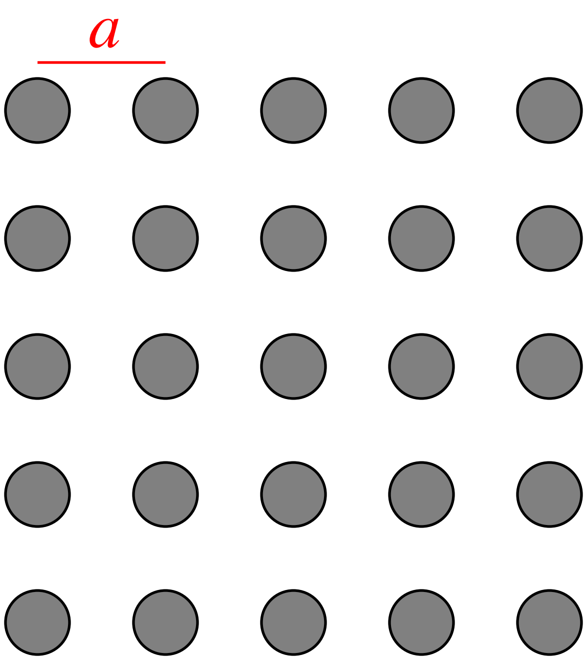
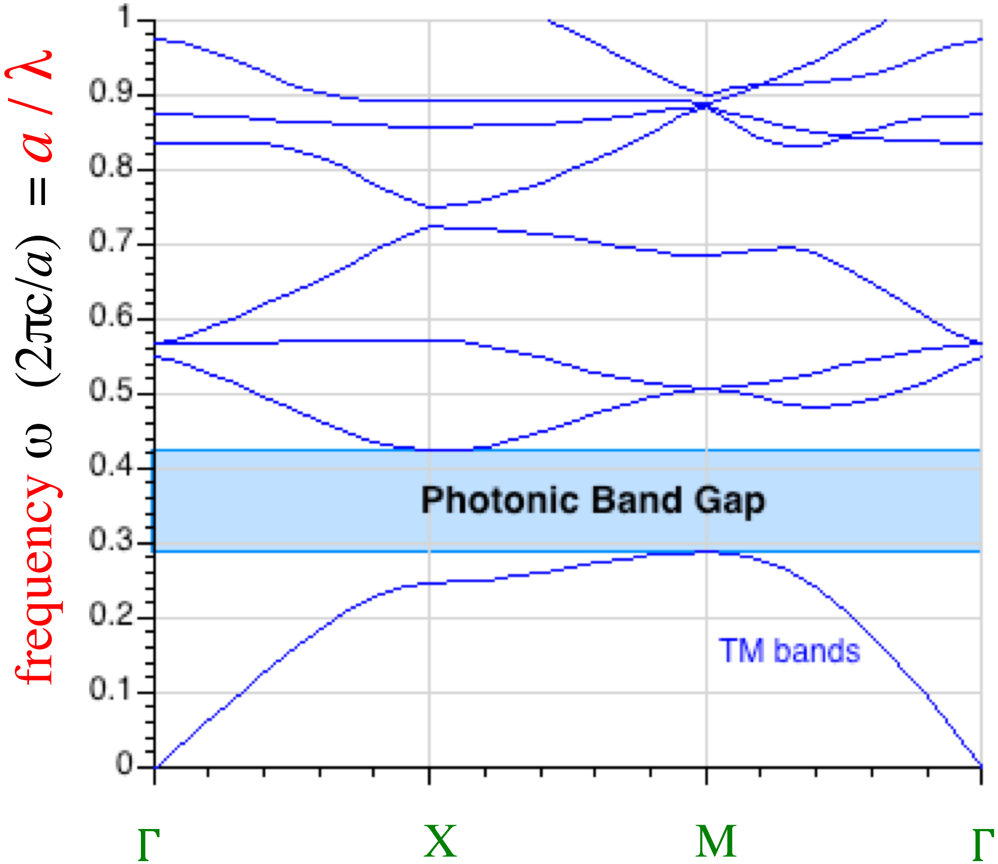
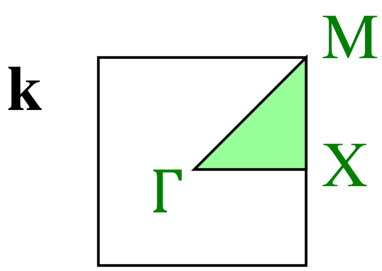
2d periodicity, ε=12:1

