Pokerbots 2025

Lecture 4: Game Theory

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New Sponsor: Codeium

About Codeium

- AI dev tools company founded in 2021 by MIT Alumni
- Launched the Windsurf Editor (AI-powered IDE) in 2024
- We're hiring! codeium.com/careers
- Tech Talk: "How We Built Windsurf"
    - Thu 11:30am, 26-100, co-hosted by web.lab
    - RSVP at tinyurl.com/codeium-iap
- Free month of Pro Ultimate subscription ($60/mo)
    - Promo Code "mit-pokerbots-2025"
- Full info about Codeium shared on Piazza

Announcements

Week 1 Mini Tournament Results

WINNER [$1000]:

Encore 1/2 Regs

BIGGEST UPSET [$500]:

queen

Tournament ELO 1105, won majority matches against “Hawk Tuah” (ELO 1974)

Week 1 Mini Tournament Results

Full tournament results can be viewed at the scrimmage server

Pokerbots Scrimmage | Home | Announcements | Manage Team | Your Games | Tournaments | Charts | Playground | Log out jcob

Tournaments

Week 1 Mini
2 days ago

86 PARTICIPANTS

25 GAMES/PAIR

Done!

Name	ELO	Wins	Losses
Encore 1/2 Regs	2468	2048	76
Pineapple	2362	1992	133
a	2264	1930	188
Peter	2250	1925	200
DKE juniors	2249	1926	198
JustKeepPunting	2202	1895	230
retry of reretry	2188	1882	243
all_luck_no_skill	2164	1864	261
pocket fools	2145	1851	274
could this be, dog?	2057	1766	359
brokerpot	2011	1722	403

Make-up Progress Reports due Tuesday

- As per syllabus, if your team missed the Week 1 bot submission deadline, a progress report is required to earn credit
- Reports are half a page and must list team name and members
- Submit via email to pokerbots@mit.edu by 11:59pm Tuesday (Tomorrow)

Hackathon on Wednesday

- Sponsored by **Codeium**
- Wednesday night (1/15)
- 32-044, 7pm -- LATE
- Show up and grind
- Dinner provided
- Snacks, fun, and games
- Prizes for challenges and those that stick around
- Details will be announced on Piazza

No Class Thursday

Cancelled in light of Hackathon the night before and Codeium Tech Talk at 11:30am

Giveaway Games!

First game: 2/3 Game (“Beauty Contest”)

- Submit an integer between 0-1000 inclusive
- Winner is the closest guess to 2/3 of the average submission (a tie is broken randomly)
- Prize: JBL Charge 5 Speaker

pkr.bot/beauty

Second game: Kerb Squared Wager

- ‘Entry fee’: your kerb gets blacklisted from winning giveaways for the next week

- Winner is picked at random with probability proportional to the length of your kerb squared

- Prize: Airpods Pro

pkr.bot/lemons

Agenda

- What is a game?
- Pure and mixed strategies
- Nash equilibria
- Applications to poker

What is a game?

Definition

We generally only consider two-player zero-sum games; we will refer to these as simply *games*.

Examples: RPS, tic-tac-toe, (chess? poker??)

Formalized

A *game* between players 1 and 2 consists of a pair of strategy sets S_1 and S_2, and a utility function $u : S_1 \times S_2 \longrightarrow \mathbf{R}$. Players submit strategies simultaneously. Player 1 seeks to maximize u, and player 2 seeks to minimize u. If randomness is involved, then expected utility is used.

Think of utility as chips. This means players want to submit actions that have the highest expected payout.

Player 2 is minimizing since we're talking about zero-sum games.

Pure and mixed strategies

Pure and mixed strategies

- A mixed strategy is described by a probability distribution over pure strategies:
    - Example:
    - P(rock) = 0.4
    - P(paper) = 0.3
    - P(scissors) = 0.3

- There are only 3 pure strategies in RPS, but infinitely many mixed strategies

- Mixed strategies show us the power of randomness

Matrix form games

Suppose the strategy sets S_1 and S_2 consist of probability distributions over a finite list of pure strategies. Also, suppose that the utility function is linear. Then, we can write our game in *matrix form*, with utility function shown.

