Pokerbots 2025

Lecture 2: Probability and Statistics

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Register & Resume Drop:
pkr.bot/register

Giveaway!

pkr.bot/people

Guess the number of people in class (measured by submissions).

Feel free to be precise with decimals.

Ties are broken by order of submission

Prize: Sony XM4 Headphones

Announcements

Engine Updates

- Some bugs have been patched
- Keep letting us know about issues
- Latest version at **pkr.bot/engine**
- Important to always use most updated version of engine as it is what we use on the scrimmage server and in tournaments

Poker Social TODAY!

- Recitation block (26-168, 2-4PM)
    - Office hours will still run concurrently
- Great place to meet teammates!
- We will be giving a way a pair of Airpods Pro
- Also try pkr.bot/teammates

Week 1 Deadlines

- First pokerbot submission due Friday 1/10, 11:59PM EST on scrimmage server
- Mini-tournament 1 will occur shortly after

Hackathon Next Week!

- Next Wednesday night (1/15)
- Show up and work on your pokerbot!
- Dinner provided, plan to go late into the night
- Snacks, fun, and games
- Prizes for challenges and those that stick around
- Location TBA

Outline of Today

- Probability Foundations
- Random Variables and Distributions
- Expected Value
- Law of Large Numbers

Probability Foundations

What is Probability?

- We say “probability of [event] happening is *x*”
- *x* is usually a number from 0 to 1 or a percent
- But what does *x*’s value <u>mean</u>?

Interpretations of Probability

- Frequentist
  - How often an event occurs over repeated trials
  - Ex: When rolling a dice repeatedly, we notice 1/6th of rolls are a 2
  - What about events that aren't repeated? Eg: 2024 election
- Subjectivist (Bayesian)
  - Degree of belief, or 'credence' of an event occurring from the perspective of an individual with a given set of information
  - Different pieces of information 'update' the probability to be higher or lower depending on how much evidence they provide for the event
  - However, this requires some assumptions for which default probability values to start with before any updates
- This is mostly a philosophical question within epistemology
- Regardless of interpretation, probability is used as a quantitative metric for modeling uncertainty in things we don't know

Motivation: Finite Possibilities with Equal Chance

- Wish to find/define probability of some event that occurs in some outcomes
- An intuitive definition:

  Probability of event = (# favorable outcomes) / (# total outcomes)
- Ex: Randomly draw card from standard deck, probability of Spades
  - 13 cards with spades
  - 52 cards total
  - 13/52 → ¼
- This reduces any question of probability into a counting problem
- What about outcomes with unequal chance?
- Or infinite outcomes? (e.g. continuous spectrum of possibilities)
- We need some general way to define their events and probabilities

Kolmogorov Axioms

Events are viewed as sets of outcomes, and every set is a subset of the largest set Ω, which can be viewed as the 'universal' set encompassing all outcomes. We define a function P to give the probability of an outcome falling within an event set , satisfying the following axioms:

1. $P(\Omega) = 1$
2. $P(E) \geq 0$ for any event set E
3. $P(A \cup B) = P(A) + P(B)$ for any disjoint (aka nonoverlapping) event sets A, B

Kolmogorov Axioms

These axioms can prove all other results which we'd expect to make sense:

- If $A \subseteq B$, then $P(A) \leq P(B)$
- P(empty set) = 0
- $P(E) \leq 1$
- $P(\Omega - E) = 1 - P(E)$

As long as we can define a universal set Ω and an idea of probability that follows the axioms, then we can use any of the results.

Example

- Universe of possibilities representing opponent's bounty card
- P(bounty card is 2) = P(bounty card is 3) = … = 1/13

Questions?

Conditional Probability

"probability of A given B"

$$P(A \mid B) = P(A \cap B) / P(B)$$

Another way to write: $P(A \cap B) = P(B) \mid P(A \mid B)$

The basis of inference aka updating beliefs based on information.

