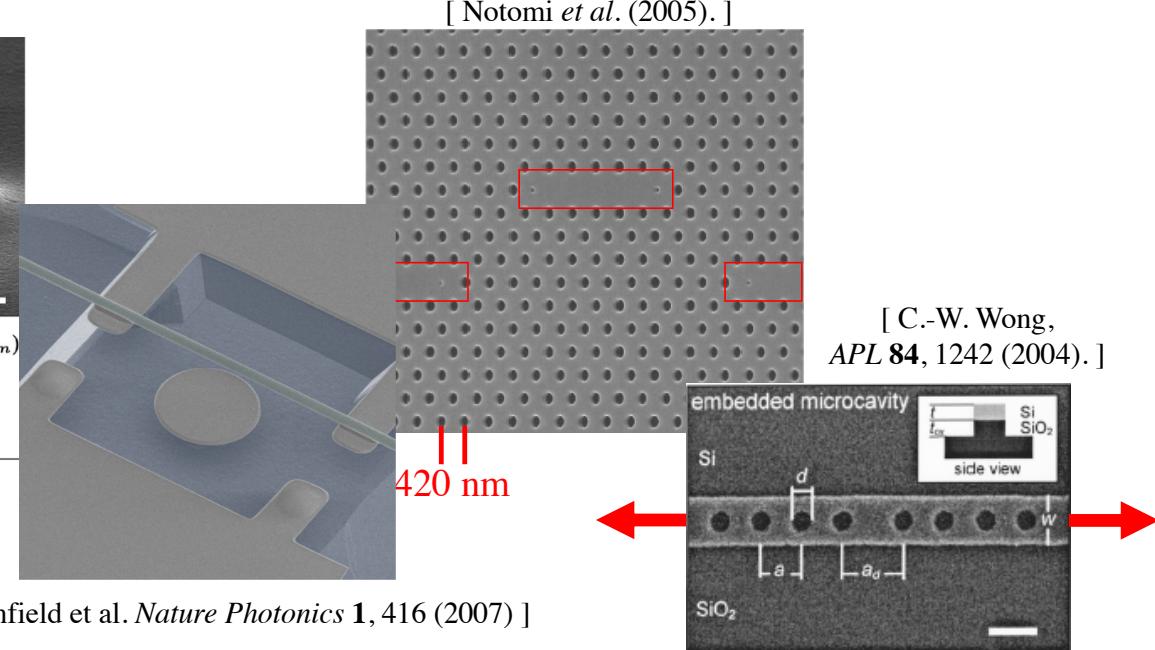


# Resonance

an **oscillating mode** trapped for a long time in some volume  
 (of light, sound, ...)      lifetime  $\tau \gg 2\pi/\omega_0$   
 frequency  $\omega_0$       quality factor  $Q = \omega_0 \tau / 2$   
 energy  $\sim e^{-\omega_0 t / Q}$       modal  
 volume  $V$



[ Schliesser et al.,  
*PRL* **97**, 243905 (2006) ]



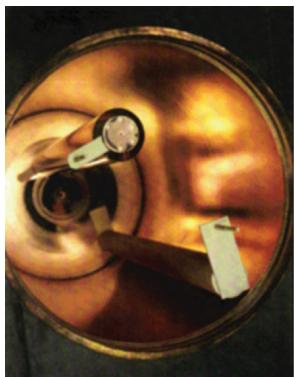
# Why Resonance?

an oscillating mode trapped for a long time in some volume

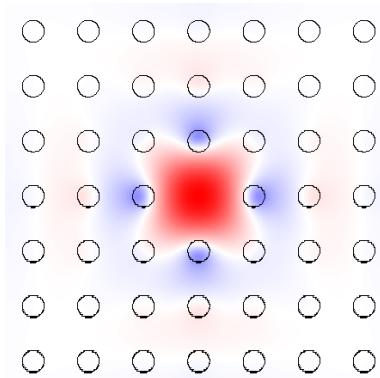
- long time = narrow bandwidth ... filters (WDM, etc.)
  - $1/Q$  = fractional bandwidth
- resonant processes allow one to “impedance match” hard-to-couple inputs/outputs
- long time, small  $V$  ... enhanced wave/matter interaction
  - lasers, nonlinear optics, opto-mechanical coupling, sensors, LEDs, thermal sources, ...

# How Resonance?

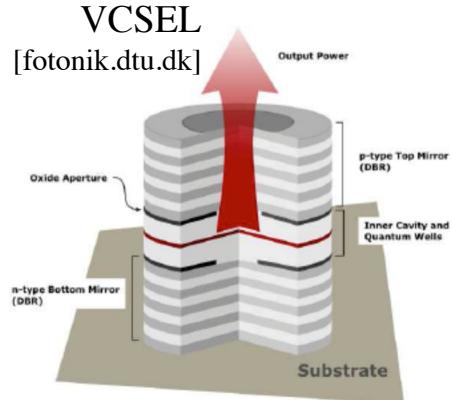
need **mechanism** to trap light for long time



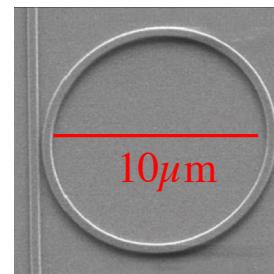
[ llnl.gov ]



metallic cavities:  
good for microwave,  
**dissipative** for infrared



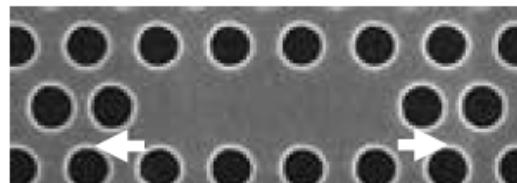
**photonic bandgaps**  
(complete or partial  
+ index-guiding)



[ Xu & Lipson  
(2005) ]

ring/disc/sphere resonators:  
a waveguide bent in circle,  
bending loss  $\sim \exp(-\text{radius})$

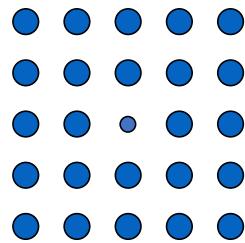
[ Akahane, *Nature* **425**, 944 (2003) ]



Why do defects in crystals  
trap resonant modes?

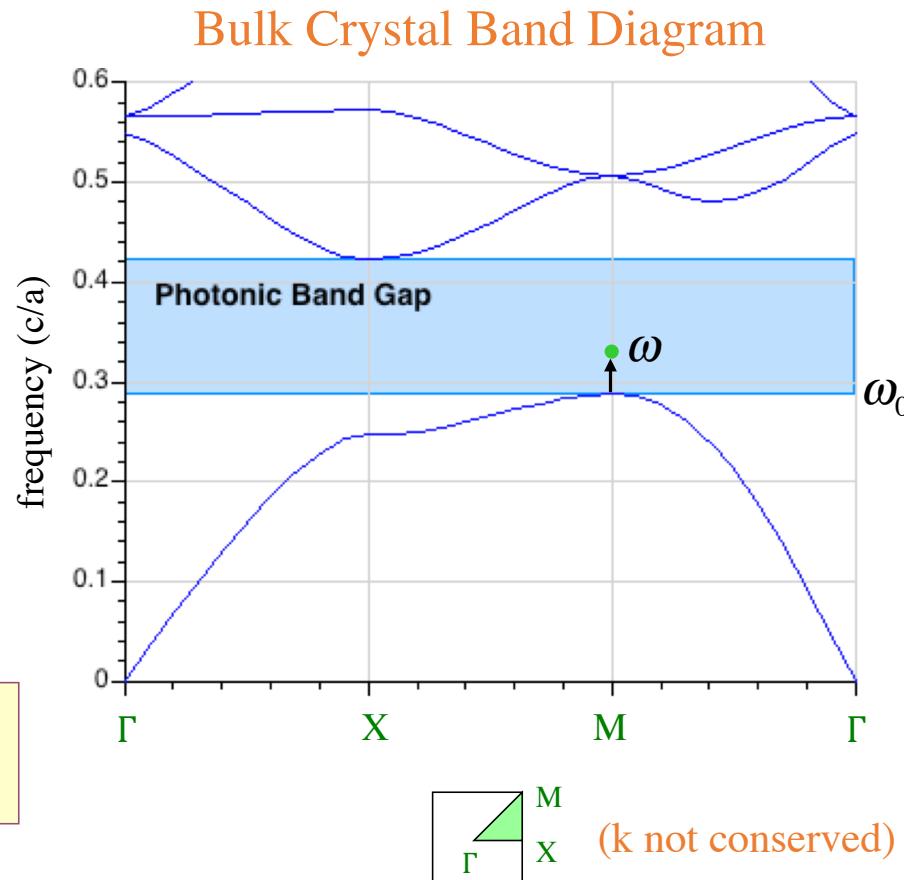
What do the modes look like?