$S_1 \setminus S_2$	Rock	Paper	Scissors
Rock	0	-1	+1
Paper	+1	0	-1
Scissors	-1	+1	0

Revisiting the first game

The Keynesian beauty contest

- No rational player would play above $2/3 \cdot 1000 = 667$
- Hence, no rational player would play above $2/3 \cdot 667 = 445$
- What happens if we continue this logic?
- What would "rational" players play?

Dominance

- We say that strategy A *dominates* strategy B if playing A is always a better idea
- u(A, O) ≥ u(B, O) for all player 2 strategies O
- Second-order dominance: replace “is always a better idea” with “is always a better idea, if our opponent does not play dominated strategies”
- And so on…

Questions?

Dominance Example

	C	D
A	+3	+2
B	+1	-1

Dominance Example

	C	D
A	+3	+2
B	+1	-1

Dominance Example

	C	D
A	+3	+2
B	+1	-1

Dominance Example

	C	D
A	+3	+2
B	+1	-1

Equilibrium

A *Nash equilibrium* is
a set of strategies,
one for each player,
such that nobody has
an incentive to switch.

Nash equilibrium examples

- 0 is an equilibrium for the Keynesian beauty contest
- 1/3 rock, 1/3 paper, 1/3 scissors is an equilibrium for RPS

Solving the equilibrium for RPS

$r + p + s = 1$; $r, p, s \geq 0$

Expected utility given each case of opponent's possible play

	Rock (r')	Paper (p')	Scissors (s')
Rock (r)	0	-1	+1
Paper (p)	+1	0	-1
Scissors (s)	-1	+1	0
EV | Opponent	p-s	s-r	r-p

Solve $p - s = s - r = r - p$ to get $r = p = s = 1/3$. This guarantees us at least 0

Why p - s = s - r = r - p?

- Suppose opponent plays the optimal r’, p’, s’ counterstrategy against our r, p, s strategy

- Opponent minimizes the utility function amongst their three options:

$$\min_{r', p', s'}(p - s, s - r, r - p)$$

- We play the strategy that maximizes the utility function *even against the optimal counterstrategy*:

$$\max_{r, p, s}(\min_{r', p', s'}(p - s, s - r, r - p))$$

Questions?