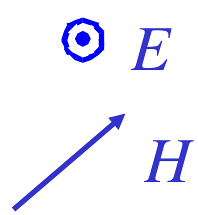




irreducible Brillouin zone

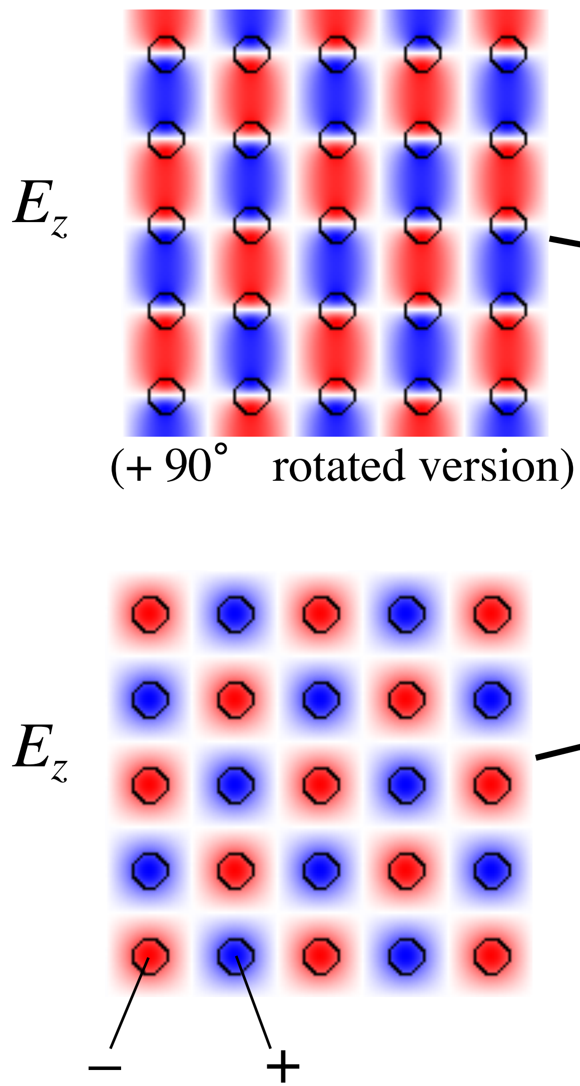


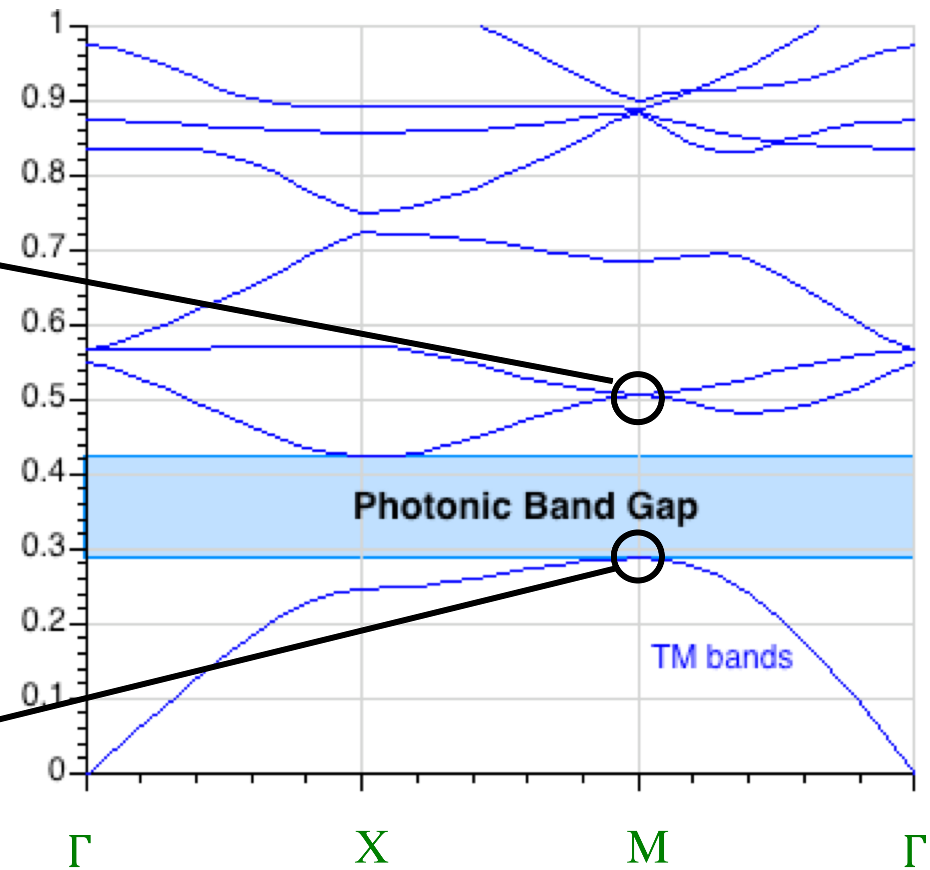
TM



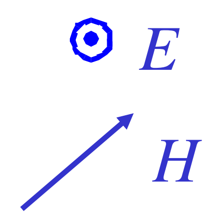
gap for
n > ~1.75:1

2d periodicity, ε=12:1





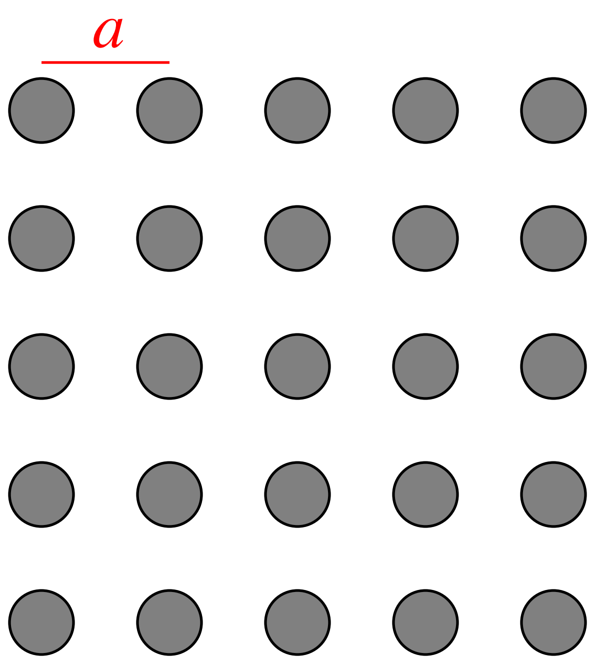
TM

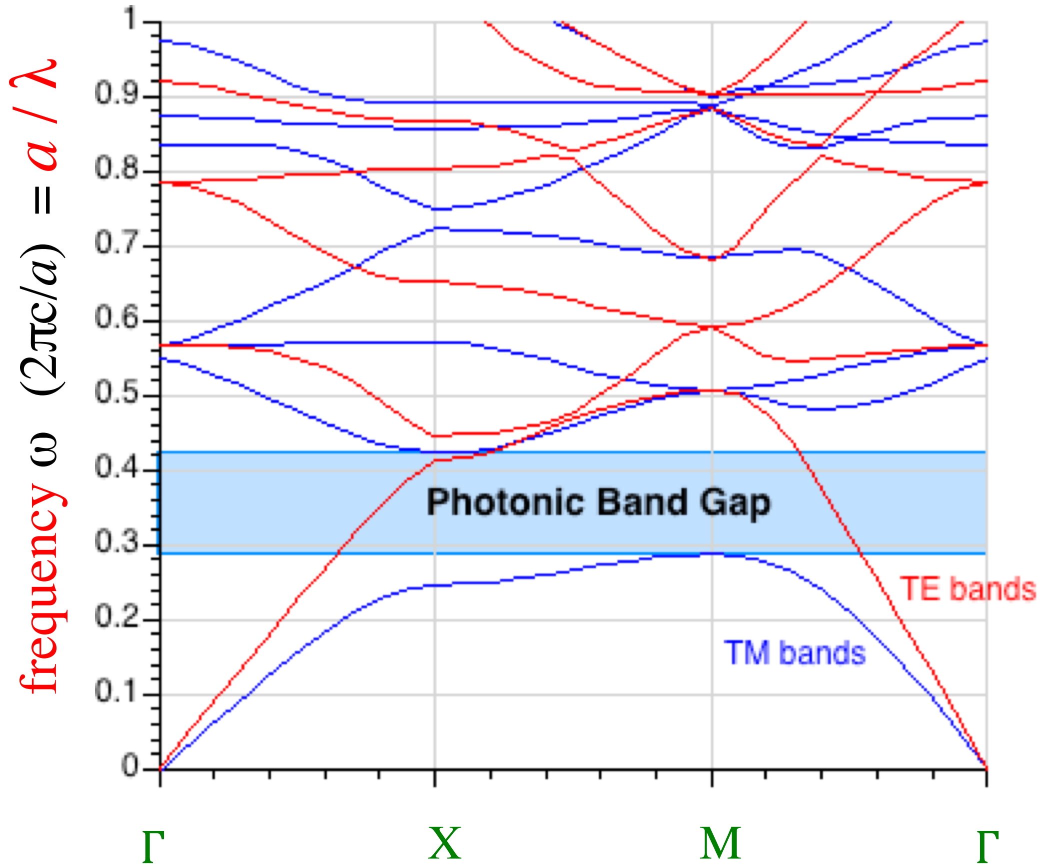


gap for
$n >$ ~1.75:1

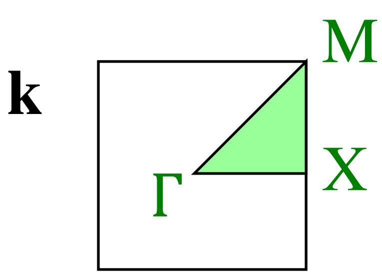
min–max theorem: 1st band "wants" to concentrate in high ε, higher bands forced out by orthogonality

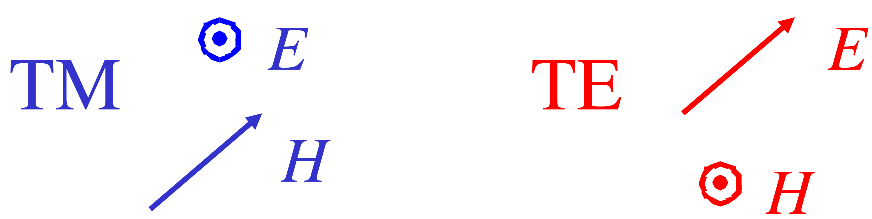
2d periodicity, ε=12:1: TM gap, no TE gap



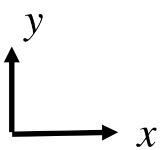


irreducible Brillouin zone





What a difference a boundary condition makes…



ε_1 ε_2

$$\nabla\times\mathbf{E} = i\omega\mathbf{H} \Longrightarrow$$
$$\frac{\partial E_y}{\partial x} = i\omega H_z \text{ is finite, so}$$
E_y continuous

$$\mathbf{E}_{1,\parallel} = \mathbf{E}_{2,\parallel}$$

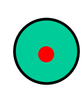
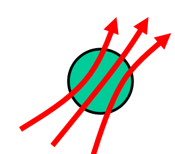
$\mathbf{E}_{\parallel}$ is continuous:
energy density $\varepsilon|\mathbf{E}|^2$
more in larger ε

$$\nabla\cdot(\varepsilon\mathbf{E}) = 0 \Longrightarrow$$
εE_x continuous
(since no discontinuity in x)

$$\varepsilon_1\mathbf{E}_{1,\perp} = \varepsilon_2\mathbf{E}_{2,\perp}$$

$\varepsilon\mathbf{E}_{\perp}$ is continuous:
energy density $|\varepsilon\mathbf{E}|^2/\varepsilon$
more in smaller ε

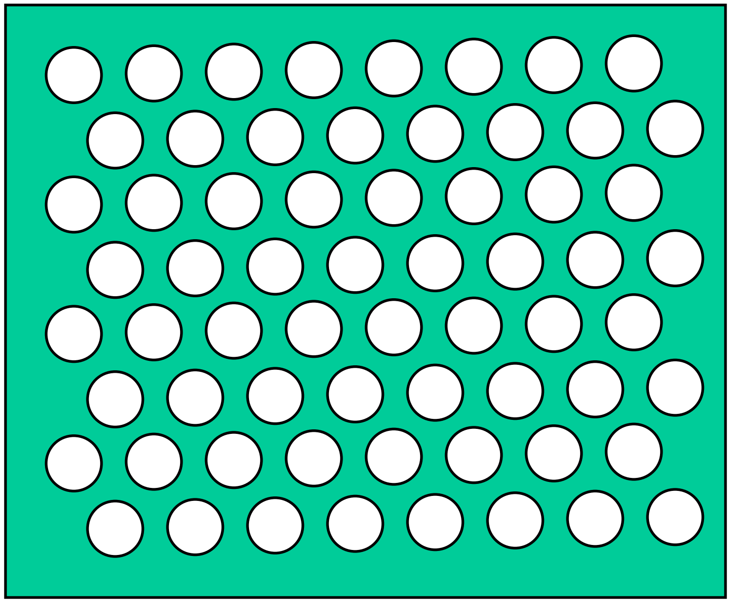
To get strong confinement & gaps,
want **E** mostly parallel to interfaces

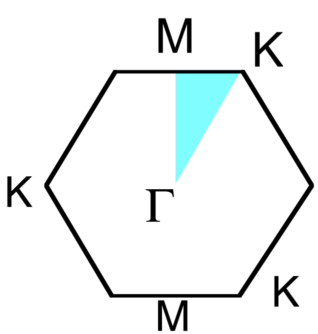
TM: $\parallel$  TE: $\perp$ 

2d photonic crystal: TE gap, ε=12:1

(C$_{6v}$ symmetry of hexagon)

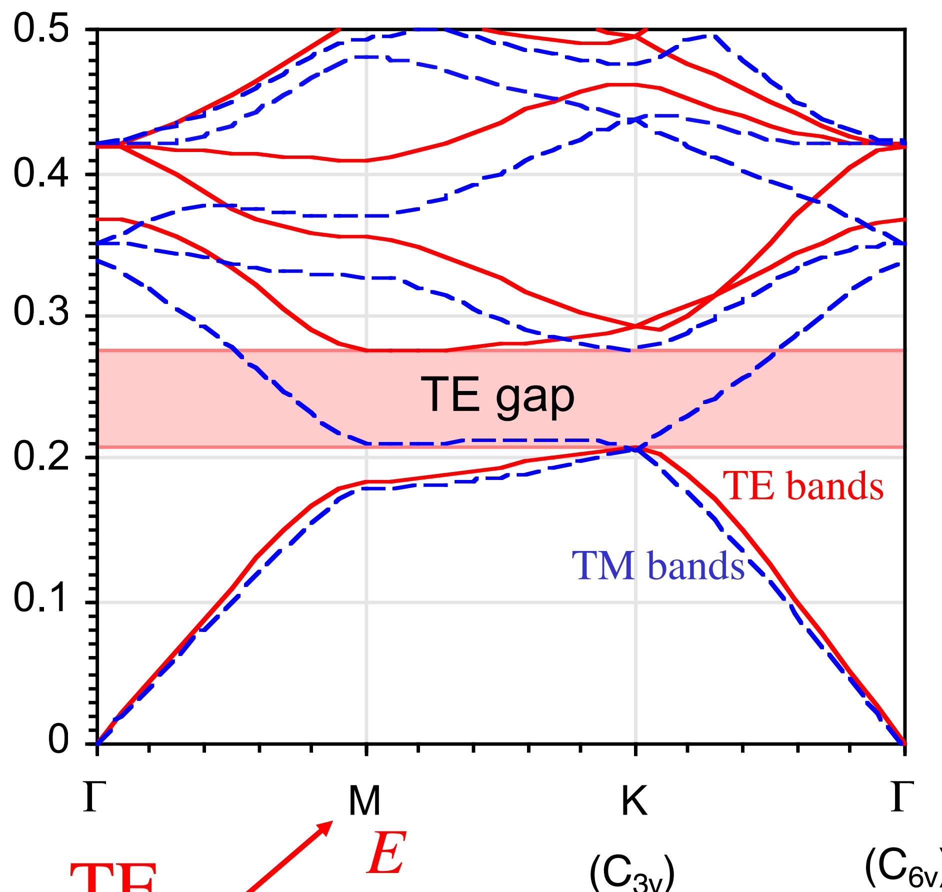
triangular / hexagonal lattice of air holes in ε=12