# Single-Mode Cavity

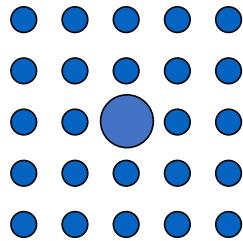


A *point defect* can **push up** a **single** mode from the **band edge**

$$\text{field decay} \sim \sqrt{\frac{\omega - \omega_0}{\text{curvature}}}$$

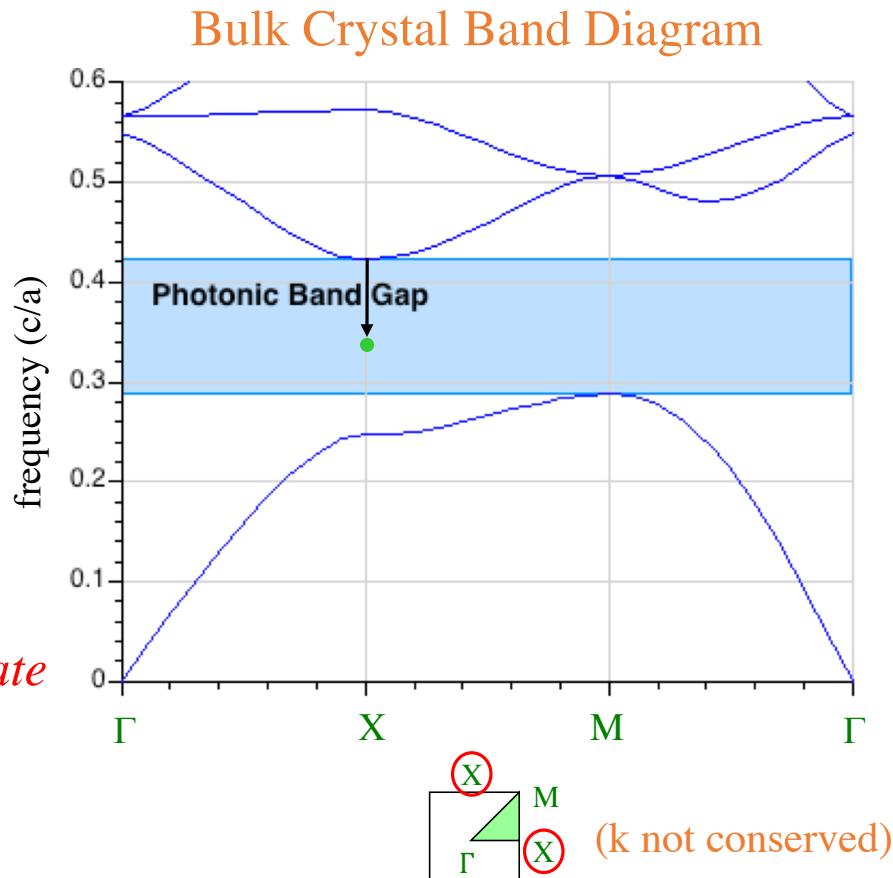


# “Single”-Mode Cavity

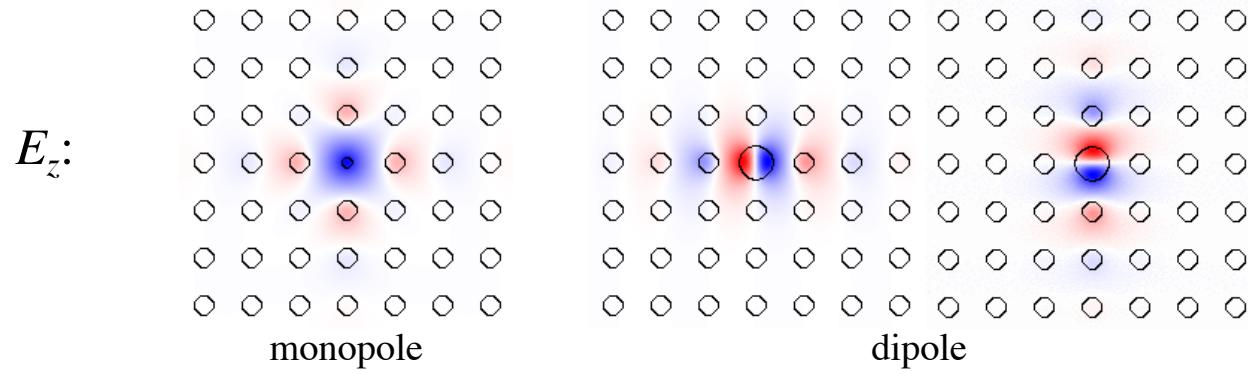
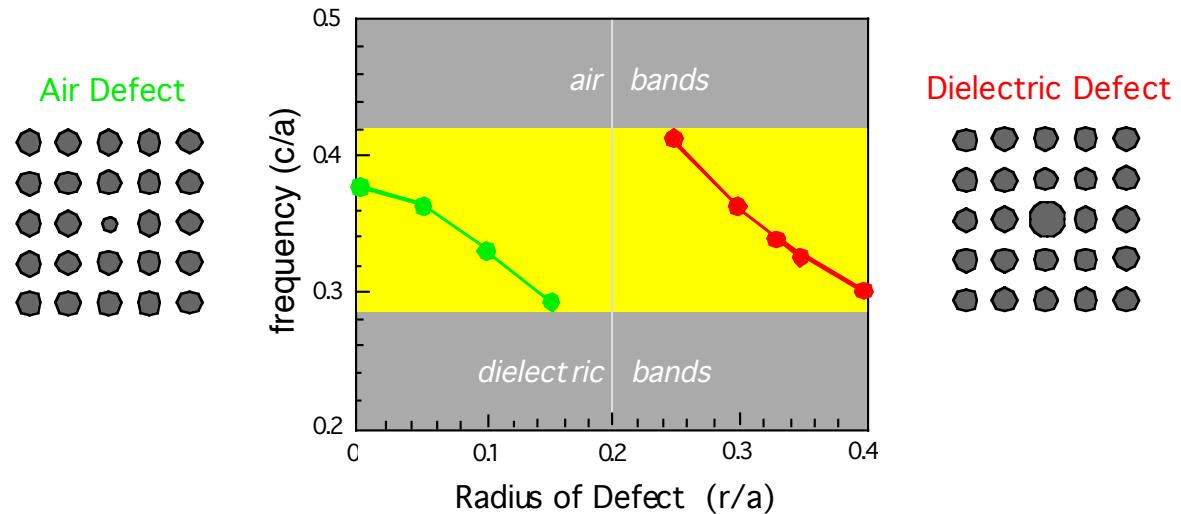


A *point defect*  
can **pull down**  
a “single” mode

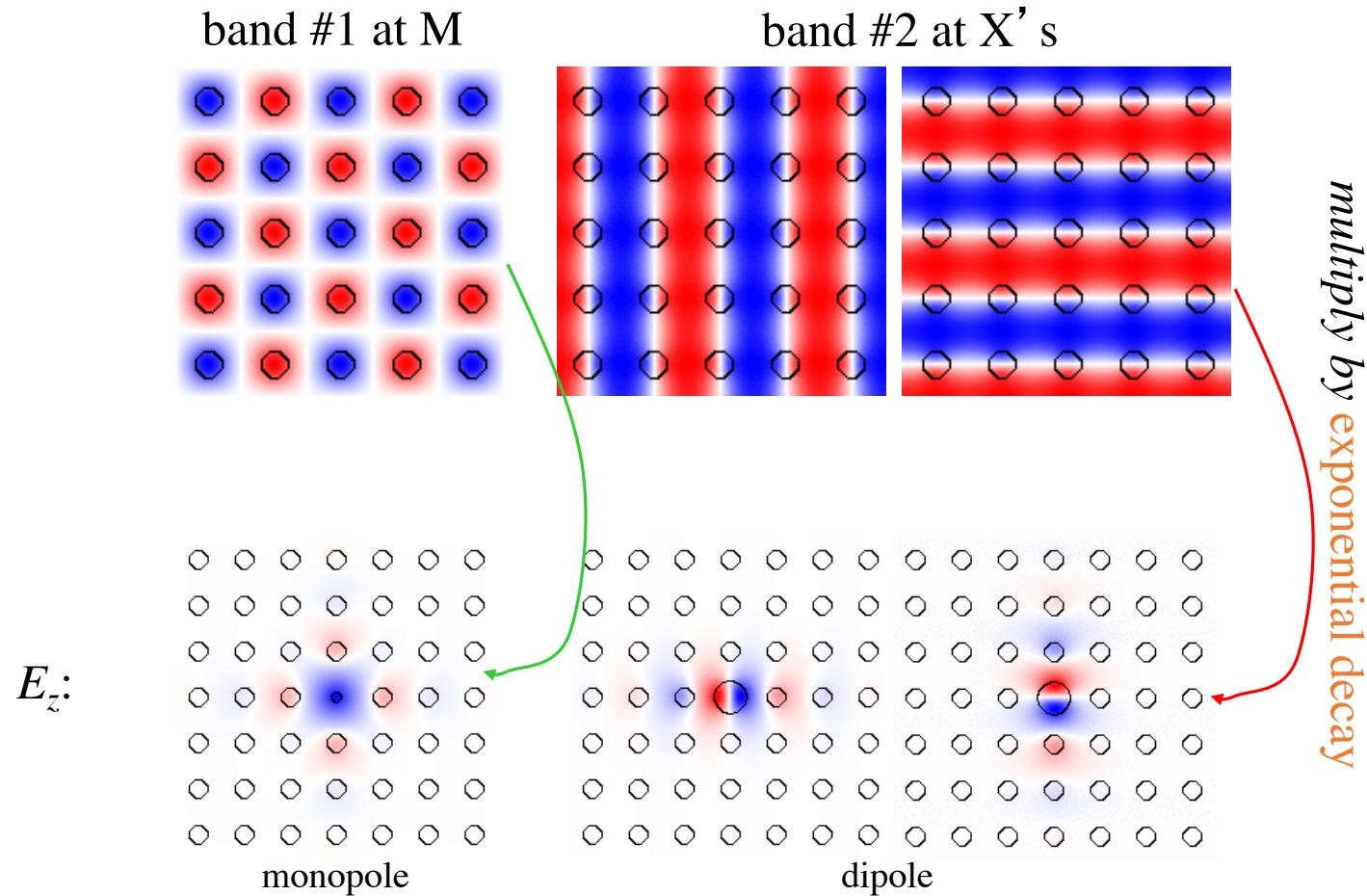
...here, **doubly-degenerate**  
(two states at *same*  $\omega$ )