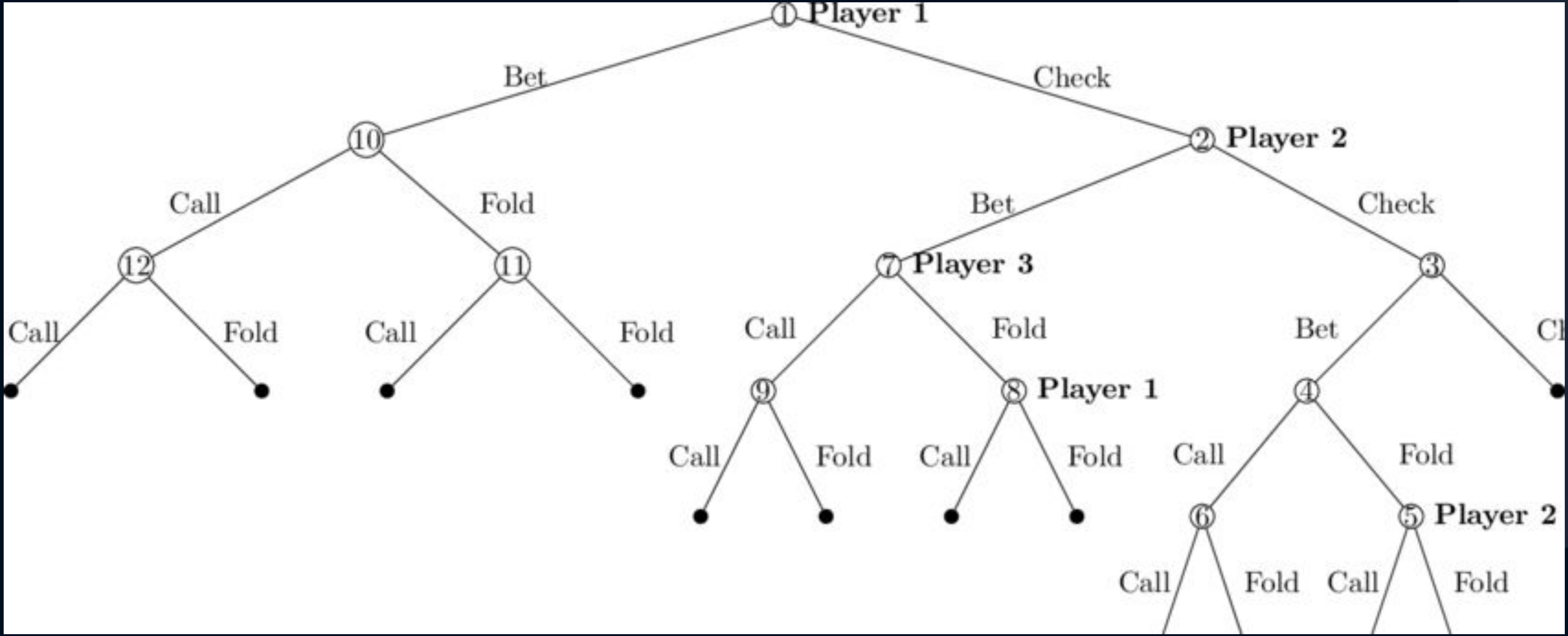
Existence of Nash equilibria

Theorem (Nash, 1951): Every finite game has a Nash equilibrium in mixed strategies.

Proof: Topology (uses Brouwer fixed-point theorem).