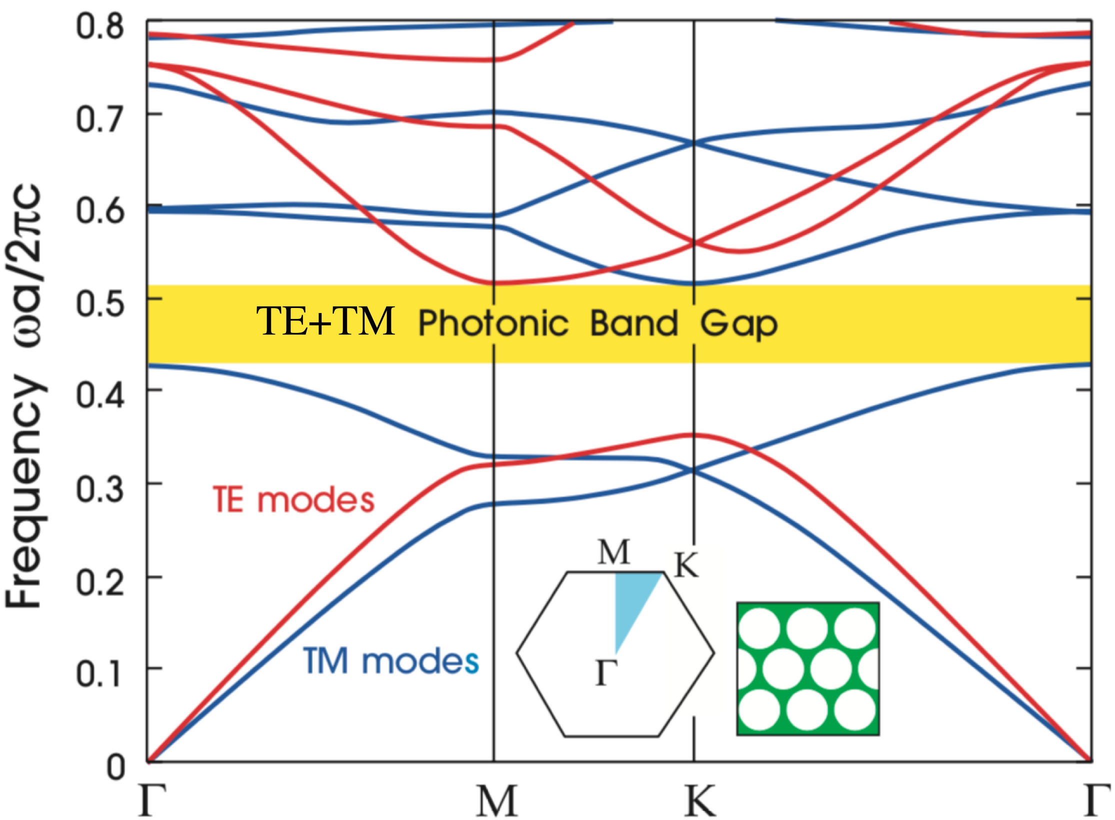
1st Brillouin zone
+ irreducible B.Z.

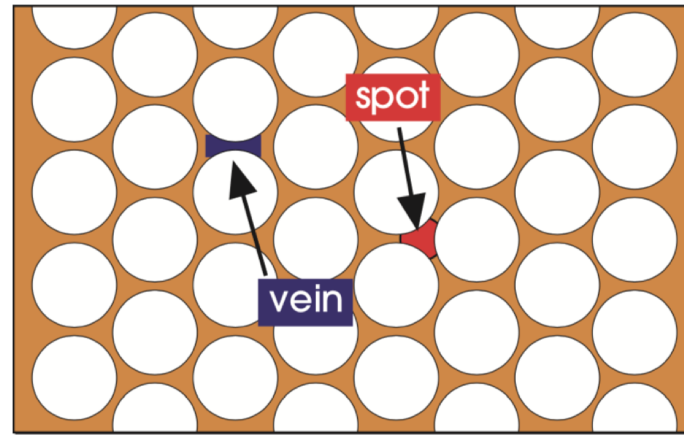
Origin of TE gap: 1st band's TE (E_x,E_y) electric field lines can "loop around" cylinders



gap for n > ~1.4:1

"Complete" overlapping TE+TM gap



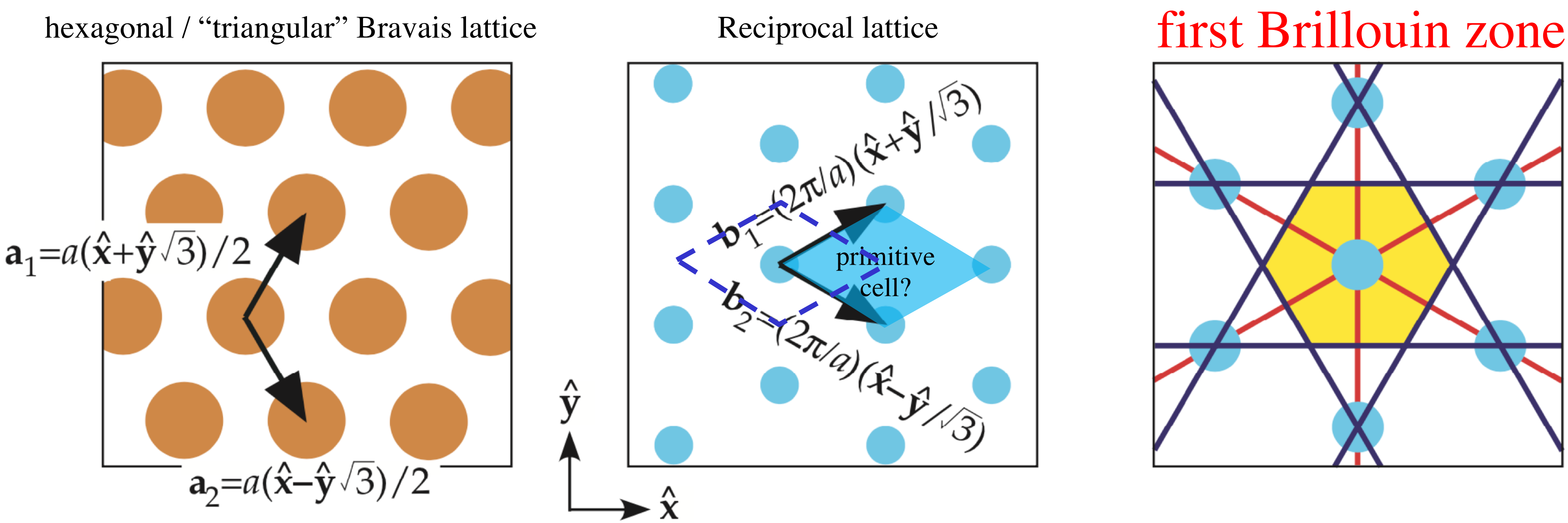


(In 2d, however, we can generally choose one polarization to work with, so an overlapping TE+TM gap is rarely needed. In 3d, though, modes are no longer purely polarized and we will have to handle every field orientation simultaneously.)

TE gap: 1st band loops around veins, 2nd band forced out by orthogonality.

TM gap: 1st two bands concentrated in interstitial "spots," 3rd band forced out by orthogonality.

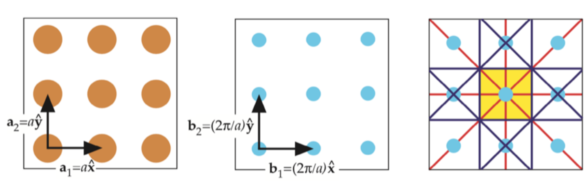
Brillouin zones: A better unit cell in **k**



- Problem: obvious “primitive cell” in **k** space breaks C_{6v} symmetry.

- Solution: define “first Brillouin zone” as points *closer* to $\mathbf{k}=\mathbf{0}$ *than to any other reciprocal lattice* vector $\mathbf{G}$.

gives square B.Z. in square lattice:

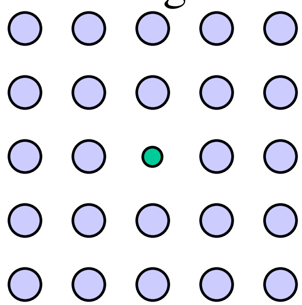


(also called “Wigner–Seitz cell” of Bravais lattice, or “Voronoi cell for any set of points.)