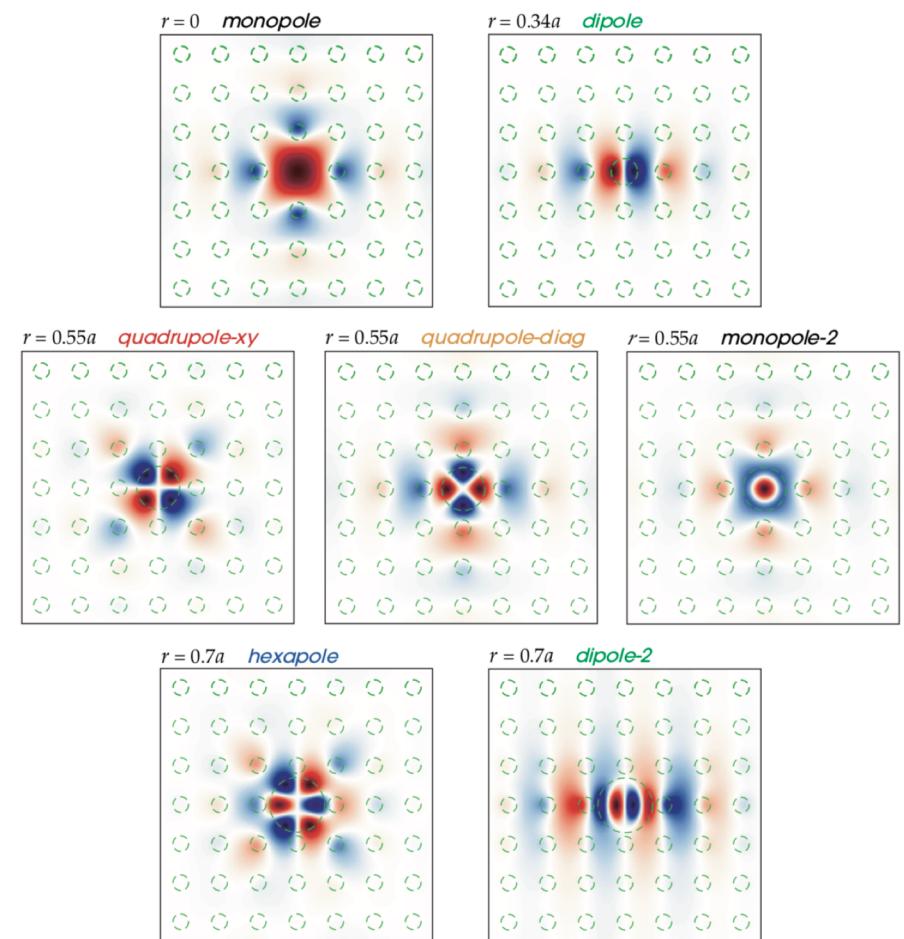
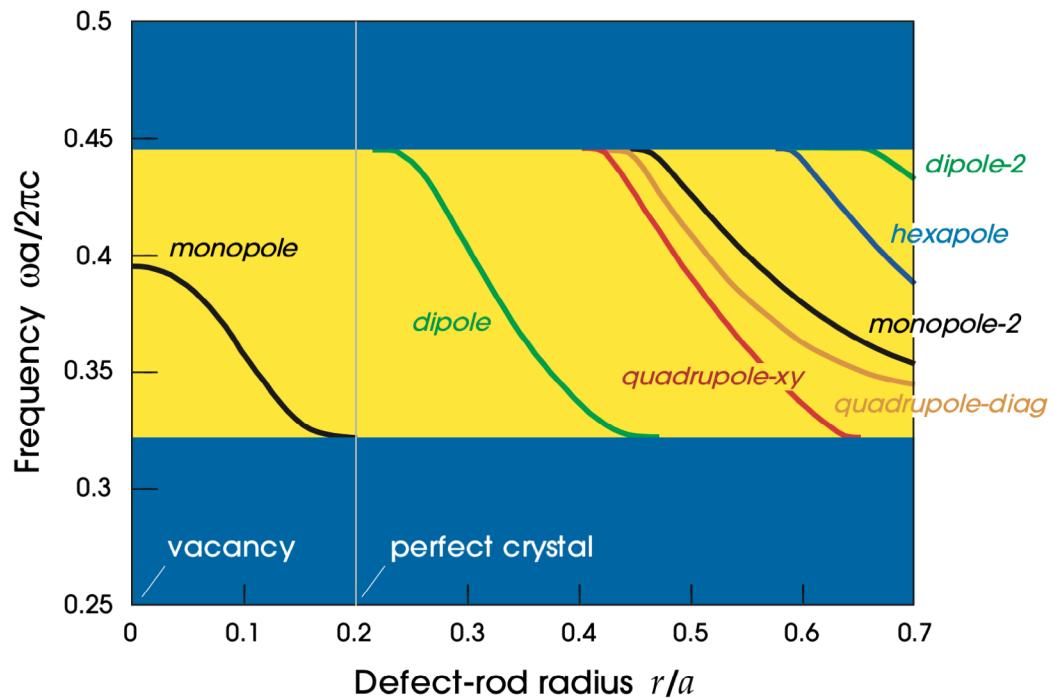
# Tunable Cavity Modes



# Tunable Cavity Modes

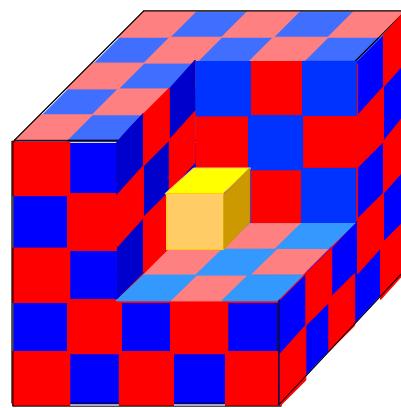


# More defect modes (4 out of 5 $C_{4v}$ irreps here)

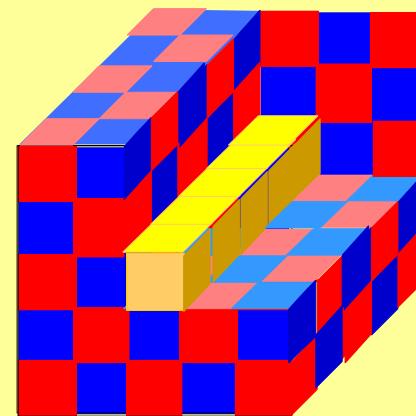


Intentional “defects” are good

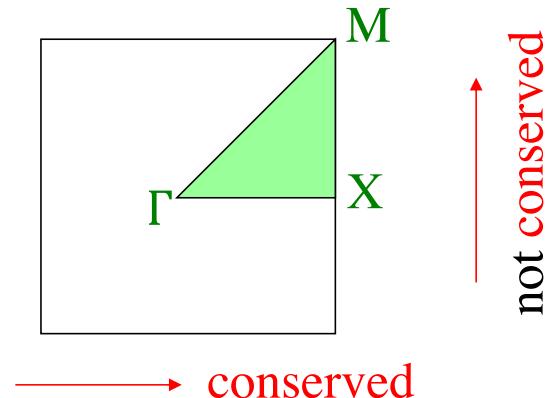
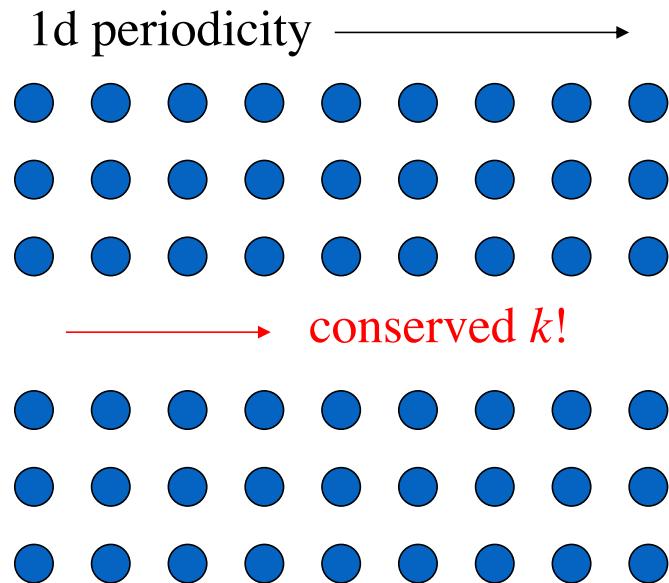
microcavities



waveguides (“wires”)