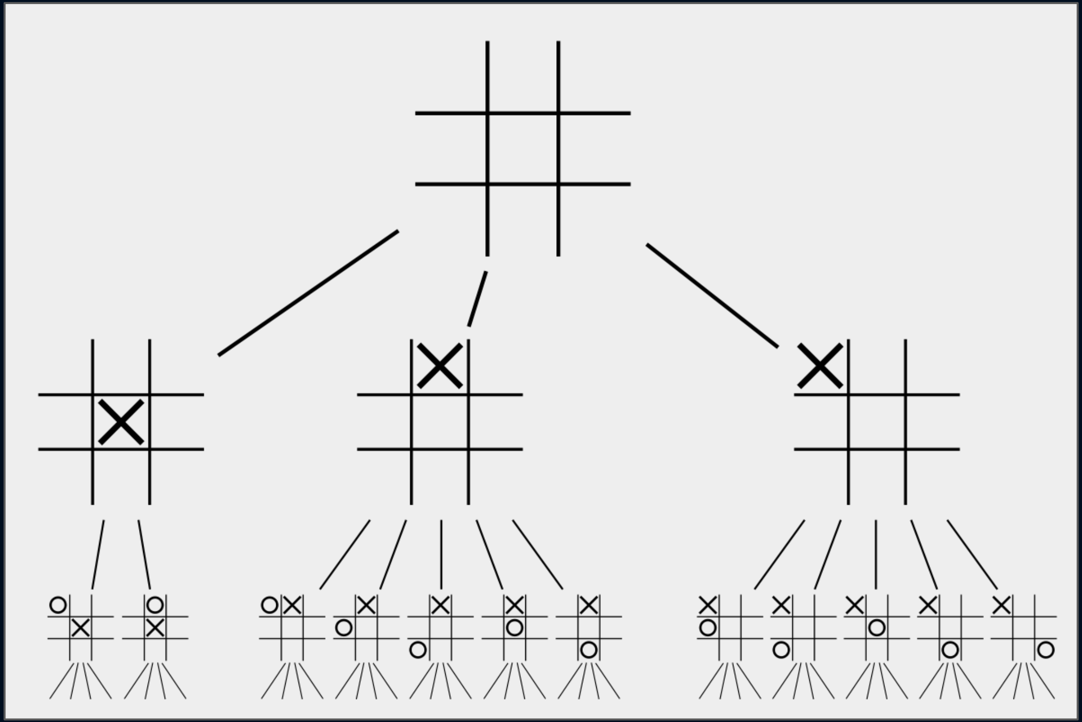
Applications to poker

Game Tree

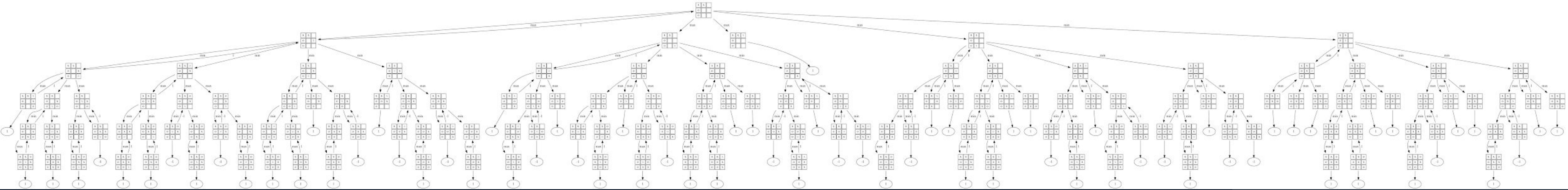


① Player 1
Bet
Check
⑩
② Player 2
Call
Fold
Bet
Check
⑫
⑪
⑦ Player 3
③
Call
Fold
Call
Fold
Call
Fold
Bet
⑨
⑧ Player 1
④
Call
Fold
Call
Fold
Call
Fold
⑥
⑤ Player 2
Call
Fold
Call
Fold

