Computation on NumPy Arrays: Universal Functions

The reasons that NumPy is so important in the Python data science world. Namely, it provides an easy and flexible interface to optimized computation with arrays of data.

Computation on NumPy arrays can be very fast, or it can be very slow. The key to making it fast is to use *vectorized* operations, generally implemented through NumPy's *universal functions* (ufuncs). This section motivates the need for NumPy's ufuncs, which can be used to make repeated calculations on array elements much more efficient. It then introduces many of the most common and useful arithmetic ufuncs available in the NumPy package.

The Slowness of Loops

Python's default implementation (known as CPython) does some operations very slowly. This is in part due to the dynamic, interpreted nature of the language: the fact that types are flexible, so that sequences of operations cannot be compiled down to efficient machine code as in languages like C and Fortran. Recently there have been various attempts to address this weakness: well-known examples are the PyPy project, a just-in-time compiled implementation of Python; the Cython project, which converts Python code to compilable C code; and the Numba project, which converts snippets of Python code to fast LLVM bytecode. Each of these has its strengths and weaknesses, but it is safe to say that none of the three approaches has yet surpassed the reach and popularity of the standard CPython engine.

The relative sluggishness of Python generally manifests itself in situations where many small operations are being repeated – for instance looping over arrays to operate on each element. For example, imagine we have an array of values and we'd like to compute the reciprocal of each. A straightforward approach might look like this:

In [2]:

```
import numpy as np
np.random.seed(0)

def compute_reciprocals(values):
    output = np.empty(len(values))
    for i in range(len(values)):
        output[i] = 1.0 / values[i]
    return output
```

```python
values = np.random.randint(1, 10, size=5)
compute_reciprocals(values)
```

Out[2]:
```
array([0.16666667, 1.        , 0.25      , 0.25      , 0.125     ])
```

This implementation probably feels fairly natural to someone from, say, a C or Java background. But if we measure the execution time of this code for a large input, we see that this operation is very slow, perhaps surprisingly so! We'll benchmark this with IPython's `%timeit` magic :

In [2]:
```python
big_array = np.random.randint(1, 100, size=1000000)
%timeit compute_reciprocals(big_array)
```

```
1 loop, best of 3: 2.91 s per loop
```

It takes several seconds to compute these million operations and to store the result! When even cell phones have processing speeds measured in Giga-FLOPS (i.e., billions of numerical operations per second), this seems almost absurdly slow.

Introducing UFuncs

For many types of operations, NumPy provides a convenient interface into just this kind of statically typed, compiled routine. This is known as a *vectorized* operation. This can be accomplished by simply performing an operation on the array, which will then be applied to each element. This vectorized approach is designed to push the loop into the compiled layer that underlies NumPy, leading to much faster execution.

Compare the results of the following two:

In [3]:
```python
print(compute_reciprocals(values))
print(1.0 / values)
```

```
[ 0.16666667  1.          0.25        0.25        0.125     ]
[ 0.16666667  1.          0.25        0.25        0.125     ]
```

Looking at the execution time for our big array, we see that it completes orders of magnitude faster than the Python loop:

In [4]:
```python
%timeit (1.0 / big_array)
```

```
100 loops, best of 3: 4.6 ms per loop
```

Vectorized operations in NumPy are implemented via *ufuncs*, whose main purpose is to quickly execute repeated operations on values in NumPy arrays. Ufuncs are extremely flexible – before we saw an operation between a scalar and an array, but we can also operate between two arrays:

In [5]:
```
np.arange(5) / np.arange(1, 6)
```

Out[5]:
```
array([ 0.        ,  0.5       ,  0.66666667,  0.75      ,  0.8       ])
```

And ufunc operations are not limited to one-dimensional arrays–they can also act on multi-dimensional arrays as well:

In [6]:
```
x = np.arange(9).reshape((3, 3))
2 ** x
```

Out[6]:
```
array([[  1,   2,   4],
       [  8,  16,  32],
       [ 64, 128, 256]])
```

Computations using vectorization through ufuncs are nearly always more efficient than their counterpart implemented using Python loops, especially as the arrays grow in size. Any time you see such a loop in a Python script, you should consider whether it can be replaced with a vectorized expression.

Exploring NumPy's UFuncs

Ufuncs exist in two flavors: *unary ufuncs*, which operate on a single input, and *binary ufuncs*, which operate on two inputs. We'll see examples of both these types of functions here.

