

# 1 Prescriptive Modeling - Linear Programming

## 1.1 Transportation Problem

```
1 # We are using the JuMP Package for Optimization
2 using JuMP
3
4 # Start with a new Model
5 m = Model()
6
7 # List of Plants
8 plants = ["P1", "P2", "P3"]
9
10 # List of Regions
11 regions = ["D1", "D2"]
12
13 # Variables for the Linear Program
14 @variable(m, x[plants, regions] >= 0);
15
16 # Capacity for each plant
17 capacity = Dict([("P1", 4000), ("P2", 4000), ("P3", 4000)])
18
19 # Demand for each region
20 demand = Dict([("D1", 4000), ("D2", 4000)])
21
22
23 # Transportation costs from plants to regions per unit of
    supply
24 costs = Dict([("P1", Dict([("D1", 22), ("D2", 22)])),
25               ("P2", Dict([("D2", 22), ("D3", 22)])),
26               ("P3", Dict([("D1", 22), ("D3", 22)]))])
27
28 # Objective Function Definition - Minimize Transportation
    cost
29 @objective(m, Min,
30 sum{x[i,j]*costs[i][j], i=plants, j= regions})
31
32 # Demand Constraint
33 for j in regions
34     @constraint(m, sum{x[i,j], i=plants} >= demand[j])
35 end
36
37 # Supply Constraint
38 for i in plants
39     @constraint(m, sum{x[i,j], j=regions} <= capacity[i])
40 end
41
42 print(m)
43
44 # Solve
45 solve(m)
46 getvalue(x)
```

```
47 print(m.objVal)
```

Listing 1: Transportation Problem

## 1.2 Transshipment Problem

```
1 using JuMP
2
3 m = Model()
4
5 plants = ["p1", "p2"]
6 dcs = ["a", "b"]
7 regions = ["r1", "r2", "r3"]
8
9 plants_dc = Dict([("plant1", Dict([("a", 44), ("b", 55)])),
10 ("plant2", Dict([("a", 43), ("b", 11)]))])
11 dc_regions = Dict([("a", Dict([("r1", 55), ("r2", 55), ("r3",
12 , 44)])),
13 ("b", Dict([("r1", 33), ("r2", 44), ("r3", 55)]))])
14
15 supply = Dict([("plant1", 222), ("plant2", 444)])
16 demand = Dict([("r1", 444), ("r2", 444), ("r3", 444)])
17
18 # From plants to DCs
19 @variable(m, x[plants, dcs] >= 0)
20 # From DCs to regions
21 @variable(m, y[dc, regions] >= 0)
22
23 # Include both costs
24 @objective(m, Min, sum{x[i,j]*plants_dc[i][j], i=plants, j=
25 dcs} + sum{y[i,j]*dc_regions[i][j], i=dc, j=regions})
26
27 # Supply Constraint
28 for i in plants
29     @constraint(m, sum{x[i,j], j=dc} <= supply[i])
30 end
31
32 # Demand Constraint
33 for j in regions
34     @constraint(m, sum{y[i,j], i=dc} >= demand[j])
35 end
36
37 # Conservation Constraint
38 for i in dc
39     @constraint(m, sum{x[k,i], k=plants} - sum{y[i,j], j=
40 regions} == 0)
41 end
42
43 # Solve
44 solve(m)
45 getvalue(x)
46 getvalue(y)
47 print(m.objVal)
```

Listing 2: Transshipment Problem

### 1.3 Fixed costs for setting up DCs

```
1 using JuMP
2
3 m = Model()
4 plants = ["P1", "P2", "P3"]
5 dcs = ["D1", "D2"]
6 M = 1000000000
7
8 # Linear variable
9 @variable(m, x[plants, dcs] >= 0)
10 # Binary Variable
11 @variable(m, y[plants], Bin)
12
13 # Transportation Costs
14 costs = Dict([("P1", Dict([("D1", 22), ("D2", 22)])),
15              ("P2", Dict([("D2", 22), ("D3", 22)])),
16              ("P3", Dict([("D1", 22), ("D3", 22)]))])
17
18 # Supply Constraint values
19 capacities = Dict([("P1", 22), ("P2", 22), ("P3", 22)]);
20
21 # Demand Constraint values
22 demands = Dict([("D1", 22), ("D2", 22)])
23
24 # Fixed costs
25 fixed_costs = Dict([("P1", 22), ("P2", 22), ("P3", 2)])
26
27
28 # Minimize total costs + Fixed cost
29 @objective(m, Min, sum{x[i,j]*costs[i][j], i=plants, j=dcs} +
30               sum{y[i]*fixed_costs[i], i=plants})
31
32 # Supply Constraint
33 for (i,k) in capacities
34     @constraint(m, sum{x[i,j], j=dcs} <= capacities[i])
35 end
36
37 # Demand Constraints
38 for (j,k) in demands
39     @constraint(m, sum{x[i,j], i=plants} >= k)
40 end
41
42 # Linking Constraints
43 for i in plants
44     @constraint(m, sum{x[i,j], j=dcs} - M*y[i] <= 0)
45 end
46
47 # Solve
48 print(m)
49 solve(m)
50 getvalue(x)
```

```
50  getvalue(y)
```

Listing 3: Fixed Cost Problem - MILP

## 2 Managing Uncertainty - Distributions and Random Variables

### 2.1 Summary Statistics

```
1 # Read Data
2 df <- read.csv("data.csv")
3
4 # Show structure of data
5 str(df)
6
7 # Show summary
8 summary(df)
9
10 # Standard deviation
11 sd(df$col)
```

Listing 4: Read data from csv file, general stats

### 2.2 Distributions

- $p$  for "probability", the cumulative distribution function (c. d. f.)
- $q$  for "quantile", the inverse c. d. f.
- $d$  for "density", the density function (p. f. or p. d. f.)
- $r$  for "random", a random variable having the specified distribution

```
1
2
3 # Calculate p.f.
4 dnorm(x, mean=mu, sd=sigma)
5
6 # Calculate cdf
7 pnorm(p, mean=mu, sd=sigma)
8
9 # Get the q quantile from probability p
10 qnorm(p, mean=mu, sd=sigma, lower.tail=FALSE)
11
12 # Generate n random numbers picked from this distribution
13 rnorm(n, mean=mu, sd=sigma)
```

Listing 5: Probability Distributions

## 3 Predictive Modeling

### 3.1 Hypothesis Tests

```
1
2 # T test for hypothesis
3 t.test(x, alternative=c("two.sided", "less", "greater"), mu=
  testMean, conf.level=0.95)
4
5 # Calculate Goodness of fit using Chi-square
6 chisq.test(x, p=probablilities)
```

Listing 6: Hypothesis Tests

### 3.2 Ordinary Least Squares Linear Regression

```
1
2 # least square
3 result <- lm(Dependent ~ Independent Variable1 + Independent
  Variable 2, data=data)
4
5 # Get the result
6 summary(result)
7
8 # Predict using model
9 predict(result, data.frame(v1=x, v2=y))
```

Listing 7: Regression

## 4 Descriptive Modeling

### 4.1 Queue Model

Indebted to *nirski* for recommending *simmer*

```
1 library(simmer)
2 library(triangle)
3 library(DT)
4
5 set.seed(1234)
6
7 # Create customer
8 customer <- create_trajectory("Customer Path") %>%
9   seize("Kiosk", 1) %>%
10   timeout(function() rtriangle(1, a=0.5, b=15, c=5)) %>%
11   release("Kiosk", 1)
12
13 logan <- simmer("Airport") %>%
14   add_resource("Kiosk", capacity=6, queue_size=16) %>%
15   add_generator("Customer", customer, function() rexp(1, 0.5))
16
17 logan %>% run(until=9000)
18
19 average_times <- function(m) {
20   m %>% get_mon_arrivals(per_resource = TRUE) %>%
21     mutate(
22       flow_time = end_time - start_time,
23       waiting_time = flow_time - activity_time
24     ) %>%
25     group_by(resource) %>%
26     summarise_each(funs(mean), flow_time, activity_time,
27                   waiting_time)
28 }
29
30 resource_popularity <- function(m) {
31   m %>% get_mon_arrivals(per_resource = TRUE) %>%
32     group_by(name) %>%
33     summarise(resource = resource %>% paste(collapse = " -> "))
34   %>%
35     count(resource)
36 }
37
38 logan %>% resource_popularity
39 logan %>% average_times
40 logan %>% plot_evolution_arrival_times("waiting_time")
41 logan %>% plot_resource_utilization("Kiosk")
```

Listing 8: Discrete Event Simulation



## 4.2 Simulation

```
1 runs <- 200
2
3 one.trial <- function() {
4   samples <- 1000
5   #define your simulation function
6 }
7
8 mc.trial <- replicate(runs, one.trial())
9
10 result <- mean(mc.trial) %>% round
11
12 t.test(mc.trial, conf.level=0.95)
```

Listing 9: Simulation