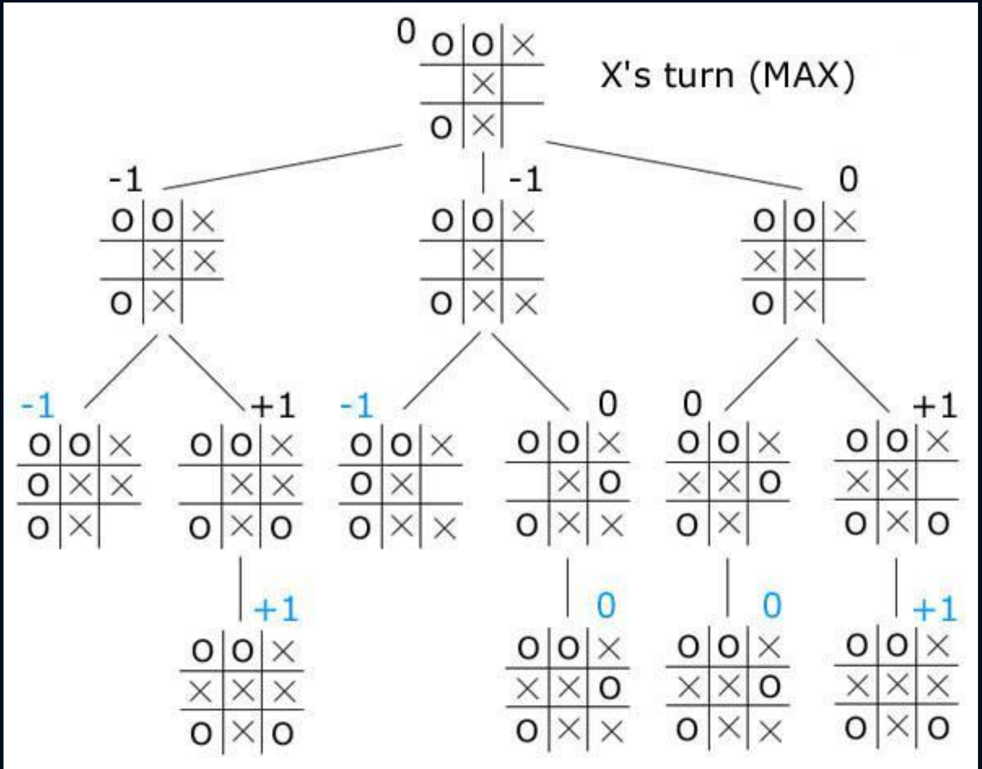
Extensive form games



Tic Tac Toe Full Extensive Form



Backwards induction

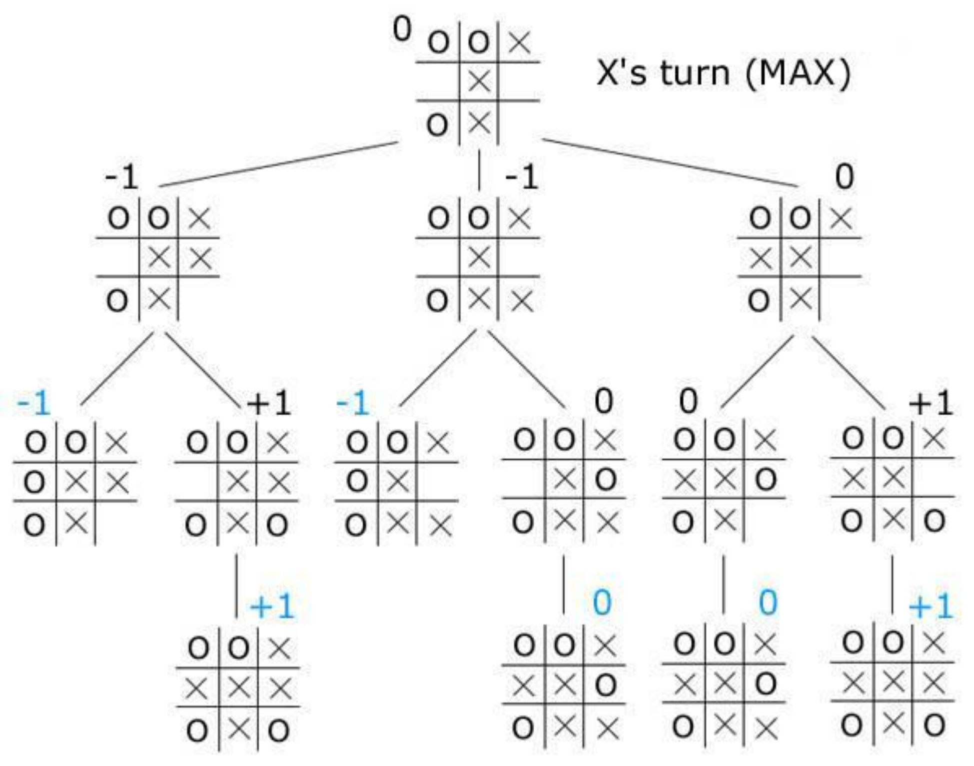


Black are the determined payouts based on players acting rationally

Blue are the leaves - the terminal states of the game

Follow the game tree up (backwards) to find the state's value

Backwards induction



Looking at the leaf states first, I see -1s in the left 2 branches, leaving their parents as -1.

The rightmost branch has payouts 0 and 1. Following this up the tree, we get 0 for the black value.

Finally, all together X should try to play for 0 (tie), and does so by putting x in left center.