Array arithmetic

NumPy's ufuncs feel very natural to use because they make use of Python's native arithmetic operators. The standard addition, subtraction, multiplication, and division can all be used:

In [7]:
```
x = np.arange(4)
print("x     =", x)
print("x + 5 =", x + 5)
```

```
print("x - 5 =", x - 5)
print("x * 2 =", x * 2)
print("x / 2 =", x / 2)
print("x // 2 =", x // 2)  # floor division
```

```
x     = [0 1 2 3]
x + 5 = [5 6 7 8]
x - 5 = [-5 -4 -3 -2]
x * 2 = [0 2 4 6]
x / 2 = [ 0.   0.5  1.   1.5]
x // 2 = [0 0 1 1]
```

There is also a unary ufunc for negation, and a `**` operator for exponentiation, and a `%` operator for modulus:

In [8]:
```
print("-x     = ", -x)
print("x ** 2 = ", x ** 2)
print("x % 2  = ", x % 2)
```

```
-x     =  [ 0 -1 -2 -3]
x ** 2 =  [0 1 4 9]
x % 2  =  [0 1 0 1]
```

In addition, these can be strung together however you wish, and the standard order of operations is respected:

In [9]:
```
-(0.5*x + 1) ** 2
```

Out[9]:
```
array([-1.  , -2.25, -4.  , -6.25])
```

Each of these arithmetic operations are simply convenient wrappers around specific functions built into NumPy; for example, the `+` operator is a wrapper for the `add` function:

In [10]:
```
np.add(x, 2)
```

Out[10]:
```
array([2, 3, 4, 5])
```

The following table lists the arithmetic operators implemented in NumPy:

Operator	Equivalent ufunc	Description
`+`	`np.add`	Addition (e.g., `1 + 1 = 2`)
`-`	`np.subtract`	Subtraction (e.g., `3 - 2 = 1`)
`-`	`np.negative`	Unary negation (e.g., `-2`)
`*`	`np.multiply`	Multiplication (e.g., `2 * 3 = 6`)
`/`	`np.divide`	Division (e.g., `3 / 2 = 1.5`)
`//`	`np.floor_divide`	Floor division (e.g., `3 // 2 = 1`)
`**`	`np.power`	Exponentiation (e.g., `2 ** 3 = 8`)
`%`	`np.mod`	Modulus/remainder (e.g., `9 % 4 = 1`)

Absolute value

Just as NumPy understands Python's built-in arithmetic operators, it also understands Python's built-in absolute value function:

In [11]:
```
x = np.array([-2, -1, 0, 1, 2])
abs(x)
```

Out[11]:
```
array([2, 1, 0, 1, 2])
```

The corresponding NumPy ufunc is `np.absolute`, which is also available under the alias `np.abs`:

In [12]:
```
np.absolute(x)
```

Out[12]:
```
array([2, 1, 0, 1, 2])
```

In [13]:
```
np.abs(x)
```

Out[13]:
```
array([2, 1, 0, 1, 2])
```

This ufunc can also handle complex data, in which the absolute value returns the magnitude:

In [14]:
```
x = np.array([3 - 4j, 4 - 3j, 2 + 0j, 0 + 1j])
np.abs(x)
```

Out[14]:
```
array([ 5.,  5.,  2.,  1.])
```

Trigonometric functions

NumPy provides a large number of useful ufuncs, and some of the most useful for the data scientist are the trigonometric functions.

In [15]:
```
theta = np.linspace(0, np.pi, 3)
```

Now we can compute some trigonometric functions on these values:

In [16]:
```
print("theta      = ", theta)
print("sin(theta) = ", np.sin(theta))
print("cos(theta) = ", np.cos(theta))
print("tan(theta) = ", np.tan(theta))
```

```
theta      =  [ 0.          1.57079633  3.14159265]
sin(theta) =  [  0.00000000e+00   1.00000000e+00   1.22464680e-16]
cos(theta) =  [  1.00000000e+00   6.12323400e-17  -1.00000000e+00]
tan(theta) =  [  0.00000000e+00   1.63312394e+16  -1.22464680e-16]
```

The values are computed to within machine precision, which is why values that should be zero do not always hit exactly zero. Inverse trigonometric functions are also available:

In [17]:
```
x = [-1, 0, 1]
print("x         = ", x)
print("arcsin(x) = ", np.arcsin(x))
print("arccos(x) = ", np.arccos(x))
print("arctan(x) = ", np.arctan(x))
```

```
x         =  [-1, 0, 1]
arcsin(x) =  [-1.57079633  0.          1.57079633]
arccos(x) =  [ 3.14159265  1.57079633  0.        ]
arctan(x) =  [-0.78539816  0.          0.78539816]
```

Exponents and logarithms

Another common type of operation available in a NumPy ufunc are the exponentials:

In [18]:
```
x = [1, 2, 3]
print("x     =", x)
print("e^x   =", np.exp(x))
print("2^x   =", np.exp2(x))
print("3^x   =", np.power(3, x))
```

```
x     = [1, 2, 3]
e^x   = [  2.71828183   7.3890561   20.08553692]
2^x   = [ 2.   4.   8.]
3^x   = [ 3  9 27]
```