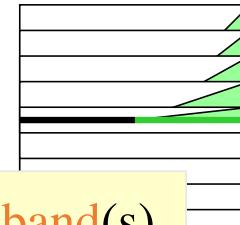
# Projected Band Diagrams



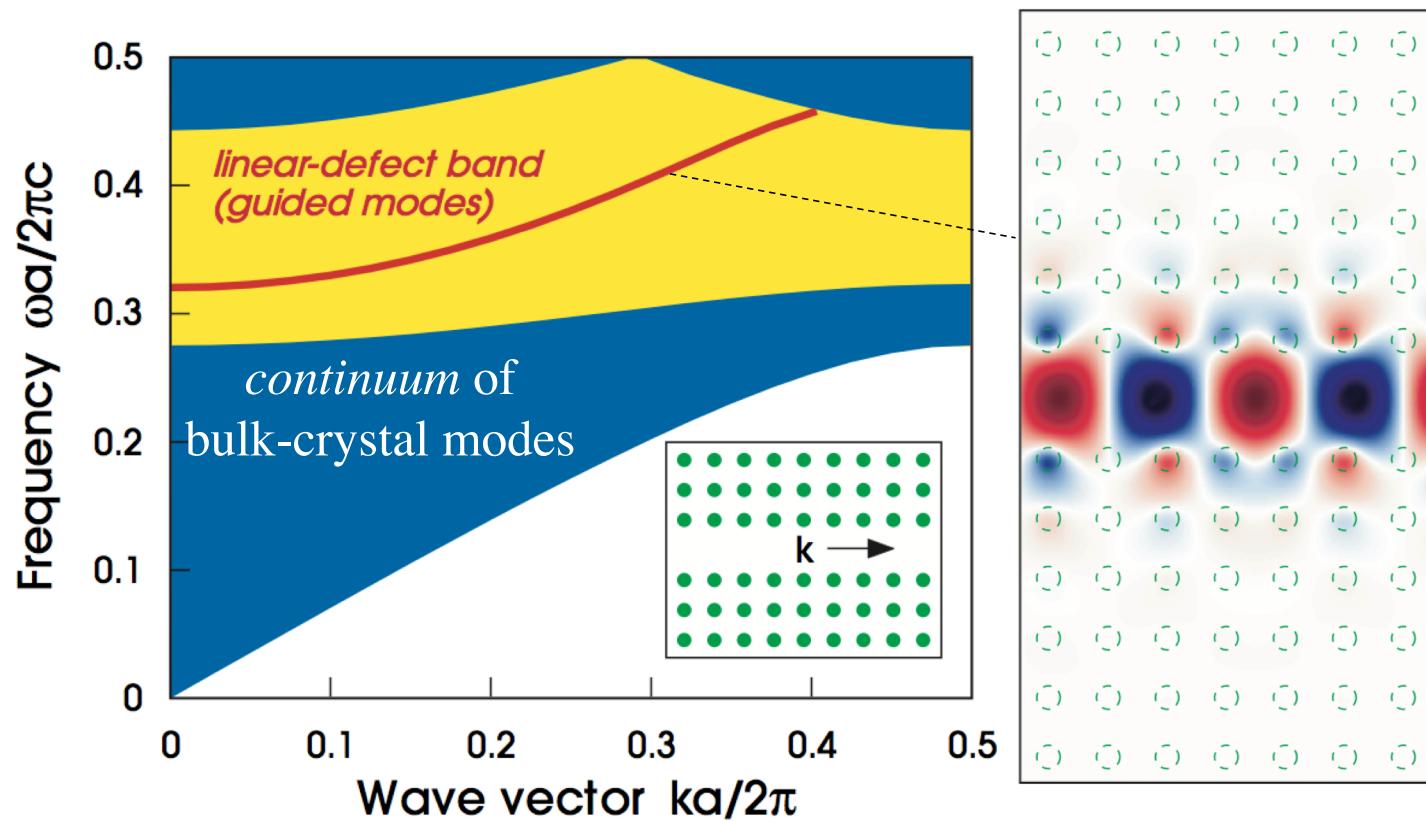
→ conserved

So, plot  $\omega$  vs.  $k_x$  only...project Brillouin zone onto  $\Gamma$ -X:

gives continuum of bulk states + discrete guided band(s)



# Air-waveguide Band Diagram



any state in the gap cannot couple to bulk crystal  $\Rightarrow$  localized

(Waveguides don't really need a *complete* gap)

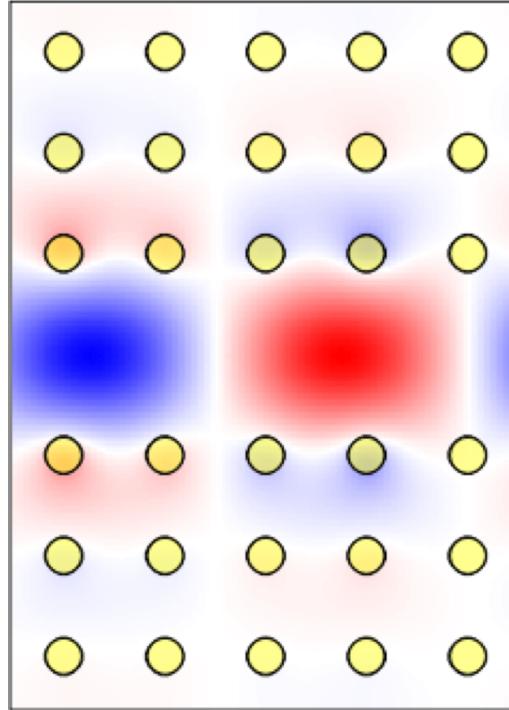
Fabry-Perot waveguide:



This is exploited *e.g.* for photonic-crystal fibers...

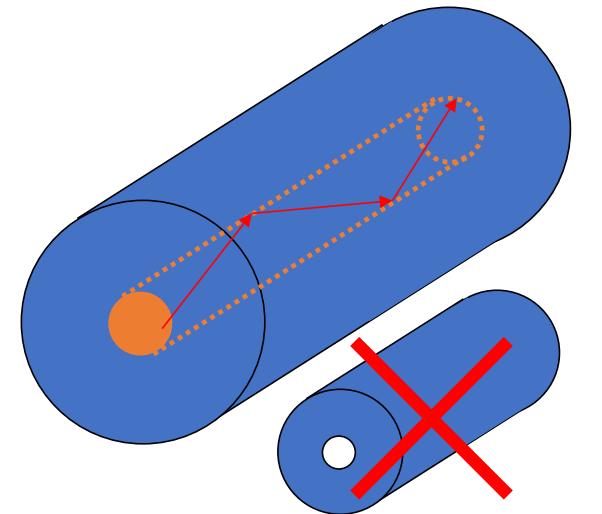
# Guiding Light in Air!

mechanism is gap only



vs. standard optical fiber:

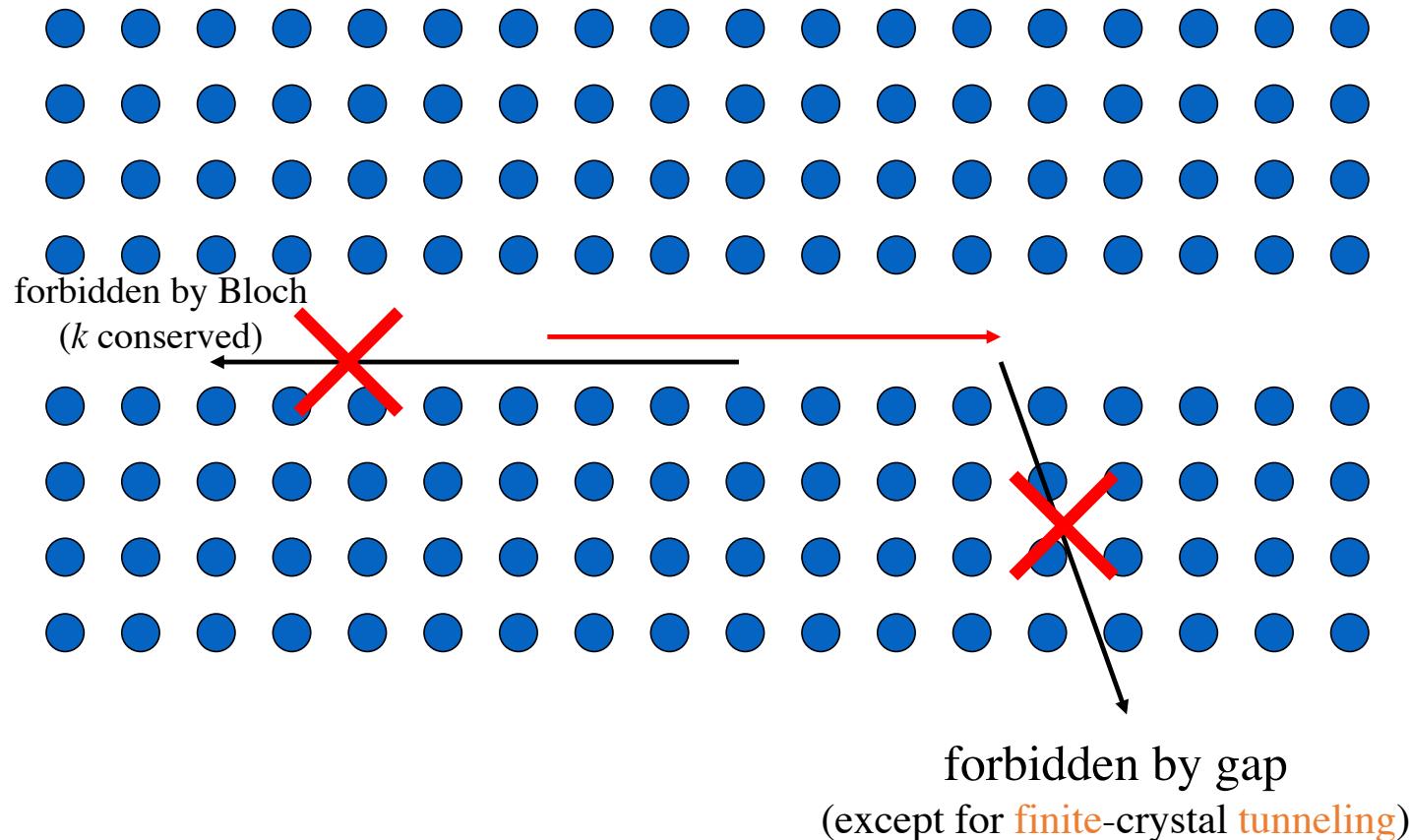
- “total internal reflection”
- requires *higher-index core*