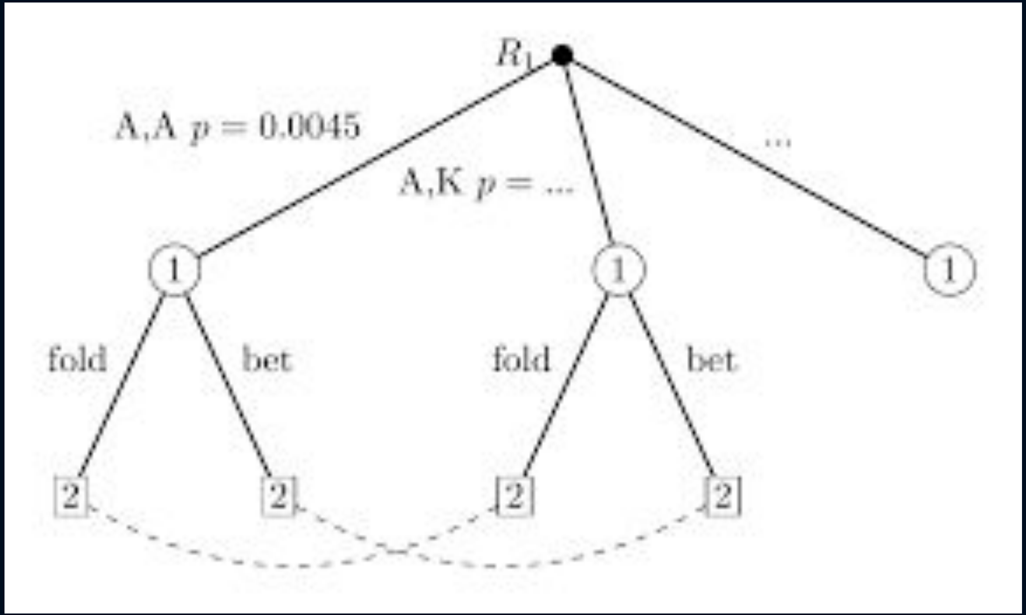
Conversion to normal form

- Every extensive form is also a normal form (matrix) game
- Mapping: each pure strategy (row/column) is a complete set of instructions for what to do at each node
- Example: in poker, each pure strategy would be a dictionary which maps the game state to a unique action

Implications

- Poker has a Nash equilibrium - it's a finite game that satisfies the conditions
- Can we use backwards induction to solve for it? Can we still use the same type of game tree?
- How is poker different from Chess? tic-tac-toe? RPS?
    - Size
    - Imperfect Information
    - Randomness

Adding in imperfect information and randomness



Imperfect Information and Randomness

- Imperfect: Don't know what node you're currently at since your opponent's hole cards are hidden (information sets)
- Randomness: Don't know where in the game tree you're moving next since future cards to be dealt are unknown
- Together, this leads to a really nice property of poker. The general game tree movements (bet/fold/call) are known, but the definite game state isn't. Imperfect information makes games fun!