The inverse of the exponentials, the logarithms, are also available. The basic `np.log` gives the natural logarithm; if you prefer to compute the base-2 logarithm or the base-10 logarithm, these are available as well:

In [19]:
```
x = [1, 2, 4, 10]
print("x        =", x)
print("ln(x)    =", np.log(x))
print("log2(x)  =", np.log2(x))
print("log10(x) =", np.log10(x))
```

```
x        = [1, 2, 4, 10]
ln(x)    = [ 0.          0.69314718  1.38629436  2.30258509]
log2(x)  = [ 0.          1.          2.          3.32192809]
log10(x) = [ 0.          0.30103     0.60205999  1.        ]
```

There are also some specialized versions that are useful for maintaining precision with very small input:

In [20]:
```
x = [0, 0.001, 0.01, 0.1]
print("exp(x) - 1 =", np.expm1(x))
print("log(1 + x) =", np.log1p(x))
```

```
exp(x) - 1 = [ 0.          0.0010005   0.01005017  0.10517092]
log(1 + x) = [ 0.          0.0009995   0.00995033  0.09531018]
```

When `x` is very small, these functions give more precise values than if the raw `np.log` or `np.exp` were to be used.

Advanced Ufunc Features

Many NumPy users make use of ufuncs without ever learning their full set of features. Below outline is a few specialized features of ufuncs here.

Specifying output

For large calculations, it is sometimes useful to be able to specify the array where the result of the calculation will be stored. Rather than creating a temporary array, this can be used to write computation results directly to the memory location where you'd like them to be. For all ufuncs, this can be done using the `out` argument of the function:

In [24]:
```
x = np.arange(5)
y = np.empty(5)
np.multiply(x, 10, out=y) #out=y tells NumPy to store the result directly into y
#This avoids allocating a new array (good for performance)
print(y)
```

```
[ 0. 10. 20. 30. 40.]
```

This can even be used with array views. For example, we can write the results of a computation to every other element of a specified array:

In [25]:
```
y = np.zeros(10)
np.power(2, x, out=y[::2]) ## fill every second element of y
print(y)
```

```
[ 1.  0.  2.  0.  4.  0.  8.  0. 16.  0.]
```

If we had instead written `y[::2] = 2 ** x`, this would have resulted in the creation of a temporary array to hold the results of `2 ** x`, followed by a second operation copying those values into the `y` array. This doesn't make much of a difference for such a small computation, but for very large arrays the memory savings from careful use of the `out` argument can be significant.

Aggregates

For binary ufuncs, there are some interesting aggregates that can be computed directly from the object. For example, if we'd like to *reduce* an array with a particular operation, we can use the `reduce` method of any ufunc. A reduce repeatedly applies a given operation to the elements of an array until only a single result remains.

For example, calling `reduce` on the `add` ufunc returns the sum of all elements in the array:

In [3]:
```
x = np.arange(1, 6)
np.add.reduce(x)
#It performs a reduction operation using the np.add function (i.e., summing values), similar to: np.sum(x)
#((((1 + 2) + 3) + 4) + 5) = 15
```

Out[3]:
```
15
```

Similarly, calling `reduce` on the `multiply` ufunc results in the product of all array elements:

In [4]:
```
np.multiply.reduce(x)
#Product: 1×2×3×4×5 = 120
```

Out[4]:
```
120
```

If we'd like to store all the intermediate results of the computation, we can instead use `accumulate`:

In [28]:
```
np.add.accumulate(x)
#This performs a cumulative sum (also called a prefix sum) — that is, it applies the addition operation step-by-step and keeps
'''
Step 1: 1
Step 2: 1 + 2 = 3
Step 3: 3 + 3 = 6
Step 4: 6 + 4 = 10
Step 5: 10 + 5 = 15
'''
```

Out[28]:
```
array([ 1,  3,  6, 10, 15])
```

In [29]:
```
np.multiply.accumulate(x)
```

Out[29]:
```
array([  1,   2,   6,  24, 120])
```

Note that for these particular cases, there are dedicated NumPy functions to compute the results (`np.sum` , `np.prod` , `np.cumsum` , `np.cumprod`), which we'll explore in Aggregations: Min, Max, and Everything In Between.

Outer products

Finally, any ufunc can compute the output of all pairs of two different inputs using the `outer` method. This allows you, in one line, to do things like create a multiplication table:

In [6]:
```
x = np.arange(1, 5)
np.multiply.outer(x, x)
#This creates a multiplication table using the outer product of the array with itself. Let’s break it down.
#result[i,j]=x[i]×x[j]
```

Out[6]:
```
array([[ 1,  2,  3,  4],
       [ 2,  4,  6,  8],
       [ 3,  6,  9, 12],
       [ 4,  8, 12, 16]])
```