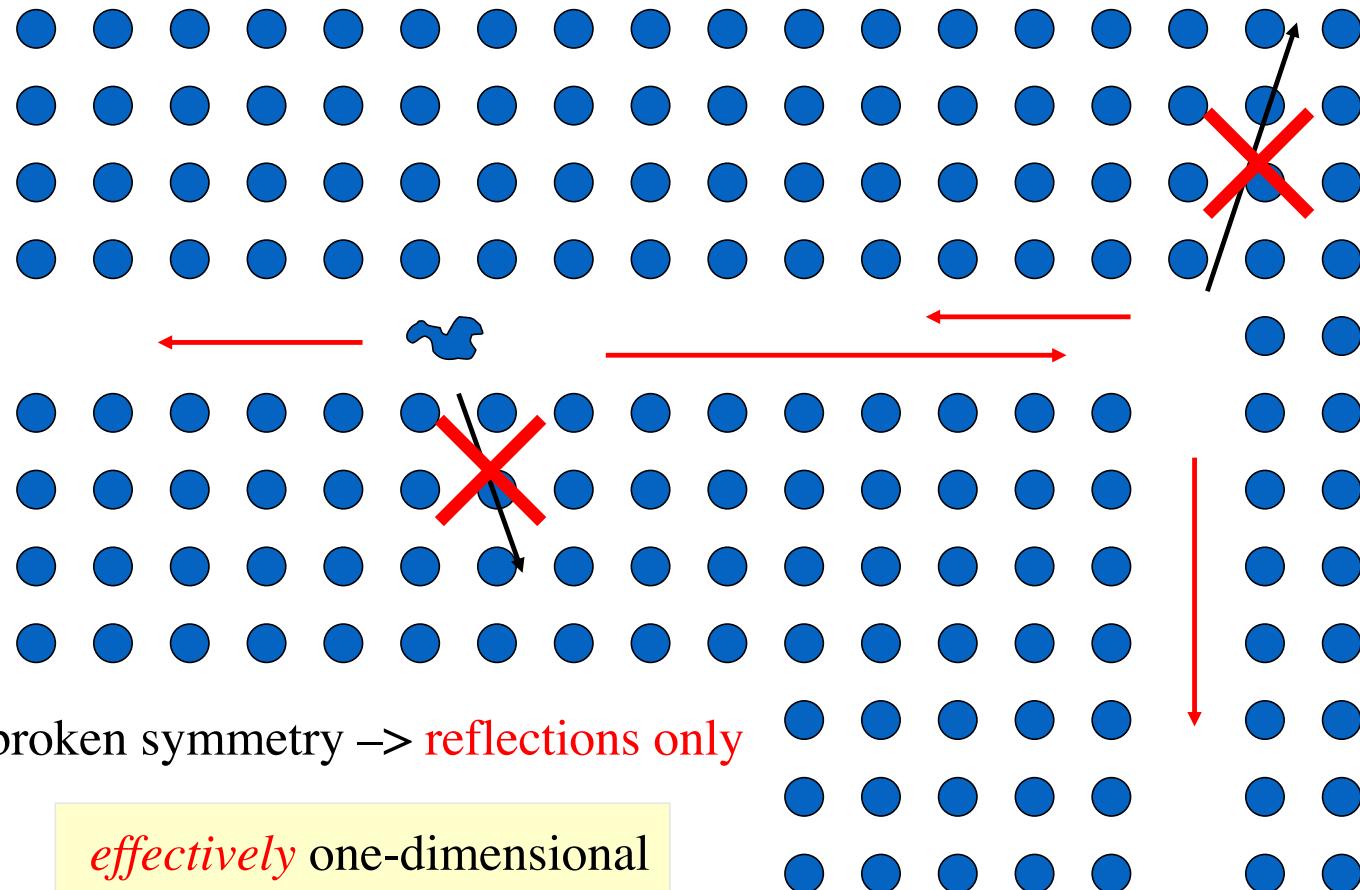
no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

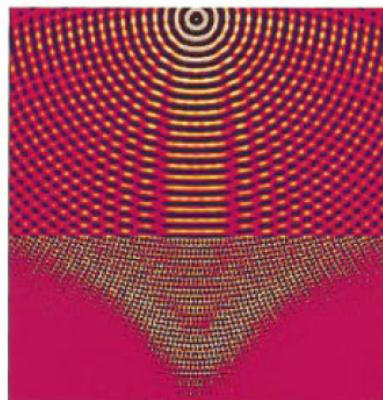
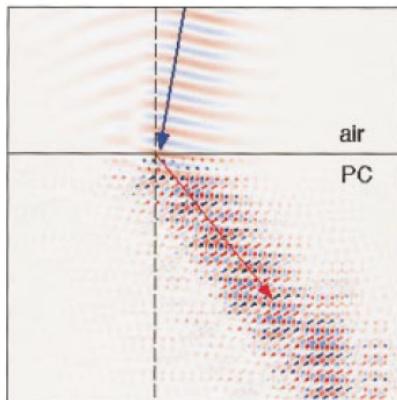
# Review: Why no scattering?



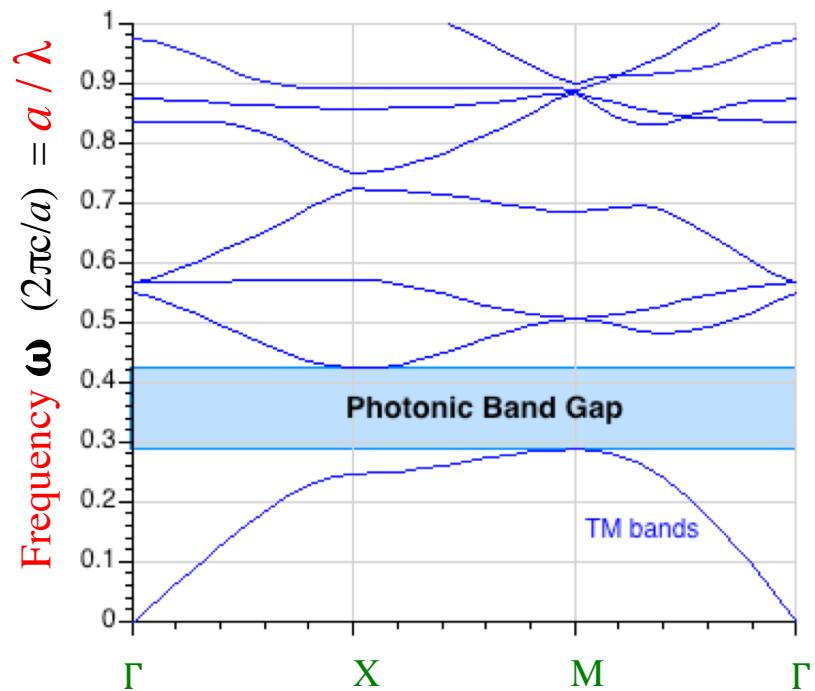
# Benefits of a complete gap...