Kuhn Poker

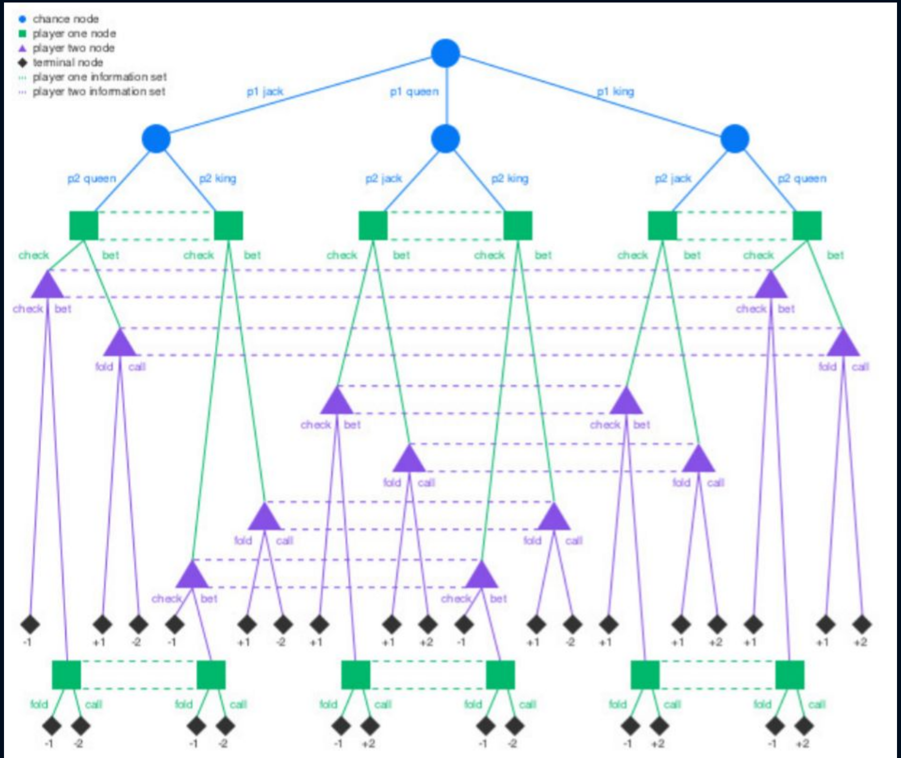
Kuhn poker is a simplified form of poker developed by Harold W. Kuhn as a simple model zero-sum two-player imperfect-information game, amenable to a complete game-theoretic analysis. In Kuhn poker, the deck includes only three playing cards, for example a King, Queen, and Jack. One card is dealt to each player, which may place bets similarly to a standard poker. If both players bet or both players pass, the player with the higher card wins, otherwise, the betting player wins.

Game description [edit]

In conventional poker terms, a game of Kuhn poker proceeds as follows:

- Each player antes 1.
- Each player is dealt one of the three cards, and the third is put aside unseen.
- Player one can check or bet 1.
  - If player one checks then player two can check or bet 1.
    - If player two checks there is a showdown for the pot of 2 (i.e. the higher card wins 1 from the other player).
    - If player two bets then player one can fold or call.
      - If player one folds then player two takes the pot of 3 (i.e. winning 1 from player 1).
      - If player one calls there is a showdown for the pot of 4 (i.e. the higher card wins 2 from the other player).
  - If player one bets then player two can fold or call.
    - If player two folds then player one takes the pot of 3 (i.e. winning 1 from player 2).
    - If player two calls there is a showdown for the pot of 4 (i.e. the higher card wins 2 from the other player).

Kuhn Poker Game Tree



Poker Nash equilibrium and use

- Poker has a Nash equilibrium that has EV 0
- The matrix is doubly exponential in size - incomputable
- Does “play good poker” reduce to “play the equilibrium strategy”?
- Exploitative strategies as alternatives

Applied game theory

Adverse selection

- Occurs anytime “buyer” and “seller” have asymmetric information
- Market for cars: suppose there are used cars with private value distributed uniformly between \$1000 and \$10000. What should the market price be?
- What would car owners do? Then what happens?
- Kerb squared game

Sources of adverse selection in poker

- First action when betting (differential check to the preflop aggressor)
- Multiple bets in a row (what types of hands do this?)
- Being in the later stages of the round (why would they still be in the pot?)
- Bounty Cards

When I bet, am I happy with my bet *conditioned on my opponent calling?*

Adverse selection and determinism

- Suppose I have the following deterministic betting strategy:
- All-in when I have a top X% hand
- How can my opponent exploit me?

Second-Price Auctions

Game Theory Takeaways

- Try to be one step ahead of your opponent (dominance)
- Equilibrium strategies can be good but hard to find
- Exploitative strategies can be very powerful - think about what you're trying to optimize for when making a pokerbot
- Be careful of adverse selection

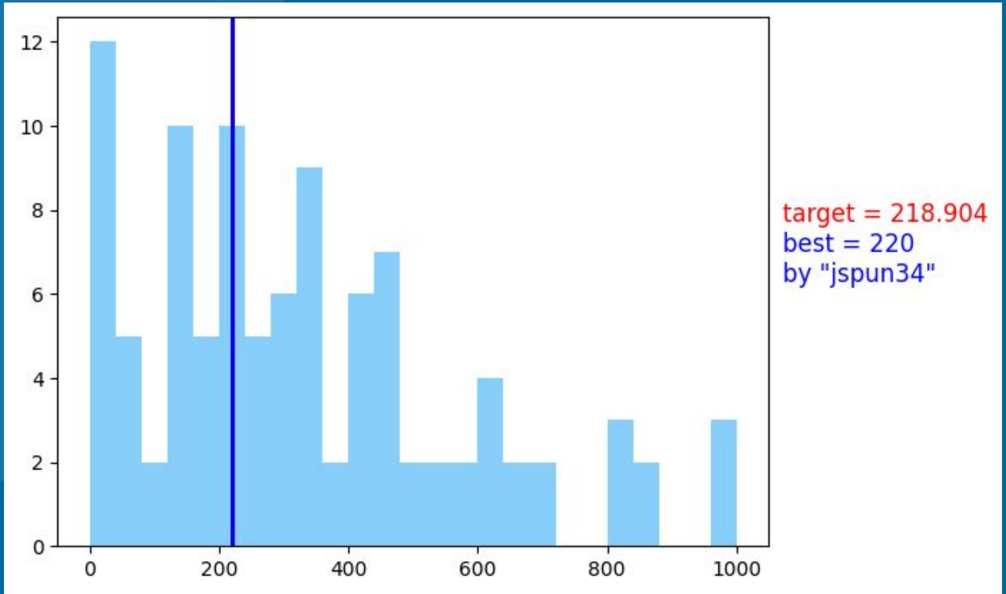
Questions?

Lunch 😋

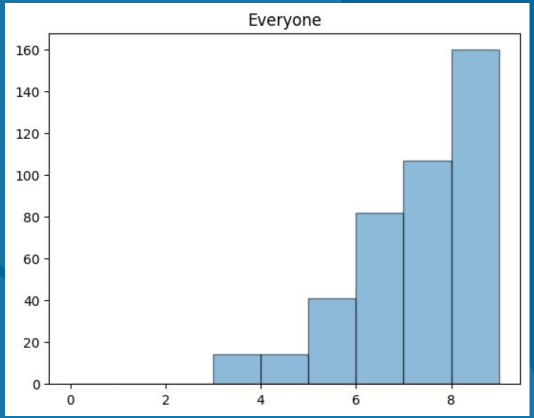
Leave any type of feedback at **pkr.bot/feedback**!

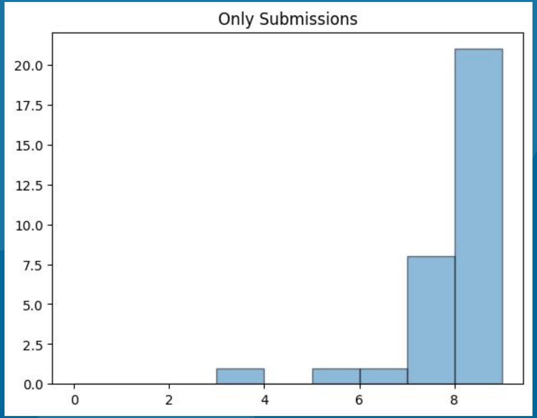
Live Coding: reference-4-2025

2/3 Contest Winner:



Kerb Squared Contest Winner: “megansun”





- 32 submissions total
- Note: MIT limits kerb length to 8 characters

Thanks for watching!

Slides/notes will be posted on pkr.bot/resources

Make sure to check pkr.bot/piazza for updates

Lecture recordings at pkr.bot/panopto

Leave feedback at pkr.bot/feedback!