Single-Mode Cavity

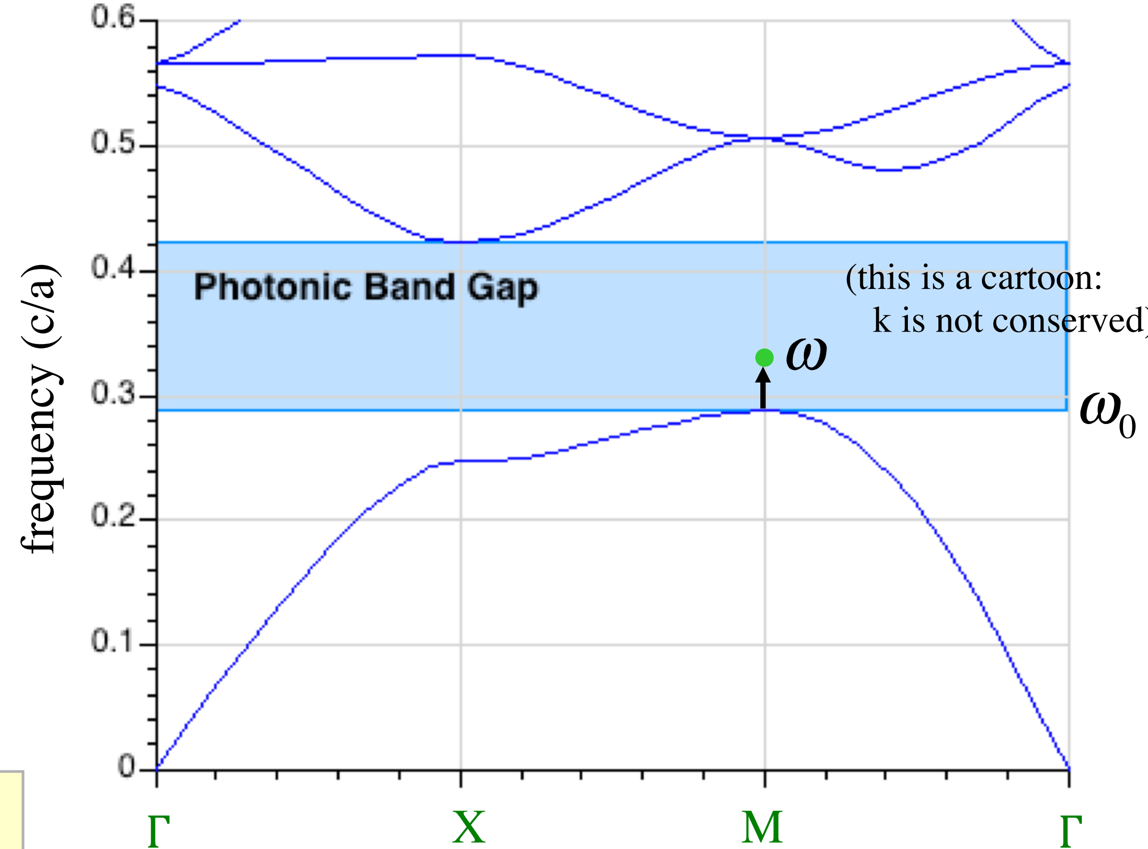
"point" defect: break all translation symmetry = localized modes in gap

reduced radius ~ reduced ε
~ higher ω



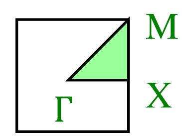
A *point defect*
can push up
a single mode
from the band edge

Bulk Crystal Band Diagram



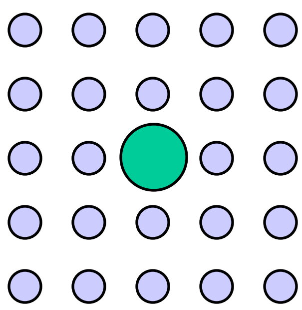
$$\text{field decay} \sim \sqrt{\frac{\omega - \omega_0}{\text{curvature}}}$$

recall Taylor expansion around M
= analytic continuation in gap: $\mathbf{k} = \mathrm{M} + i(\text{decay})$



(k not conserved)

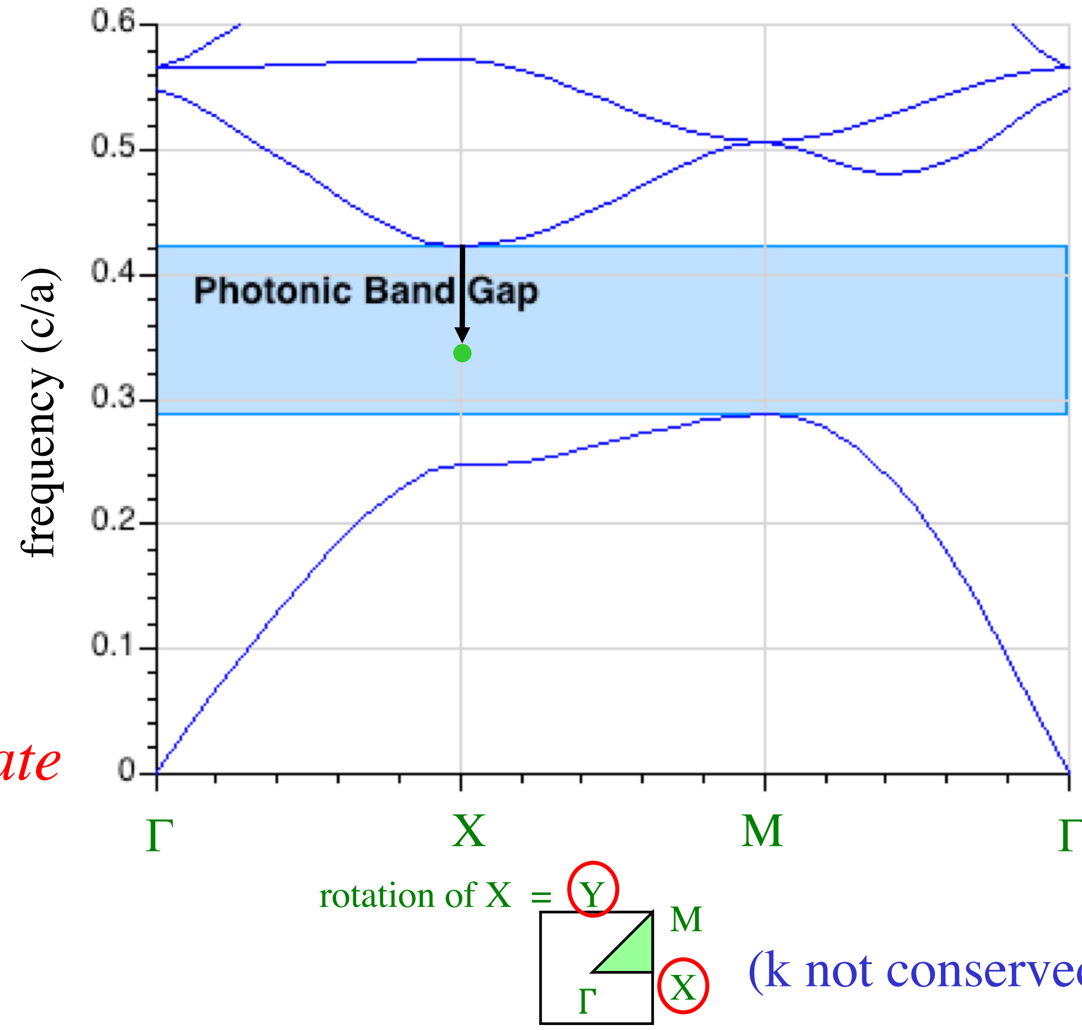
"Single"-Mode Cavity



A *point defect* can pull down a "single" mode

…here, doubly-*degenerate* (*two* states at *same* ω)

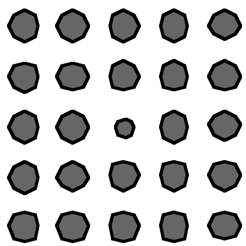
Bulk Crystal Band Diagram

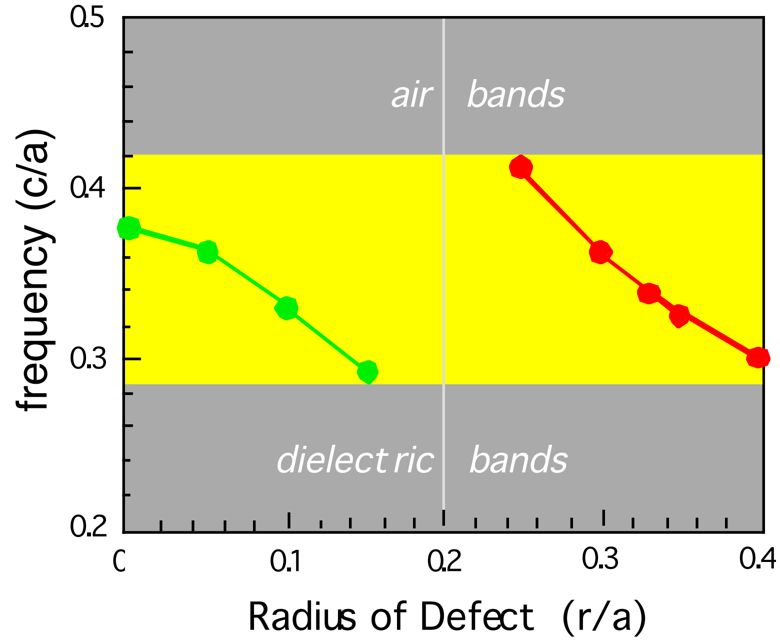


(k not conserved)

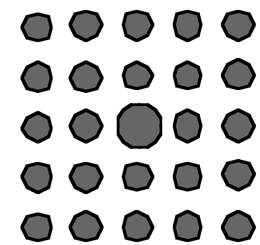
Tunable Cavity Modes

Air Defect

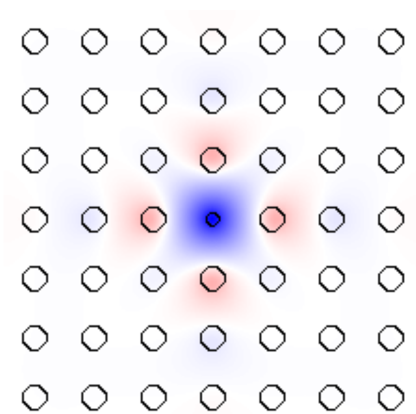




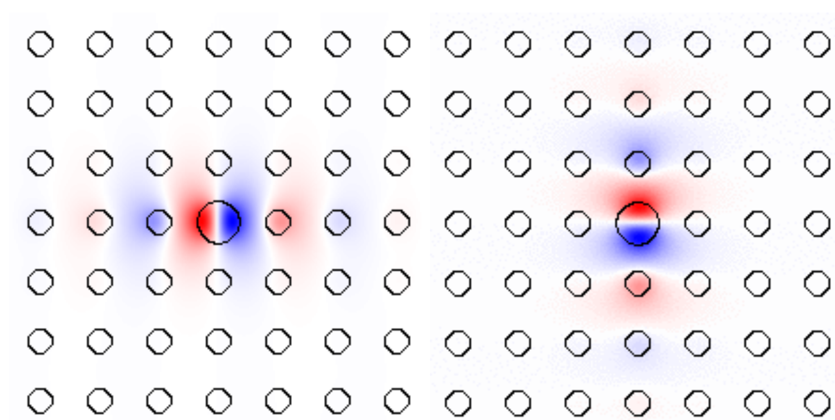
Dielectric Defect



E_z:

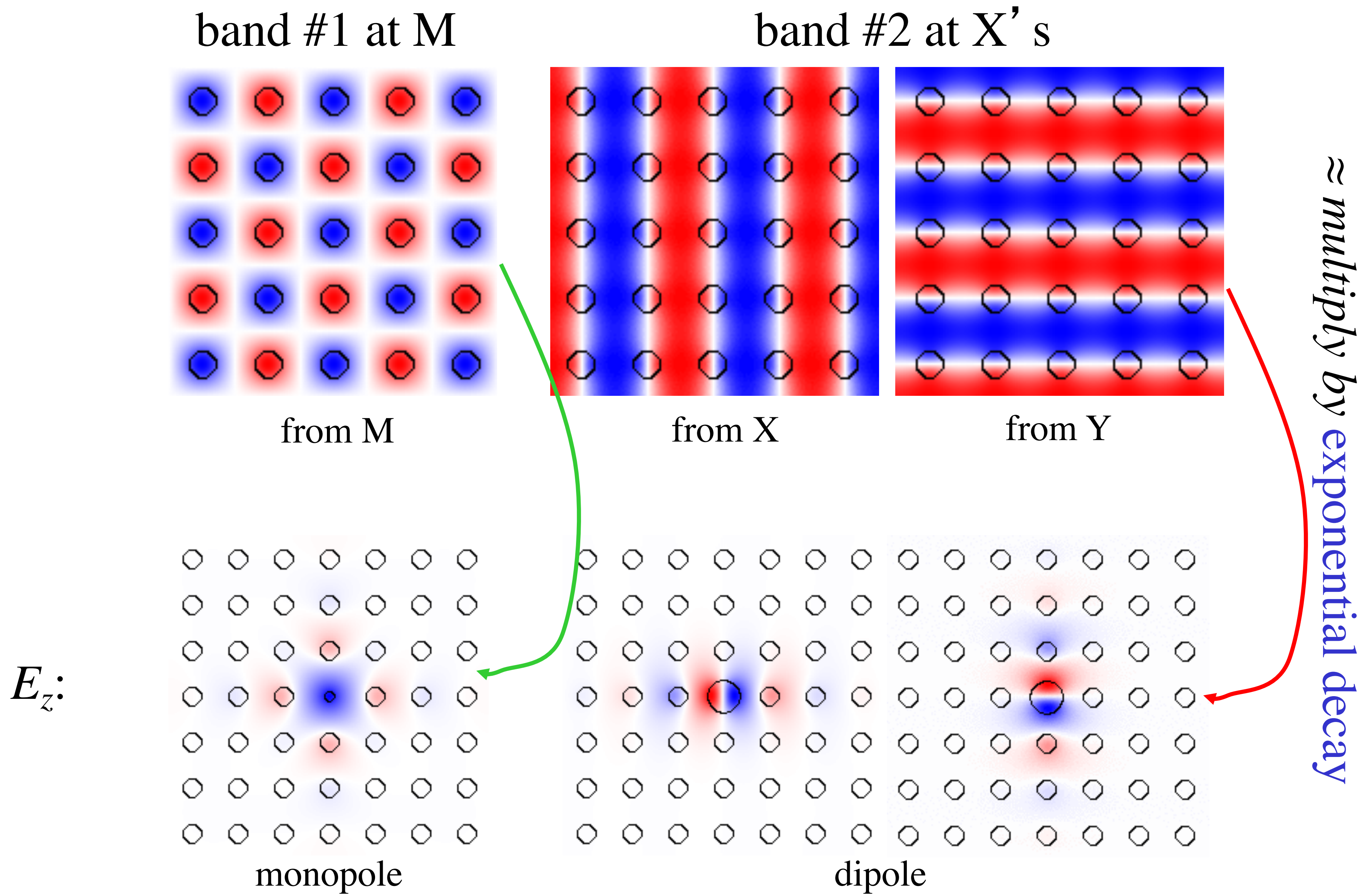


monopole



dipole

Tunable Cavity Modes

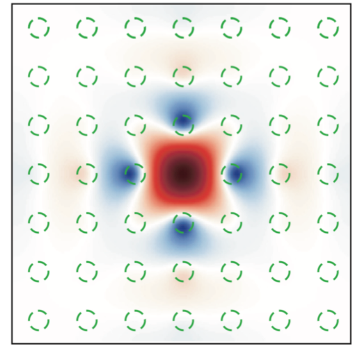


Other defects = other C_{4v} irreps

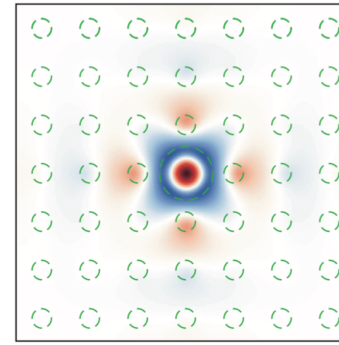
	E	$2C_4$	C_2	2σ	$2\sigma'$
Γ_1	1	1	1	1	1
Γ_2	1	1	1	-1	-1
Γ_3	1	-1	1	1	-1
Γ_4	1	-1	1	-1	1
Γ_5	2	0	-2	0	0

Γ_1

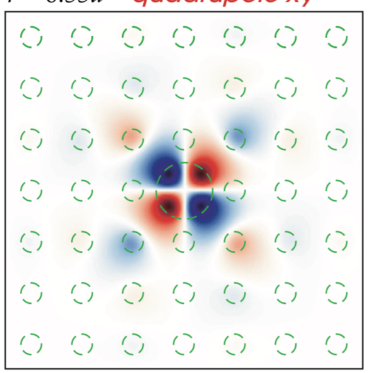
$r = 0$ *monopole*



$r = 0.55a$ *monopole-2*



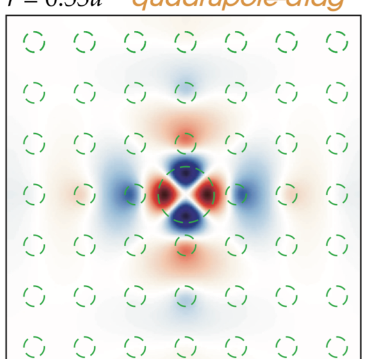
$r = 0.55a$ *quadrupole-xy*



Γ_4

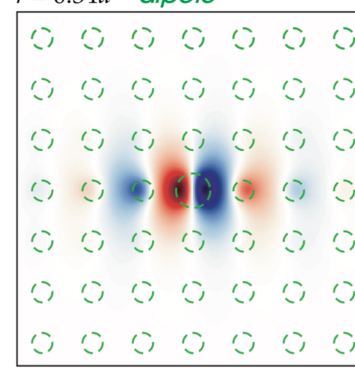
Γ_3

$r = 0.55a$ *quadrupole-diag*

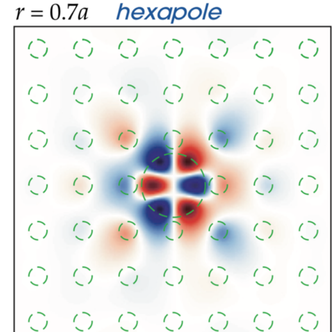


Γ_5 (doubly degenerate)

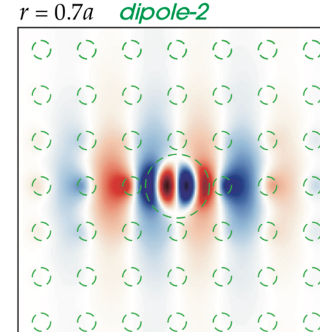
$r = 0.34a$ *dipole*



$r = 0.7a$ *hexapole*

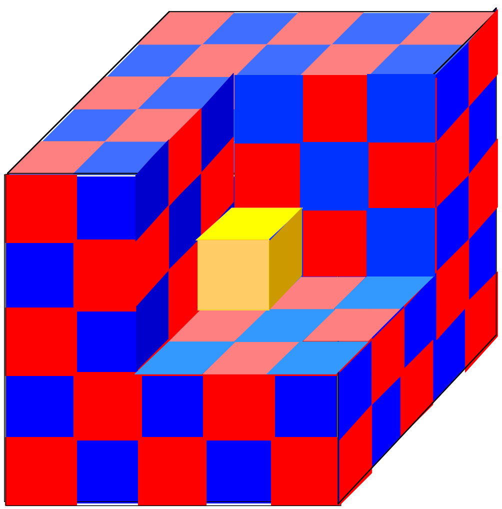


$r = 0.7a$ *dipole-2*

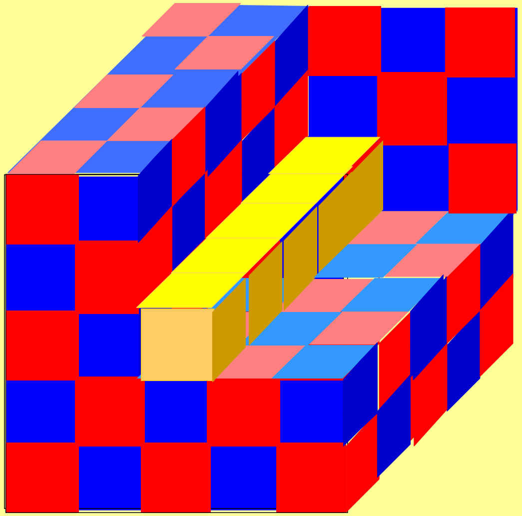


Intentional "defects" are good

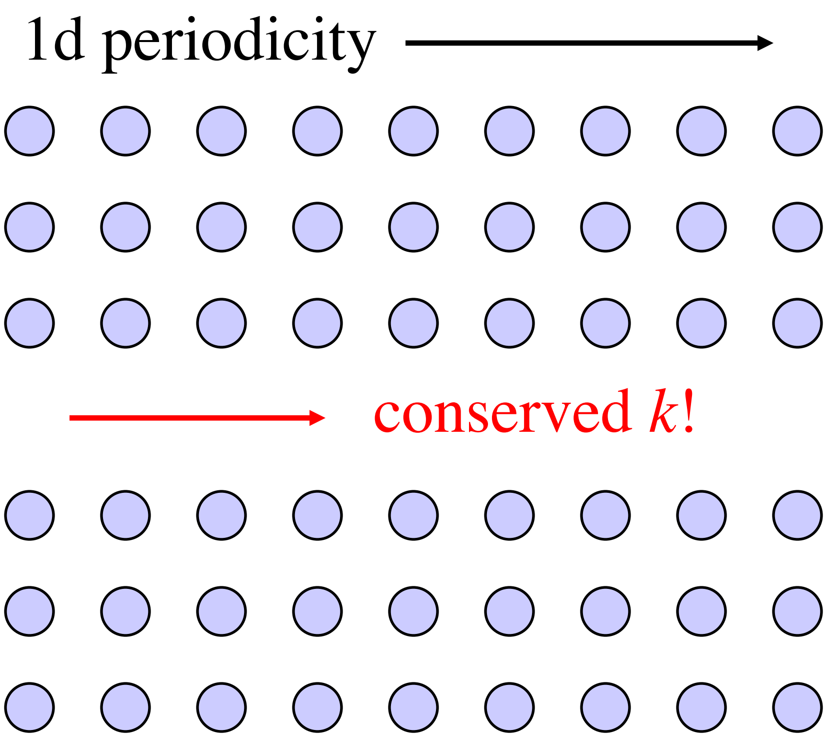
microcavities

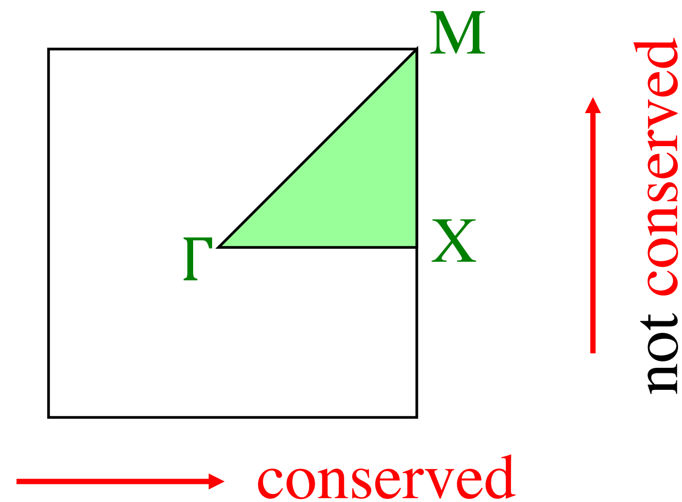


waveguides ("wires")

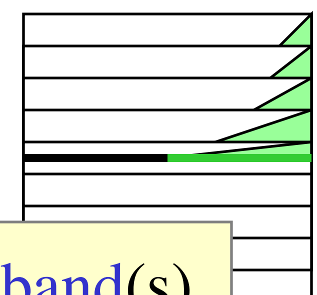


Projected Band Diagrams



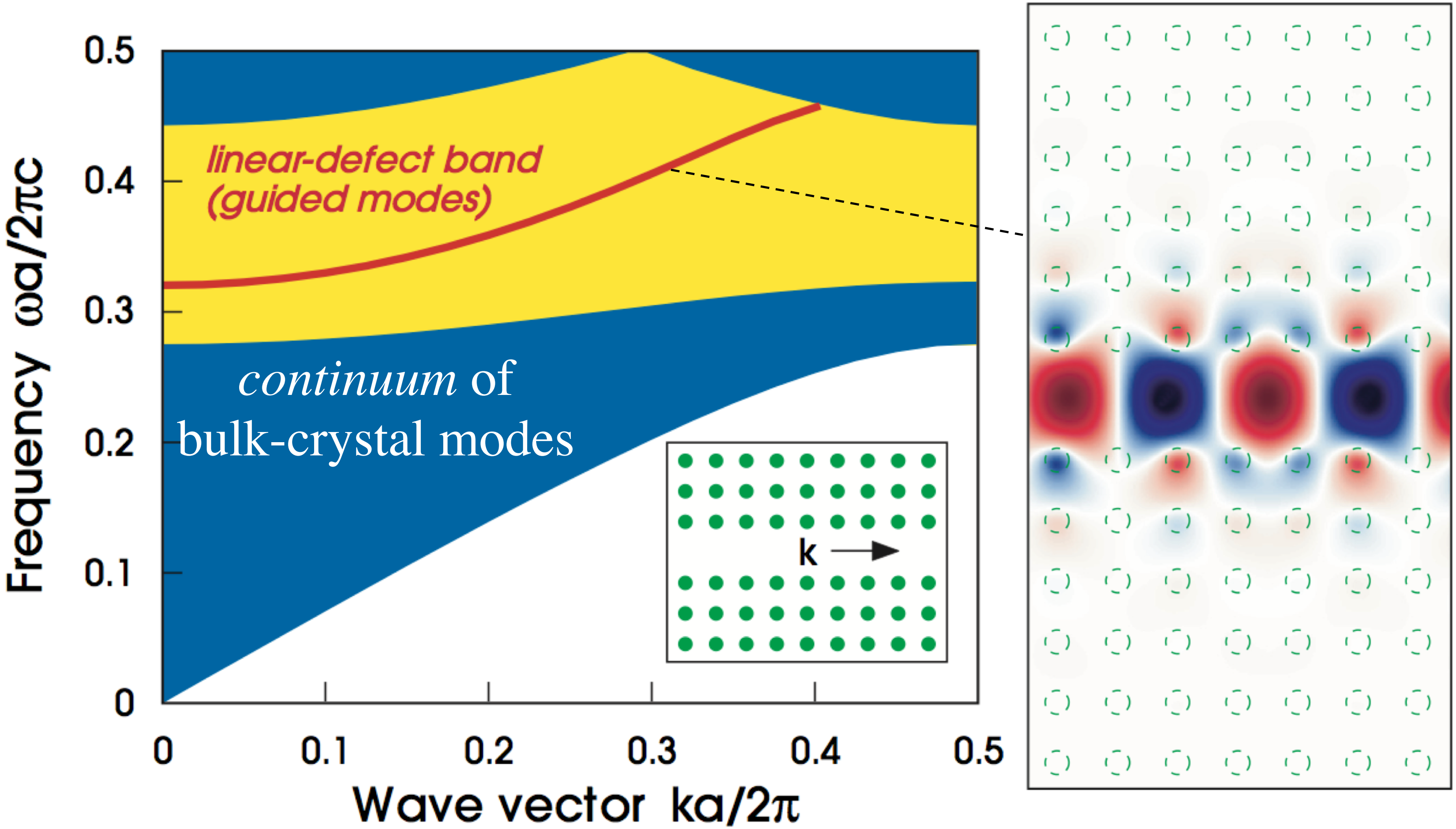


So, plot ω vs. k_x only…project Brillouin zone onto Γ–X:



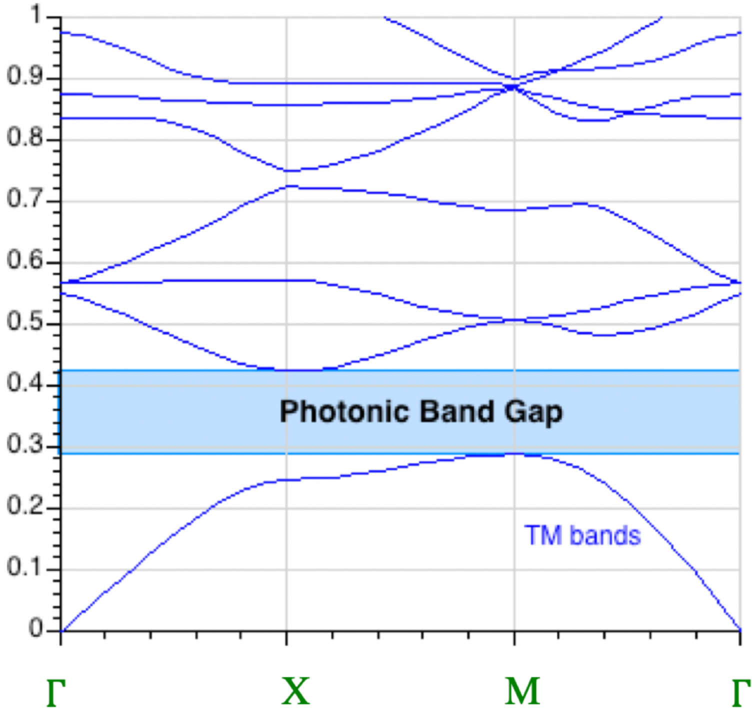
gives continuum of bulk states + discrete guided band(s)