# Band diagrams: Poor tool to understand refraction/reflection at interfaces



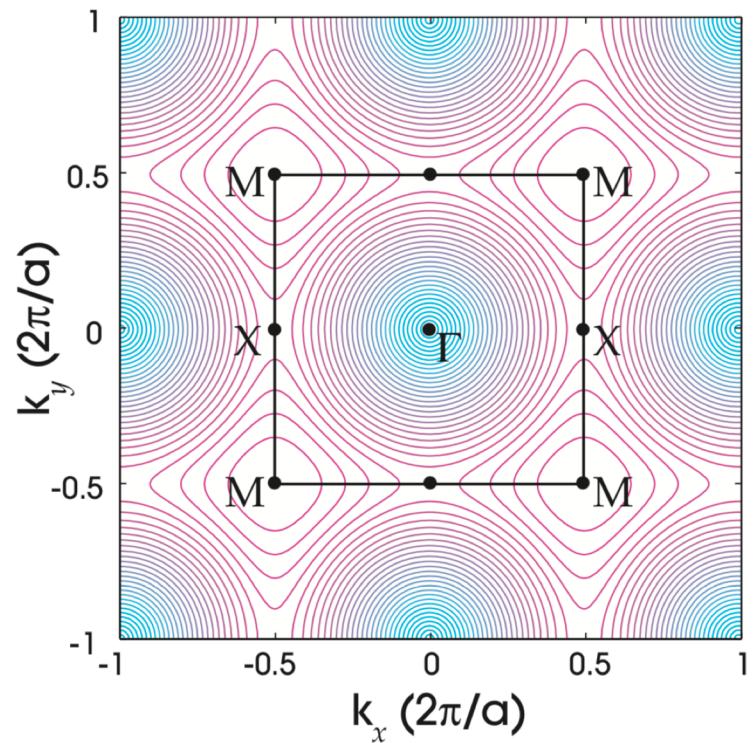
[ M. Notomi, *PRB* **62**, 10696 (2000). ]



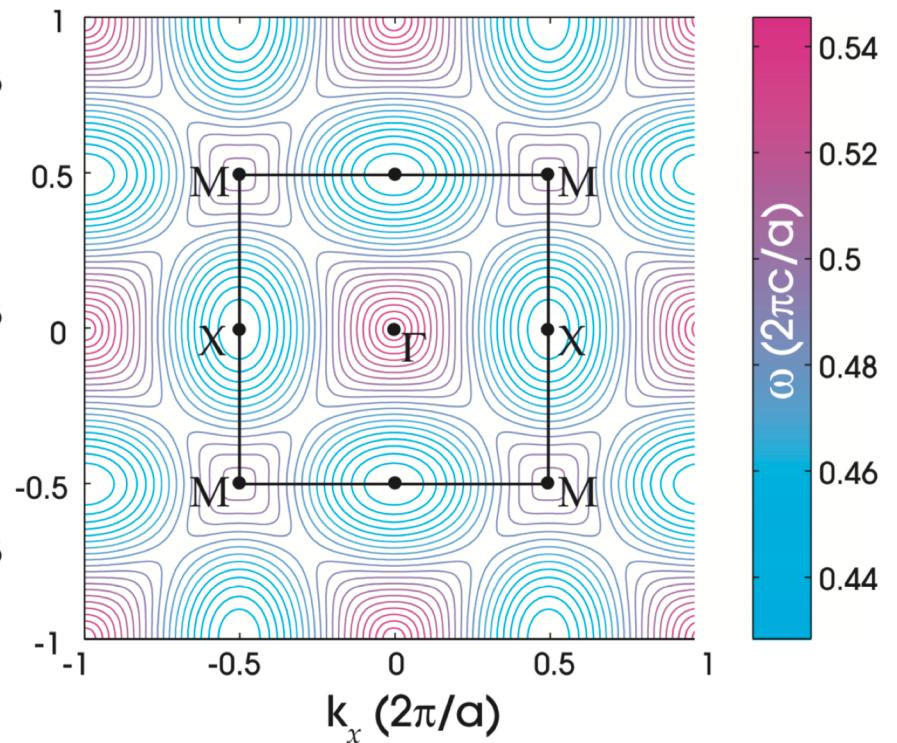
At an interface, only  $\omega$  and surface-parallel  $\mathbf{k}$  are conserved.

— we need *all the solutions* at a given  $\omega$ , not the different  $\omega$ 's at a given  $\mathbf{k}$ .

band 1:



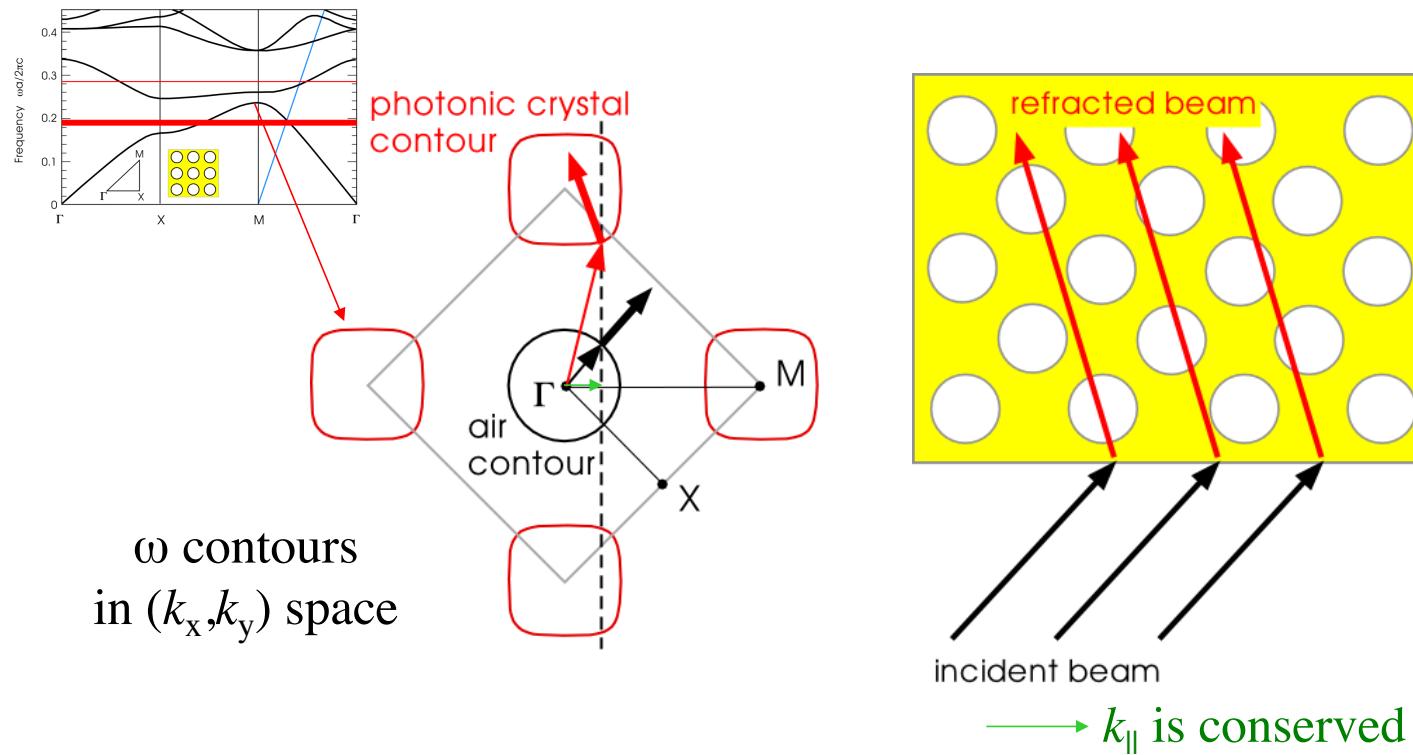
band 2:



**Figure 15:** Isofrequency diagrams: contour plots of  $\omega(k_x, k_y)$  for the first two TM bands of a square lattice of radius  $0.2a$  dielectric rods ( $\epsilon = 11.4$ ) in air. The first Brillouin zone is shown as black squares.

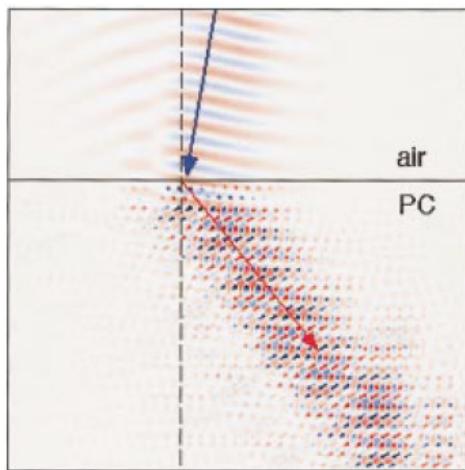
# Refraction and wavevector diagrams

[ Luo *et al*, PRB **65**, 2001104 (2002). ]

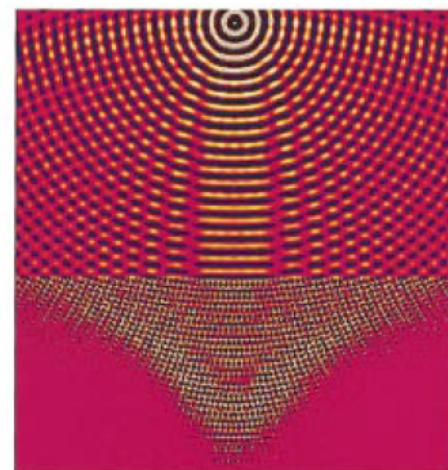


# Negative-refractive all-dielectric photonic crystals

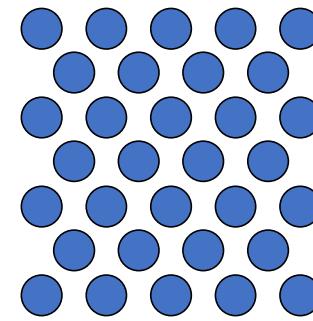
negative refraction



focussing



(2d rods in air, TE)