Example

- Universe of possibilities representing opponent's bounty card and whether or not they go all in after a 2 turns over in the flop
- We know they always go all in if their bounty card is a 2, and go all in 1/24 of the time whenever their bounty card is not a 2.
- We want to find the conditional probability their bounty card is a 2 given that they just went all in after a 2 was revealed on the flop.
- P(All in| bounty is 2) = 1
- P(All in| other bounty) = 1/24
- Want to find P(bounty is 2 | All in)

Example

- P(All in ∩ bounty is 2)

Example

- $P(\text{All in} \cap \text{bounty is } 2) = P(\text{All in} \mid \text{bounty is } 2) * P(\text{bounty is } 2)$

Example

- $P(\text{All in} \cap \text{bounty is } 2) = P(\text{All in} \mid \text{bounty is } 2) * 1/13$

Example

- $P(\text{All in} \cap \text{bounty is } 2) = 1 * 1/13$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$

Example

- P(All in ∩ bounty is 2) = 1/13
- P(All in ∩ other bounty)

Example

- $P(\text{All in} \cap \text{bounty is } 2) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = P(\text{All in} \mid \text{other bounty}) * P(\text{other bounty})$

Example

- P(All in ∩ bounty is 2) = 1/13
- P(All in ∩ other bounty) = P(All in | other bounty) * 12/13

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/24 * 12/13$

Example

- P(All in ∩ bounty is 2) = 1/13
- P(All in ∩ other bounty) = 1/26

Example

- $P(\text{All in} \cap \text{bounty is } 2) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in})$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = P(\text{All in} \cap \text{bounty is 2}) + P(\text{All in} \cap \text{other bounty})$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 1/13 + 1/26$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is 2} \mid \text{All in}) = ?$

Example

- P(All in ∩ bounty is 2) = 1/13
- P(All in ∩ other bounty) = 1/26
- P(All in) = 3/26
- P(bounty is 2 | All in) = P(bounty is 2 ∩ All in) / P(All in)

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is 2} \mid \text{All in}) = P(\text{All in} \cap \text{bounty is 2}) / P(\text{All in})$

Example

- $P(\text{All in} \cap \text{bounty is } 2) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is } 2 \mid \text{All in}) = P(\text{All in} \cap \text{bounty is } 2) / (3/26)$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is 2} \mid \text{All in}) = (1/13) / (3/26)$

Example

- $P(\text{All in} \cap \text{bounty is 2}) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is 2} \mid \text{All in}) = (2/26) / (3/26)$

Example

- $P(\text{All in} \cap \text{bounty is } 2) = 1/13$
- $P(\text{All in} \cap \text{other bounty}) = 1/26$
- $P(\text{All in}) = 3/26$
- $P(\text{bounty is } 2 \mid \text{All in}) = 2/3$

Example

- P(bounty is 2) = 1/13
- P(bounty is 2 | All in) = 2/3
- Takeaway: probability of an event increases when you observe evidence that makes it more likely
- Conditional probabilities allow us to mathematically represent this effect, which is called Bayes' Theorem

Questions?

Random Variables and Distributions

Random Variable

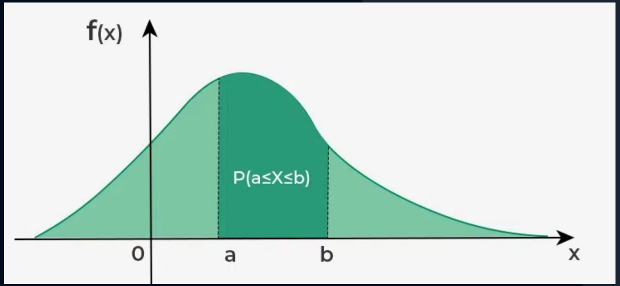
- A random variable is a quantity or event which takes on different values with different probabilities
  - Ex: A drawn card that's face down and hasn't been turned over
- The set of possible values it can take on would represent the universal set of events for probabilities concerning this variable.
  - Ex: 52 total cards that our mystery card could have
- Any event concerning this variable corresponds to set possible values it takes on
  - Ex: "card is black" <--> set of 26 cards

Distributions

- Distributions are functions that are used to define probabilities for different events

- For discrete events, this is called a "probability mass function" (think bar graph)

- For continuous events, this is called a "probability density function" (think histogram)



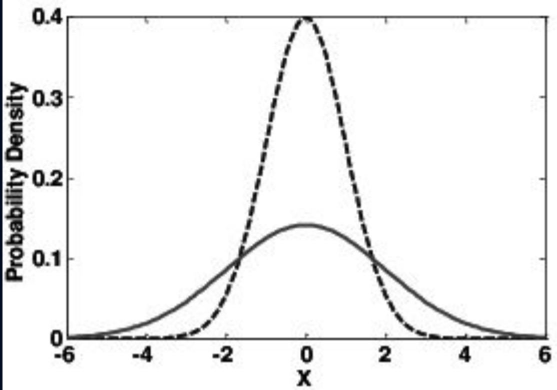
Expected Value

Expected Value

- The notion of expected value (EV) is used to summarize distributions on real numbers by some representative value
- In short, “expected value” is just another way to say “mean”
- Finite events with equal probability → take the average
  - EV of dice roll: (1+2+3+4+5+6)/6 = 3.5
- What about events with different probabilities?
- Generalize to weighted average (weighted by probability)
  - X is a random variable with possible values in set S
  - $E[X] = \Sigma_{s \in S}\, s * P(X = s)$
- For continuous distributions, this generalizes to an integral calculation

Variance

- If expected value describes where random variable “usually” is, then variance measures how much the random variable may fluctuate
- This is done by finding the average squared distance from the mean:
  - Random variable X with $E[X] = \mu$
  - $\rightarrow Var[X] = E[(X - \mu)^2]$
- Standard deviation is a similar measurement, which is simply the square root of variance



In short

- Expected Value is the average of a random variable and is often used as a “best guess” for the result
- Variance and standard deviation are used to measure how close this guess would generally be

Law of Large Numbers

Law of Large Numbers

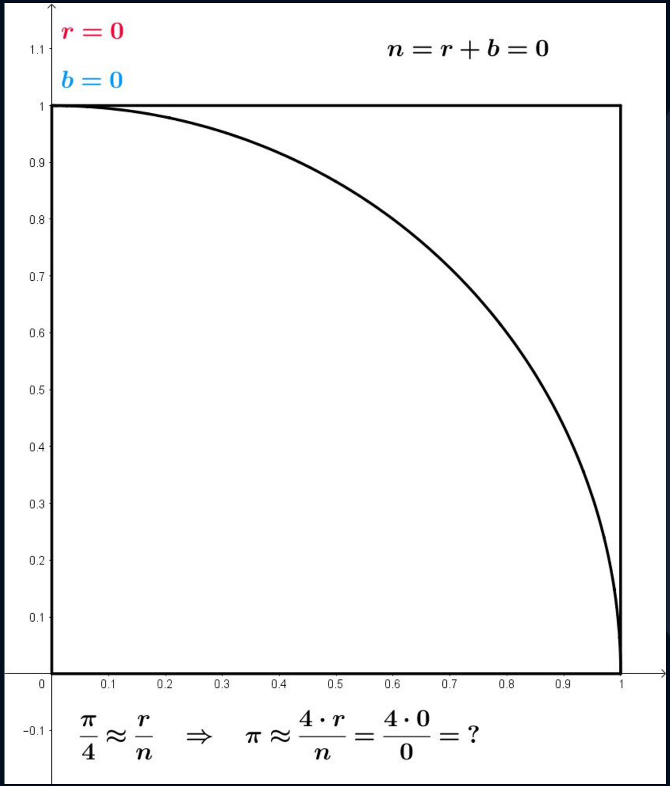
- X is a random variable with mean μ with some distribution
- Independent samples $X_1, X_2, \ldots X_n$ are drawn from the distribution.
- "Sample mean" $\bar{X}$ is the average of these samples.
- It can be shown that $E[\bar{X}] = \mu$ and $Var[\bar{X}] = Var[X]/n$
- The law of large numbers states that this sample mean $\bar{X}$ is guaranteed to approach the actual mean μ as n (the number of samples) approaches infinity

Law of Large Numbers

- This gives us another intuitive way to think of the mean - the average of infinite hypothetical trials
- The law of large numbers also conveys a powerful idea: with enough data points, you can accurately estimate properties of random processes, even if their underlying distributions are unknown
- With the ability of computation, certain quantities are now much easier to compute

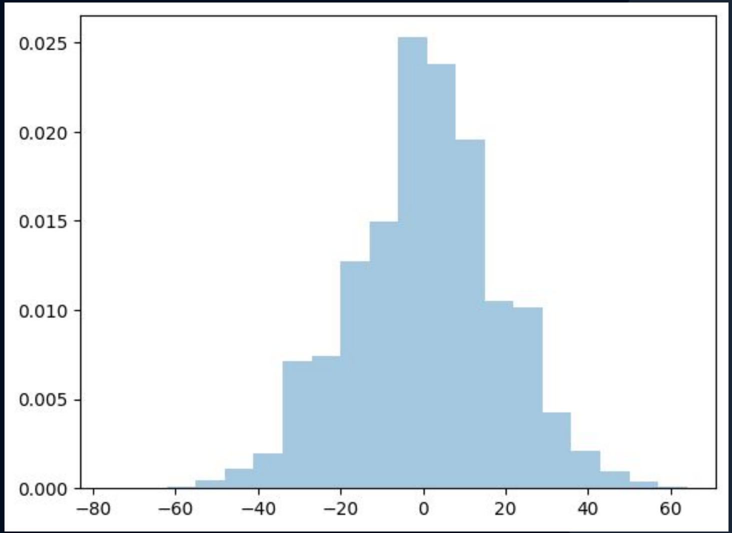
Monte Carlo Estimation

- Style of computational methods to calculate a result using repeated random sampling and averaging (aka a direct application of LLN)
- Ex: estimating area using geometric probability



Monte Carlo Estimation

- Another example is the way we run Pokerbots matches!
- Dividing the cumulative deltas by 1000 is a Monte Carlo estimate for the expected number of chips gained in a single round
- Therefore a good pokerbot should try to maximize the expected number of chips earned within a single hand



Summary

Summary

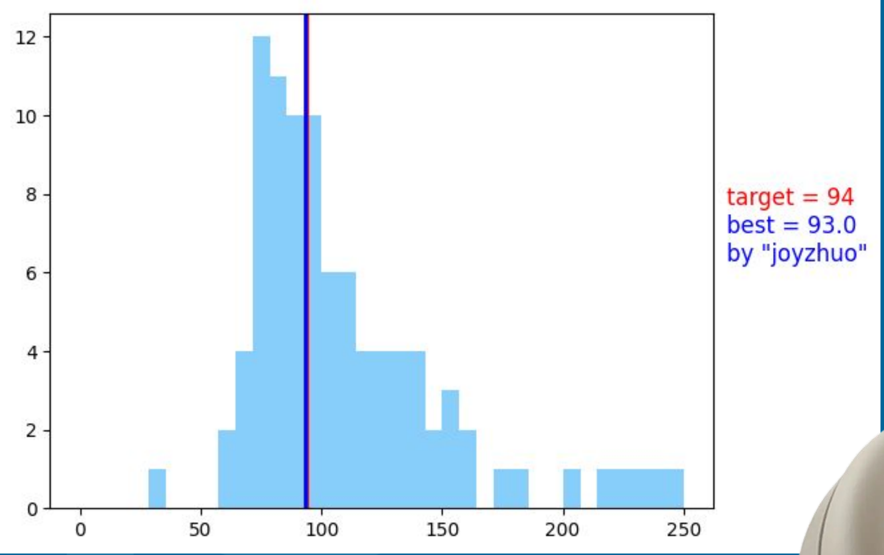
- Probability is a tool for describing uncertain situations
- In particular, Expected Value gives us a numerical estimate of an average scenario
- Law of Large Numbers and Monte Carlo use generated samples to make some calculations easy with computation
- If you'd like to learn how to use numerical/statistical packages to do some of these computations, come to recitation tomorrow!

Lunch Time!

Leave any type of feedback at **pkr.bot/feedback**!

Live Coding Session

Estimation Game results



Answer: 94
‘93’ was guessed by ‘joyzhuo’ and ‘kevinmz’ in that order, 43 seconds apart

Thanks for watching!

Slides will be posted on **pkr.bot/resources**

Make sure to check **pkr.bot/piazza** for updates

Lecture recordings at **pkr.bot/panopto**

Leave feedback at **pkr.bot/feedback**!