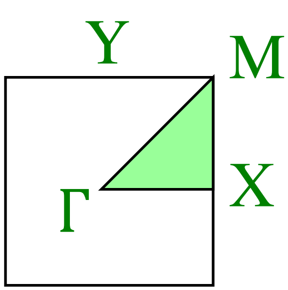
Air-waveguide Band Diagram



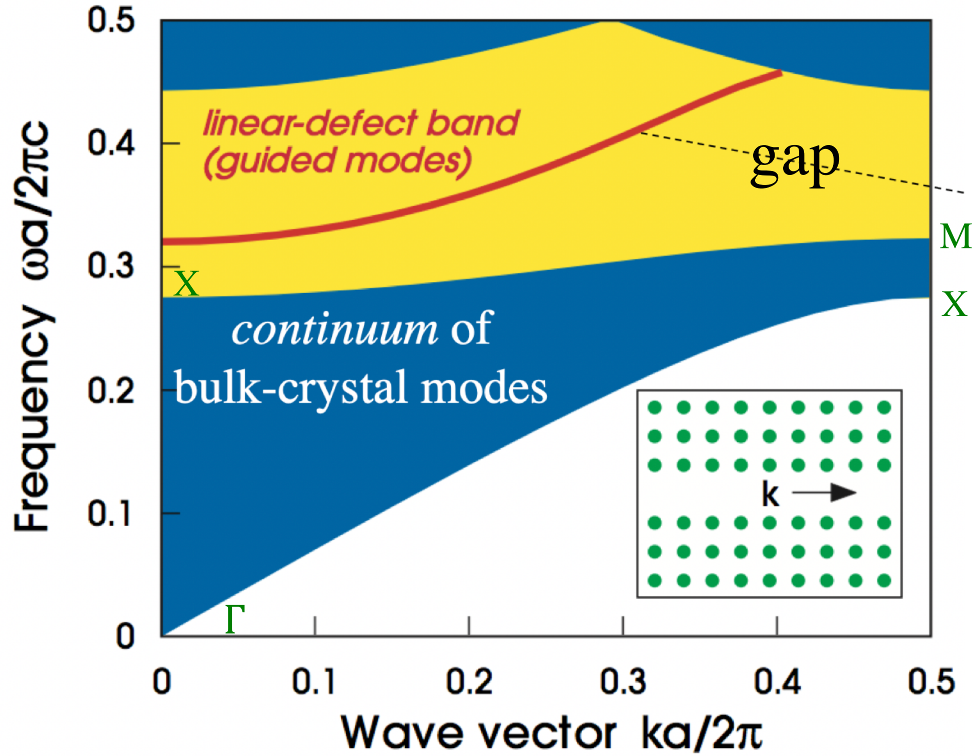
any state in the gap cannot couple to bulk crystal ⇒ localized

Projected band diagram: k_x conserved





plot all bands vs. k_x



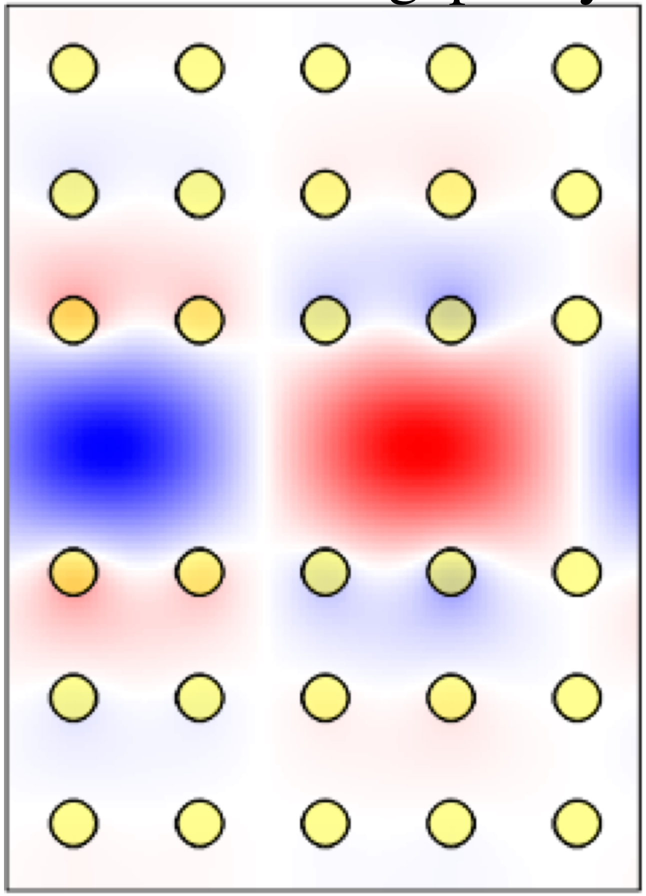
(Waveguides don't really need a *complete* gap)

Fabry-Perot waveguide:

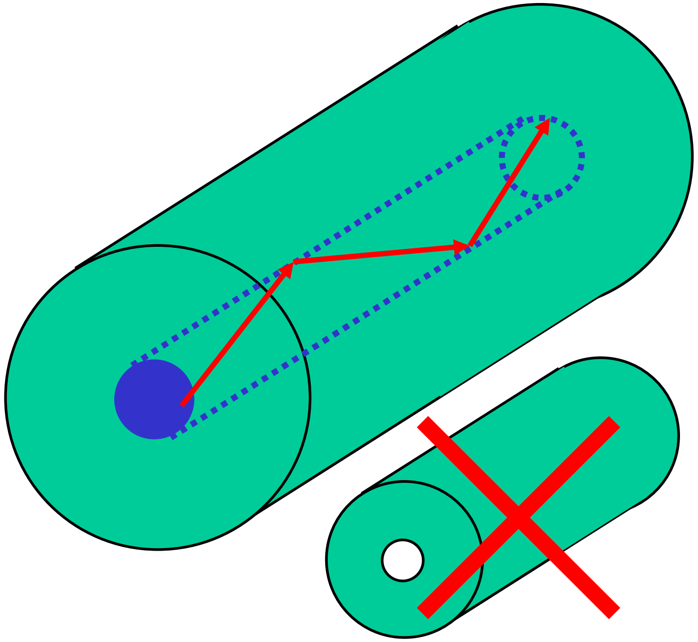
This is exploited *e.g*. for photonic-crystal fibers…

Guiding Light in Air!

mechanism is gap only



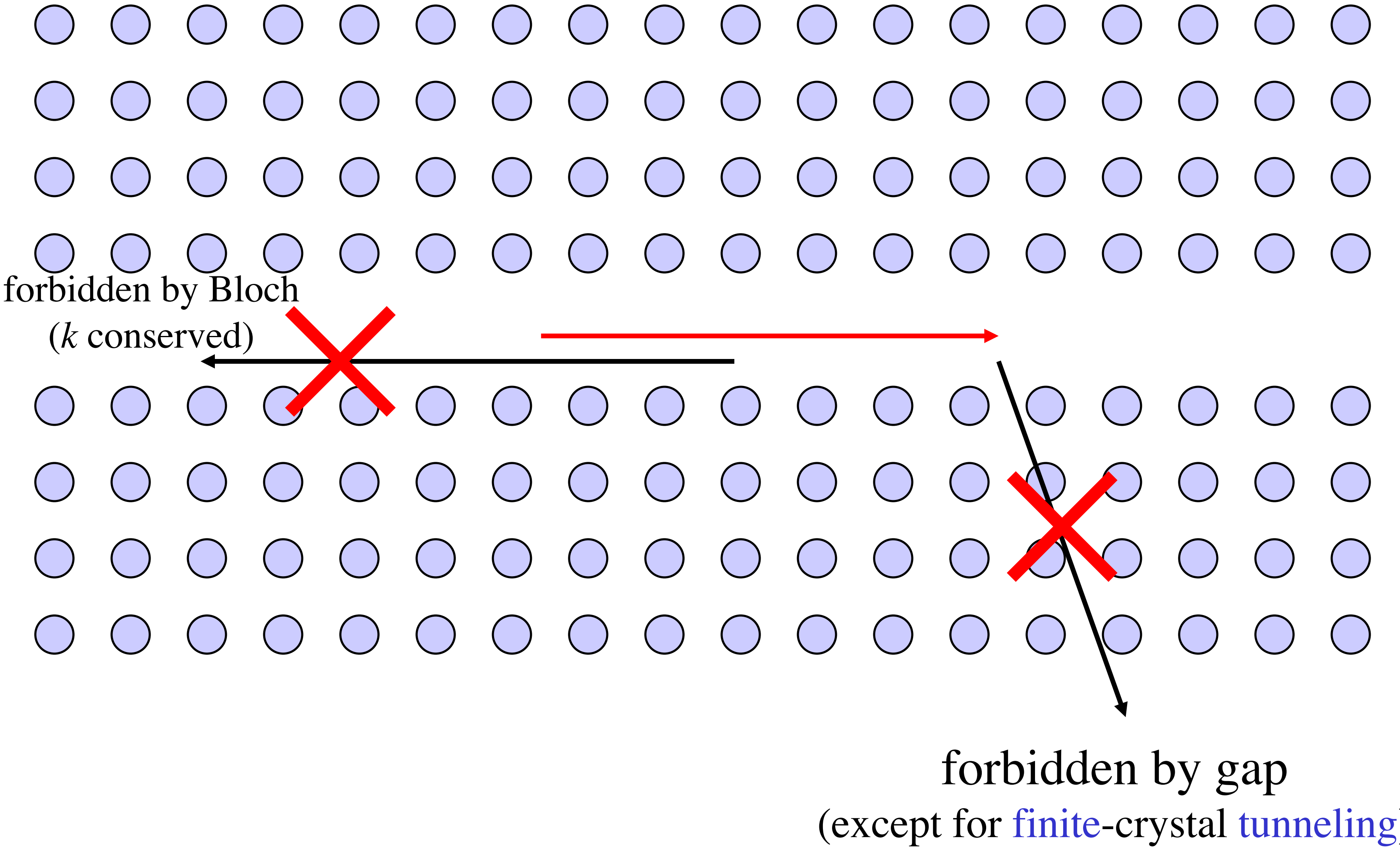
vs. standard optical fiber:
"total internal reflection"
— requires *higher*-index core



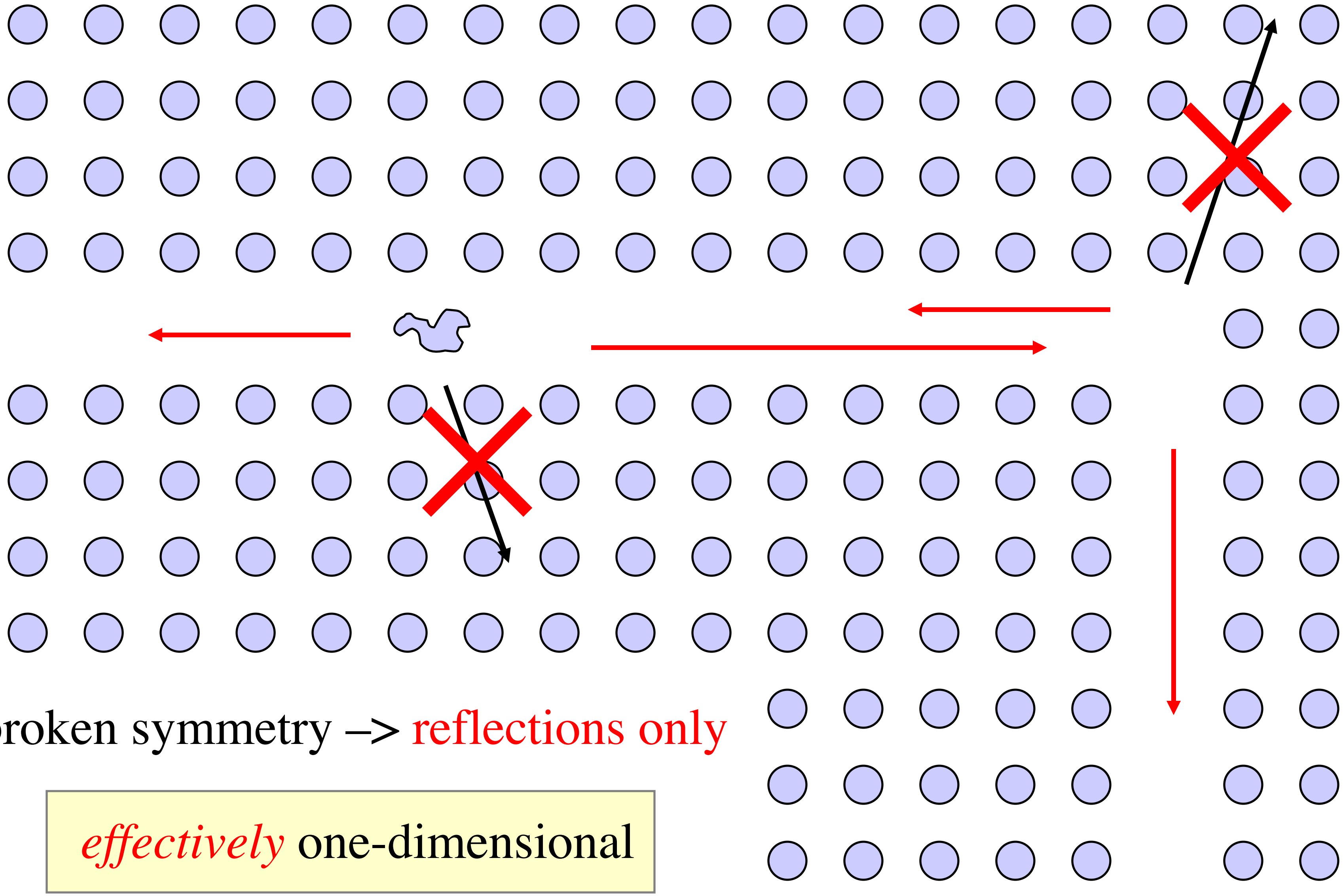
no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

Review: Why no scattering?



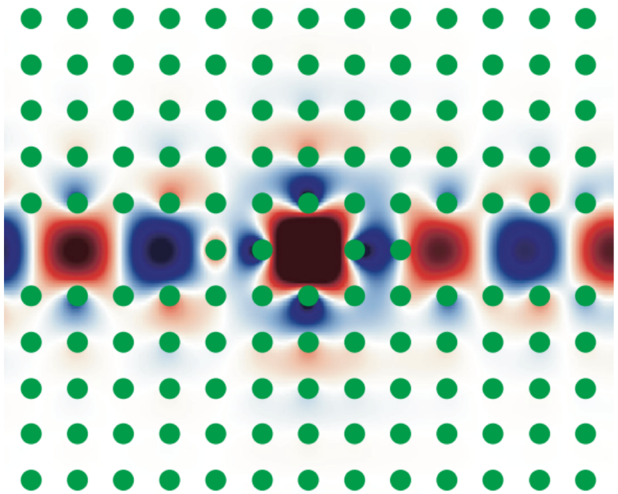
Benefits of a complete gap…



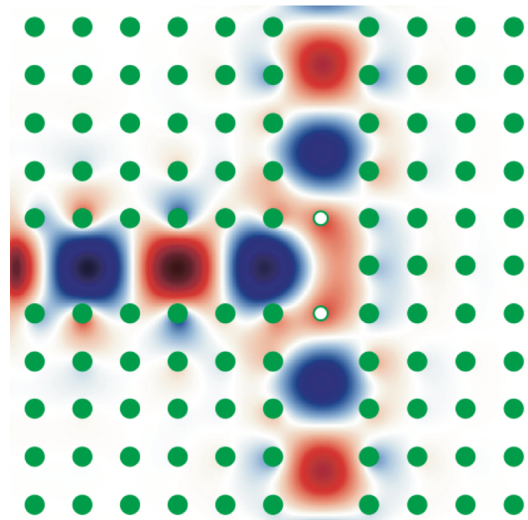
broken symmetry –> reflections only

effectively one-dimensional

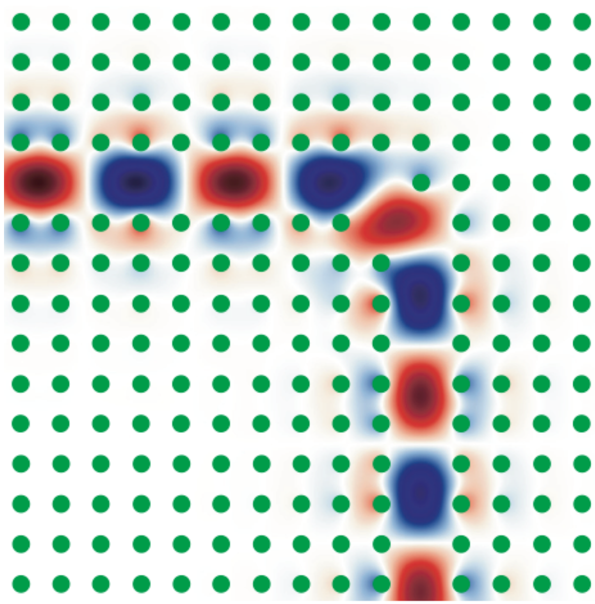
"1d" Waveguides + Cavities = Devices



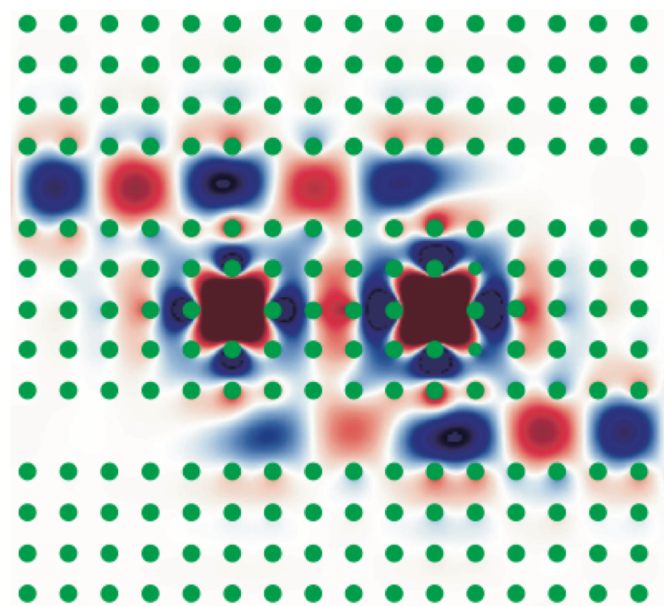
resonant filters



waveguide splitters

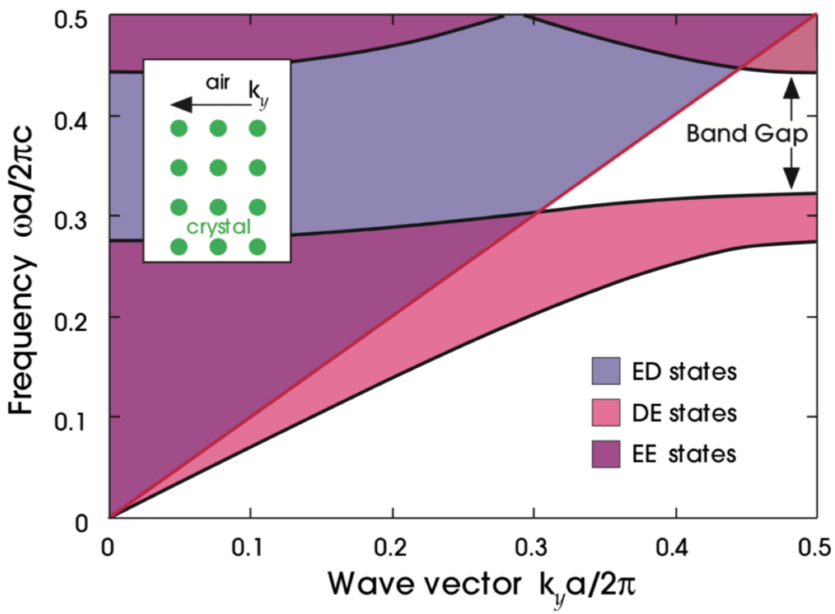


high-transmission sharp bends



channel-drop filters

Surface states in 2d



Projected band diagram: only $k_{||}$ conserved: Plot projected crystal bands + light cone.

Surface termination determines surface state solution: see effect of 2 different terminations!