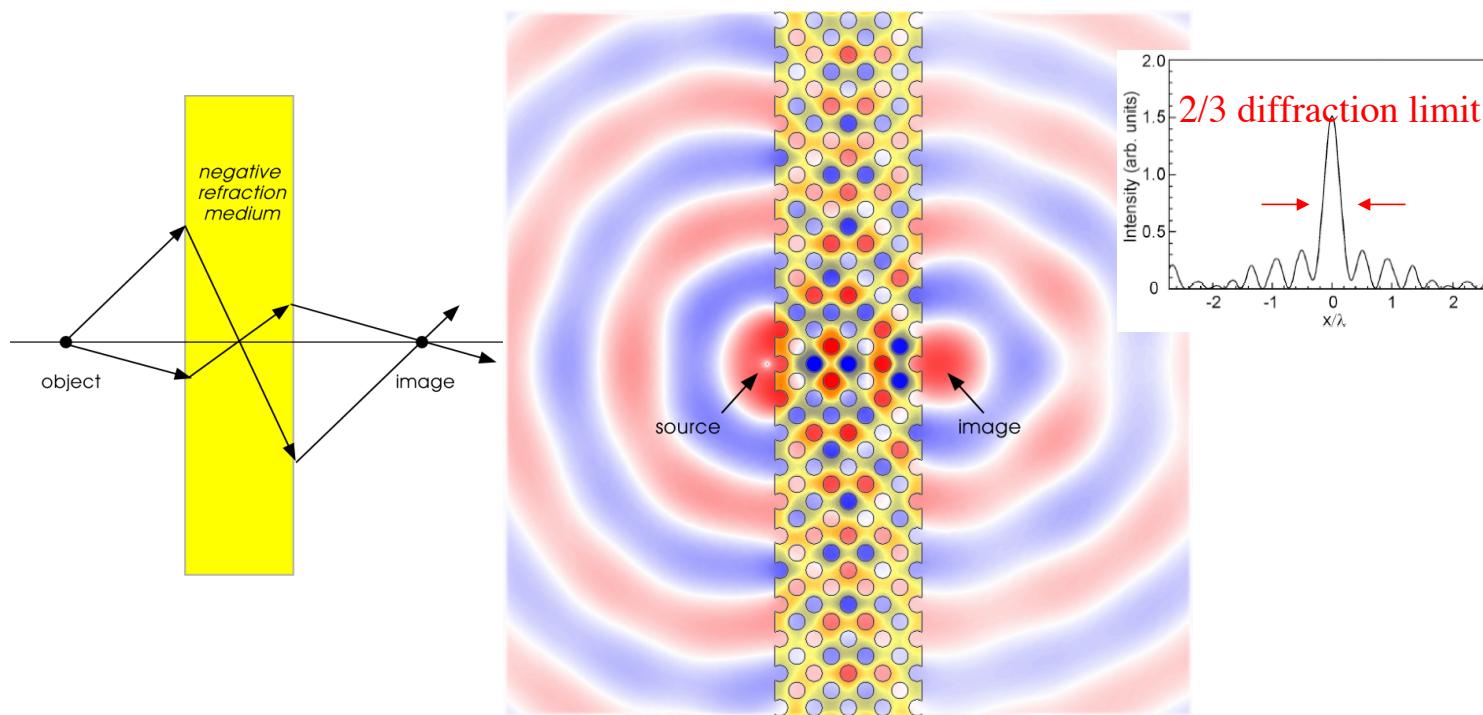


[ M. Notomi, *PRB* **62**, 10696 (2000). ]

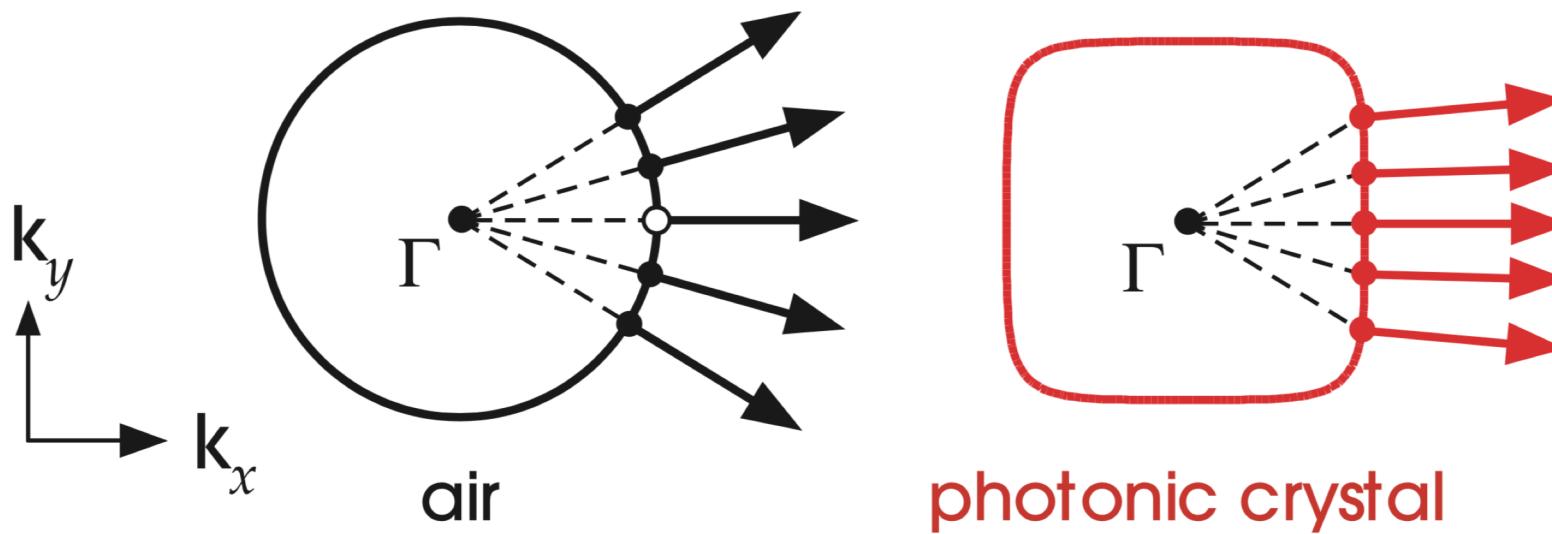
**not metamaterials:** wavelength  $\sim a$ ,  
no homogeneous material can reproduce *all* behaviors

# “Superlensing” with Photonic Crystals

[ Luo *et al*, PRB **68**, 045115 (2003). ]



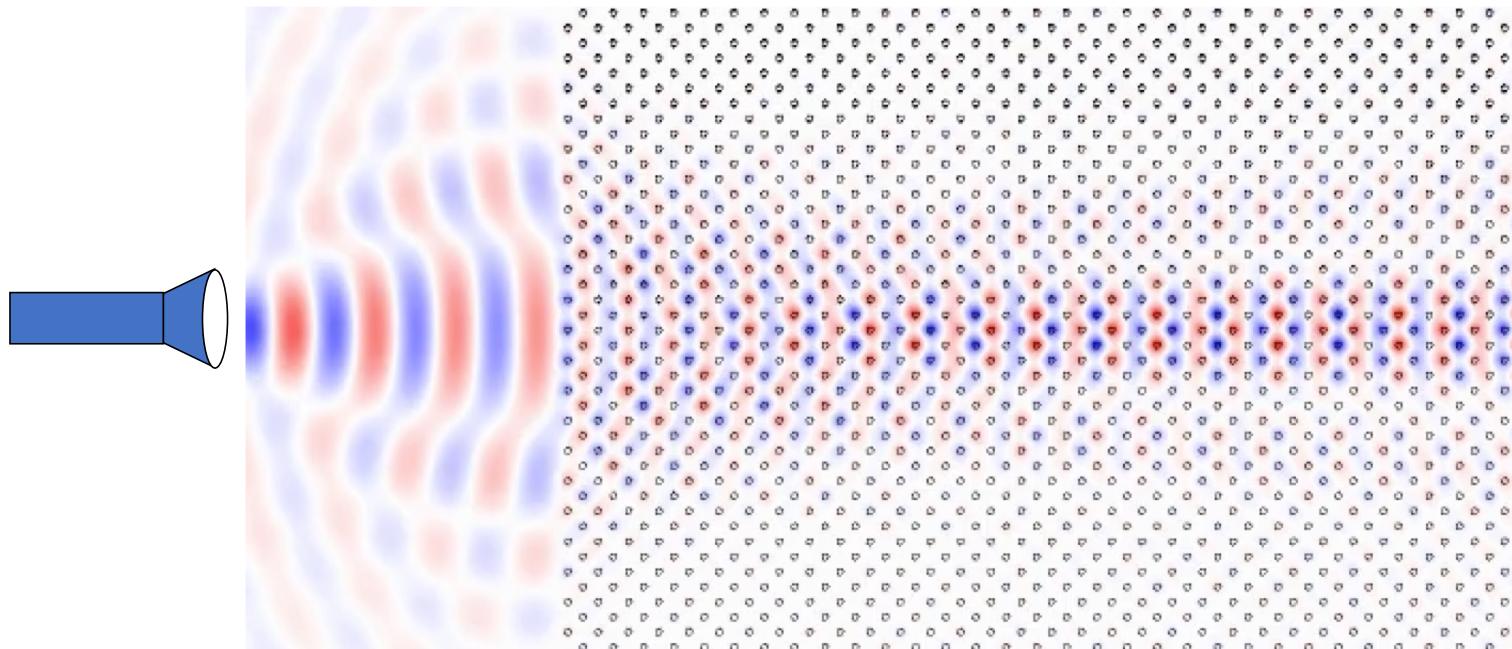
# Supercollimation



A Gaussian (etc.) beam propagating in the  $x$  direction consists of many  $k_y$  components at the same  $\omega$ . In a homogeneous medium, each  $k_y$  component travels in a different direction (group velocity). The beam therefore spreads (diffracts).

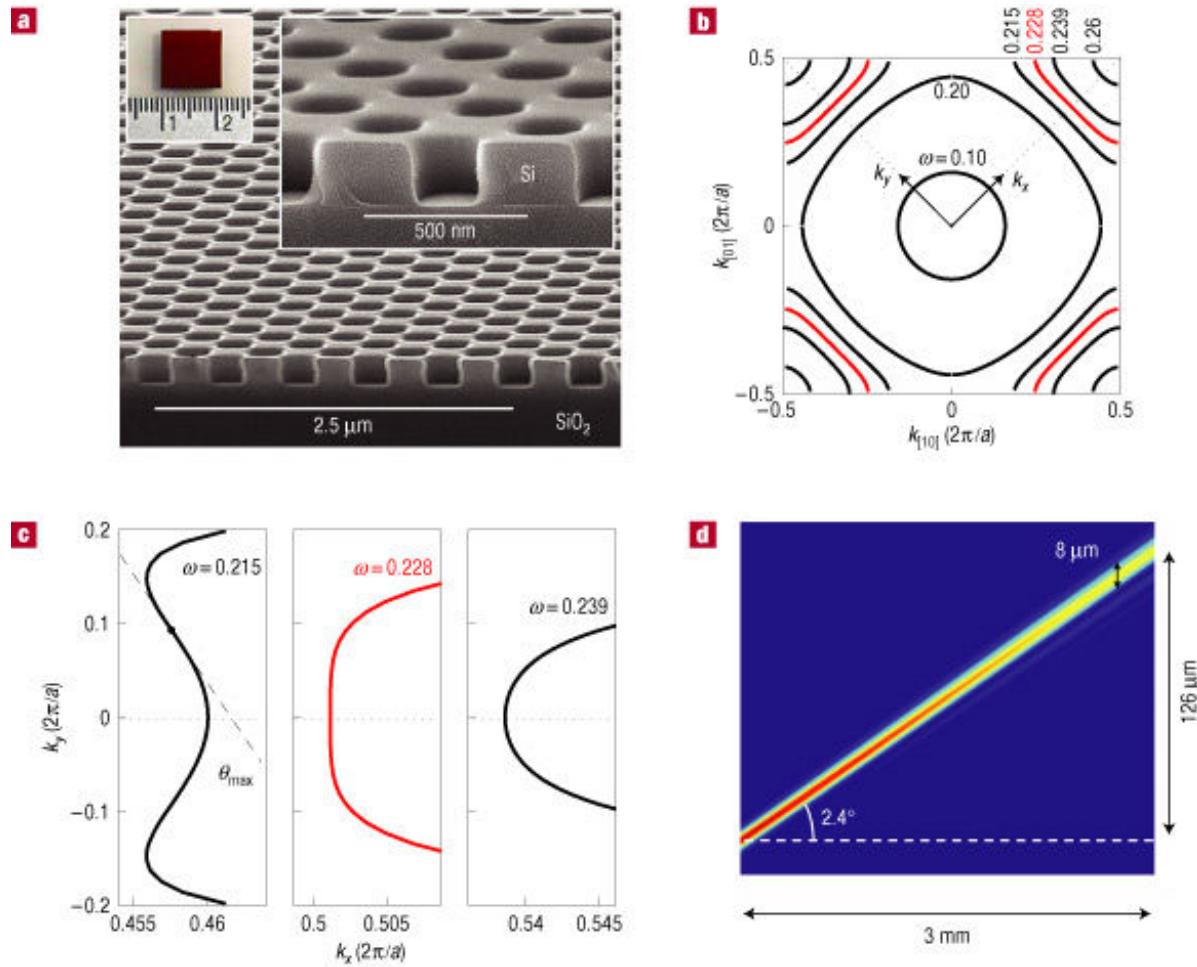
In a photonic crystal, the  $\omega$  contour can be very “flat” so beam spreading is minimized: all the  $k_y$  components travel in almost the same direction. Supercollimation!

# Supercollimation on the computer:



the light forms one or more *coherent “Bloch beams”*  
that propagate *without scattering*  
... and *almost without diffraction (supercollimation)*

# Experimental supercollimation

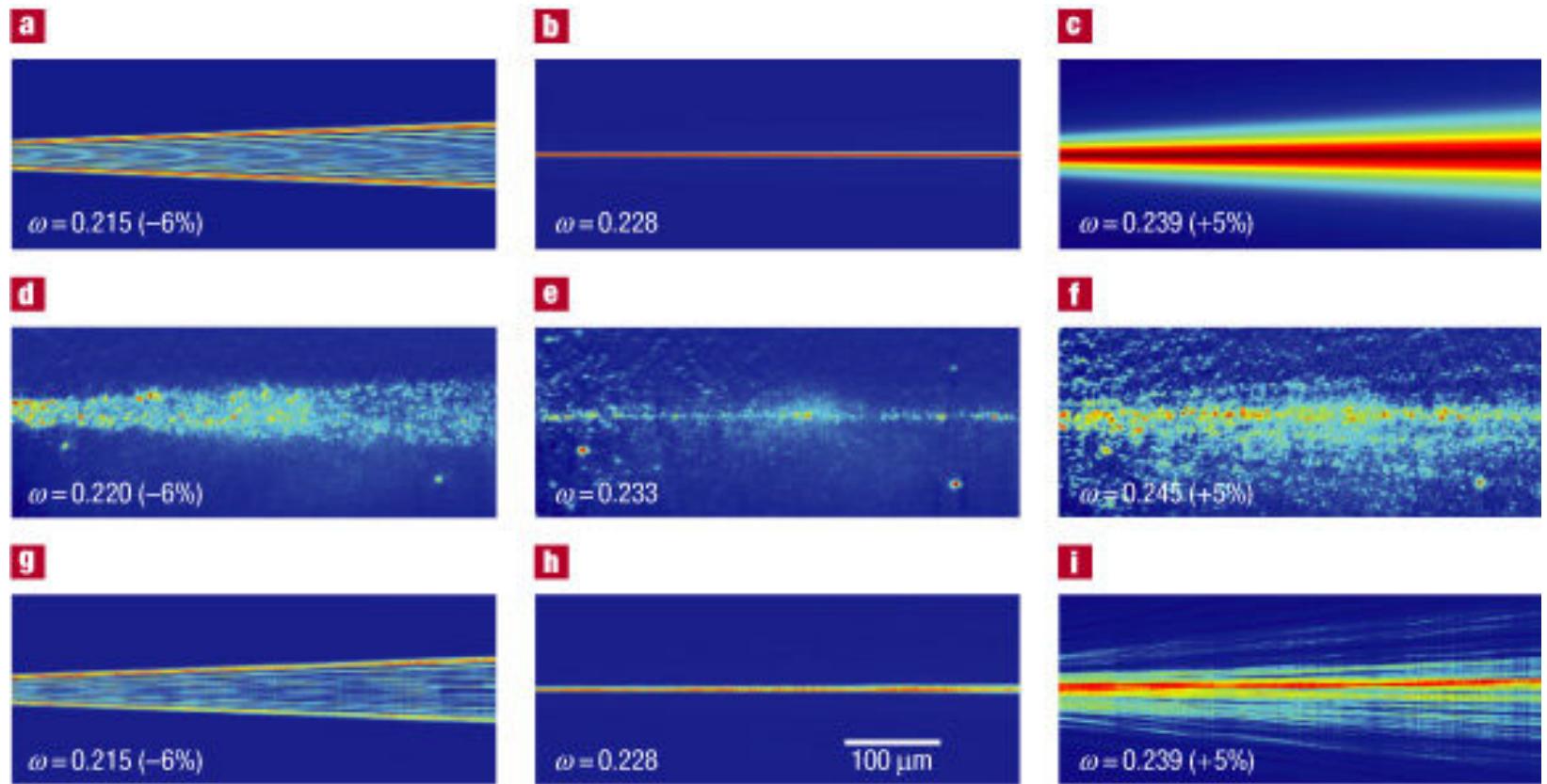


Rakich et al., “Achieving centimetre-scale supercollimation in a large-area two-dimensional photonic crystal,” *Nature Materials* **5**, 93–96 (2006).

# Experimental supercollimation at $\lambda \approx 1.5\mu\text{m}$

Rakich et al., “Achieving centimetre-scale supercollimation in a large-area two-dimensional photonic crystal,” *Nature Materials* **5**, 93–96 (2006).

Theory:



Experiment  
(measured  
vertical scattering  
from disorder)

Theory, including  
disorder: