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MINI PROJECT REPORT

ON

STOCK MARKET TREND PREDICTION USING MARKOV CHAINS

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PROJECT EVALUATION

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Sl.No.	Parameter	Max Marks	Marks Awarded
1	Background & Framing of the problem	4	
2	Approach and Solution	4	
3	References	4	
4	Clarity of the concepts & Creativity	4	
5	Choice of examples and understanding of the topic	4	
6	Presentation of the work	5	
	Total	25	

Name of the Course Instructor :

Signature of the Course Instructor :

Stock Market Trend Prediction Using Markov Chains

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Stock market or equity market refers to a place where stocks or equities are traded. Stock market trend provides the general direction of movement to stock prices. Analysis and prediction of this stock market trend has been a major topic under study and implementation in stock business. As tempting as it sounds, stock market prediction has its own barriers due to its chaotic nature, volatility, time dependence and many other variables. We have chosen Markov Chain Model in order to significantly predict the stock market trend whilst handling its variables and dependencies. A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. The states and the probabilities of transition between every pair of states is represented in the form of a matrix. This matrix is called a stochastic matrix where each entry is a non-negative real number representing a probability. Stochastic matrix or probability transition matrix provides a foundation to all the processes involved in predict the future movement of the stock index. In this paper, we present a short review of existing tools and techniques for stock market trend prediction. We also explain the simplicity of Markov Chain Model through theory and examples and analyse the results obtained.

***Keywords—*: Markov chains, stock market, stock market trend, transition matrix**

I. INTRODUCTION

Stock markets are now an integral part of the economy of the entire world. All fluctuations in this market directly influences consumers, businesses and economy of a country. Interested investors always keep a track of the stock market trend to get a picture on the movement of stocks in the global market. Given that an appropriate investment can bring huge profits to the investor, it becomes even more important to carefully analyse and understand the current stock market trend before making any investments. Hence prediction of the stock market trend has become a crucial and popular task in this business.

Stock market trend prediction is an attempt to predict the future value a particular stock or equity traded. A successful prediction shall definitely prove to be helpful to an investor. This gives the possible trend that would prevail in the market in the future. The investor can adopt a suitable investment strategy based on the prediction of stock market trend.

As helpful as it sounds, investing in stocks based on the market prediction is still risky due to its unpredictable behaviour. The predictions are mostly based on currently available information and do not include new information. There are several variables and dependencies that make stock market trend prediction complex. Many studies have been conducted in this regard. Popular techniques like Neural networks, Support Vector Machines, Fuzzy logic have been used to predict the stock market.

In this paper, we aim to predict the stock index trend of stock indices using Markov Chain Analysis. A Markov Chain is a kind of stochastic process where the outcome of an experiment depends only on the outcome of the previous experiment. Stochastic processes are of interest for describing the behaviour of a system evolving over a period of time. In this study the first order Markov Chain Model is applied to the S&P 500 index.

II. LITERATURE SURVEY

Stock Market prediction has been one of the more active research areas in the past, given the obvious interest of a lot of major companies. In this research several machine learning techniques have been applied to varying degrees of success. However, stock forecasting is still severely limited due to its non-stationary, seasonal and in general unpredictable nature. Predicting forecasts from just the previous stock data is an even more challenging task since it ignores several outlying factors (such as the state of the company, economic conditions ownership etc.). That is why is most essential to come up with statistical models to tackle this issue. There are various statistical methods to study such phenomena like; moving average, regression analysis, Markov Chain model and etc. In this paper we use Markov Chain model to analyse this problem [6].

Many machine learning techniques which have been widely applied to forecasting stock market data include Artificial Neural Networks (ANNs), Fuzzy Logic (FL), and Support Vector Machines (SVMs). Out of these ANNs have been the most successful, however even their performance is quite limited, and not reliable enough. ANNs suffer from over-fitting problem due to the large number of parameters to fix, and the little prior user knowledge about the relevance of the inputs in the analysed problem [2]. Support vector machines (SVMs) had been developed as an alternative that avoids such limitations. Their practical successes can be attributed to solid theoretical foundations based on VC-theory [1].

A study mentioned a two state Markov Chain Model for discretized returns and convergence of Markov Chain to its steady state [5]. The application of a stochastic model like Markov Chains to the stock market helped in understanding the nature of stock price movements. The primary benefits of Markov analysis are simplicity and out-of-sample forecasting accuracy. Simple models, such as those used for Markov analysis, are often better at making predictions than more complicated models. This result is well-known in econometrics [3].

III. METHODOLOGY

The age-old problem of stock market prediction, using tools and techniques of Machine Learning has been analysed thoroughly. Time dependence, volatility and other similar complex dependencies are some interesting properties which make this modelling non-trivial. To incorporate these properties, we have chosen to implement Markov Chains. The Markov process seems to be more applicable when stock market prices are analysed for futuristic prediction. Owing to its memory-less property. It is a stochastic process. In mathematics, a stochastic matrix is a square matrix used to describe the transitions of a Markov chain. Each of its entries is a non-negative real number representing a probability. It is also called a probability matrix, transition matrix, substitution matrix, or Markov matrix. The future probabilities are determined by the immediate present and not past values. This is suitable for the random nature of stock market fluctuations. In this paper, we aim to present a comprehensive review of the existing stock market prediction methods. We discuss the foundation of the techniques to analyse results, strengths and limitations. We also discuss the datasets and the evaluation metrics popularly used in stock market prediction.

Markov chain is a special type of random process. Markov model is named after Andrei A Markov, the individual who originally distributed his outcome about the model of Markov. Markovian model is a stochastic model based on the Markovian property, which states that the future is independent of the past, given the present state. The set of values taken by the Markov process is known as state space. A Markov process having discrete state space is termed as Markov chain. The fundamental difference between the Markov chain model and other statistical methods of projection like; regression model, time series analysis is that the Markov model does not require any mutual laws among the factors from complex predictor, it only requires the characteristic of evolution on the history of an event (i.e. initial probability) to estimate the transition probability for different possible states at various time to come. Markov forms are the characteristic stochastic analogues of the deterministic procedures depicted by differential and distinction conditions. They structure one of the most significant classes of arbitrary procedures. By using the Markov chain model, it is easier to predict the possibility of state value in a certain period after knowing the initial probability distribution and transition probability matrix (TPM). Markov chain model has been extensively used in predicting stock index for a group of stock as well as for a single stock.

Definition of Markov chain:

The sequence $\{X_n, n \geq 0\}$ is said to be a Markov chain if for all state values i0, i1, i2, ……, in ϵ I,

then

$$P\left\{Xn+1 = \frac{j}{X0} = i0, X1 = i1, \dots\dots\dots\dots\dots.., Xn = i\right\}$$
$$= P\left\{Xn+1 = \frac{j}{Xn} = i\right\}$$

Where, i0, i1, i2, ……, in are the states in the state space I. This type of probability is called

Markov chain probability. This indicates that regardless of its history prior to time n, the

probability that it will make a transition to another state j depends only on the previous state i.

The Markov Property:

For any positive integer n and possible states $i_0, i_1, \dots . i_n$ of the random variables,

$$P(Xn-1 = in-1)$$
$$= P(X0 = i0, X1 = i1, \dots, Xn-1$$
$$= in-1).$$

In other words, knowledge of the previous state is the only necessary requirement to determine the probability distribution of the current state. This definition is defined as such to allow for *non-stationary transition probabilities* and therefore *time-inhomogeneous Markov chains;* that is, as time goes on (steps increase), the probability of moving from one state to another may change.

The transition Matrix:

A stochastic process {Xt} is said to have the Markovian property if P{X t+1 = j/X0 = k0, X1 = k1,.........X t-1= kt-1, Xt = i } = P{ X t+1 = j/Xi = i, for t= 0,1,......and every sequence i, j, ko, kj ,.......k t-1. In words this Markovian property says that the conditional probability of any future event, given any past event and the present state Xt = i, is independent of the past event and solely depends on the present state. The conditional probabilities P(X t+1 = j/ X t =i}=pij are called transition probabilities. They can be arranged in the form of a n x n matrix known as the Transition Probability Matrix. It is given by

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,j} & \dots & P_{1,S} \\ P_{2,1} & P_{2,2} & \dots & P_{2,j} & \dots & P_{2,S} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i,1} & P_{i,2} & \dots & P_{i,j} & \dots & P_{i,S} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{S,1} & P_{S,2} & \dots & P_{S,j} & \dots & P_{S,S} \end{bmatrix}.$$

A **transition matrix** P_t for Markov chain { X } at time *t* is a matrix containing information on the probability of transitioning between states. In particular, given an ordering of a matrix's rows and columns by the state space *S*, the $(i,j)^{th}$ element of the matrix P_t is given by

$$(Pt)i,j = P(Xt + 1 = j \mid Xt = i)$$

Given below are the properties of a Markov Chain:

- A state I has period k≥1 if any chain starting at and returning to state ii with positive probability must take a number of steps divisible by k. If k=1, then the state is known as aperiodic, and if k>1, the state is known as periodic. If all states are aperiodic, then the Markov chain is known as aperiodic.
- $P_{i,j} > 0$ for all i and j.
- A Markov chain is known as irreducible if there exists a chain of steps between any two states that has positive probability.
- An absorbing state i is a state for which $P_{i,i}$ =1. Absorbing states are crucial for the discussion of absorbing Markov chains.
- A state is known as recurrent or transient depending upon whether or not the Markov chain will eventually return to it. A recurrent state is known as positive recurrent if it returns within a finite number of steps, and null recurrent otherwise.
- Each row of the matrix is a probability vector, and the sum of its entries is 1. This is so because the sum represents total probability of transition from state i to itself or any other state.
- The diagonal element represents transition from one state to the same state.

IV. OBJECTIVES OF THE STUDY

In this study, we aim to predict the stock market trend of the S&P market index using the First Order Markov Chain Model. The results of the trend prediction using Markov Chain Model is compared with the results obtained through traditional trend prediction tools. Short term (one-year data), medium term (3-year data) and long term (7-year data) are used to predict stock market trend using Markov Chain Model and the respective results are compared.

V. DATASET

In this study we will use the data from S&P 500 market Index. It is a stock market index that measures the stock performance of 500 large companies listed on stock exchanges in the US States.

In its raw form, this data represents a sequence of many events leading to the last quoted price. Its attributes include Date, Open, High, Low, Adj close and Volume.

VI. IMPLEMENTATION

We use the S&P 500 Futures historical market data taken from finance.yahoo.com. This dataset includes information of the open, high, low, close values, and volume of shares. In this analysis, we use the closing stock value for prediction.

The stock value returns at time n is given as,

$$R = \frac{sp(n) - sp(n-1)}{sp(n-1)}$$

Where

1) sp(n) is today's closing price
2) sp(n-1) is yesterday's closing price

We define a set of random variables.

Bull(indicating highs), R>0,
Bear(indicating lows), R<0

Assuming that R follows a stationary first order Markov chain and future movement of R depends only on its current state.

To apply the Markov process to stock market behaviour, the stock value can be viewed as a system toggling between highs and lows. Highs and lows are major reference points for stock traders. In daily charts they are regarded as key points. The reason for this is their close link to the trend definition. **Higher highs + higher lows** define an **upward trend**. **Lower highs + lower lows** define a **downward trend**.

Many traders pay attention to the progress of these trends. If the market price succeeds in surpassing yesterday's high, the market is called bullish. This is especially true if there is an upward trend on the daily chart, as well. Vice versa, if the market price reverses below yesterday's low, the market is bearish, particularly when complemented by a downtrend in a higher time frame. To mark this behaviour, in this analysis we implement 4 states.

These four states are denoted by the symbols HH, HL, LH, LL. Where HH indicates "Higher Highs", HL indicates "Higher Lows", LH indicates "Lower Highs" and LL indicates "Lower Lows".

The states are not directly observable. The situations in the stock market are considered hidden. Given a sequence of observations we can find the hidden state sequence that produced those observations.

We construct a state transition probability matrix from the past behaviour of the system and this transition probability matrix in conjunction with the probability values of the present state of the system is used to determine the probabilities of the next state. A 4x4 Transition probability matrix of R gives the probability of all possible transitions between all possible states.

Let this transition probability matrix of R be

$$P = \begin{array}{c} \\ HH \\ HL \\ LH \\ LL \end{array} \begin{array}{c} \begin{array}{cccc} HH & HL & LH & LL \end{array} \\ \begin{bmatrix} p11 & p12 & p13 & p14 \\ p21 & p22 & p23 & p24 \\ p31 & p32 & p33 & p34 \\ p41 & p42 & p43 & p44 \end{bmatrix} \end{array}$$

P_{ij} is the conditional probability of moving into state j from state i.

$0< P_{ij} <1$ and $P_{i1} + P_{i2} + P_{i3} + P_{i4} = 1$ for i = 1, 2, 3, 4; j = 1, 2, 3,4

Let the unique state probability vector for an observation period n be $x(n)=[x1 \quad x2 \quad x3 \quad x4]$ where, xi = probability that the system is in the i^{th} state at the time of observation.

$0< xi <1$ and $x1 + x2 + x3 + x4 = 1$ for i = 1, 2, 3, 4.

The steady-state probabilities for period n+1 is obtained by multiplying the known state probabilities for period n by the transition probability matrix. Using the vector of state probabilities and the matrix of transition probabilities, the multiplication can be expressed as follows:

x(next period) = x(current period) * P

(or)

$x(n+1) = x(n) * P$

In general ,

$x(n+1) = x(0) * P^{n+1}$

For the development of the model, the closing value of stock is divided into several intervals, using frequency distribution, which helps to find out the frequencies of its interval constructed. These intervals are distinct and non-overlapping and are essentially the different states (HH, HL, LH, LL) of Markov Chain. To formulate the transition matrix, the number of transitions from one interval to another interval is counted. This transition frequency matrix provides the transition probability matrix of Markov Chain. The transition probability matrix can be constructed by dividing each class frequency by its total class frequency.

The stock market's closing value for 252 days is taken for the study from the S&P Dataset. During this period the closing value ranges between 2237 to 3385. The frequency distribution table is constructed to establish the intervals that represent the states of Markov Chain. The frequency distribution table is represented below.

FREQUENCY DISTRIBUTION TABLE

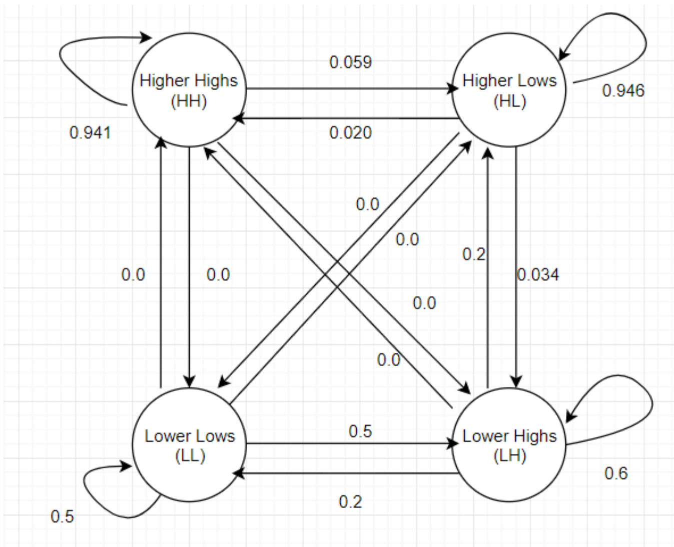
States	**Range of closing value**	**Frequency**
HH	3099-3385	68
HL	2812-3098	149
LH	2525-2811	25
LL	2237-2524	10

The transition frequencies for the closing value is counted for each interval and represented below.

	HH	**HL**	**LH**	**LL**	**Total**
HH	64	4	0	0	68
HL	3	141	5	0	149
LH	0	5	15	5	25
LL	0	0	5	5	10

From this the transition probability matrix is constructed by dividing each class frequency by each total class frequency. Thus p11 = 64/68=0.941, p12=4/68=0.059 and so on. The transition probability matrix we get is as follows.

$$P = \begin{array}{c} \\ HH \\ HL \\ LH \\ LL \end{array} \begin{array}{c} \begin{array}{cccc} HH & HL & LH & LL \end{array} \\ \begin{bmatrix} 0.941 & 0.059 & 0.0 & 0.0 \\ 0.020 & 0.946 & 0.034 & 0.0 \\ 0.0 & 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.5 & 0.5 \end{bmatrix} \end{array}$$



The initial state vector x(0) is obtained by observing the last day's price interval.From the data the start state for future prediction is found to be 'Lower Highs' i.e.

X(0)= [0 0 1 0]

Let this be the starting trend for future stock value prediction.

Prediction of Stock trends :

For Day 1:

x(1) = x(0) * P

$$x(1) = [0 \quad 0 \quad 1 \quad 0] \begin{bmatrix} 0.941 & 0.059 & 0.0 & 0.0 \\ 0.020 & 0.946 & 0.034 & 0.0 \\ 0.0 & 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.5 & 0.5 \end{bmatrix}$$

x(1) = [0.0 0.2 0.6 0.2]

The state probabilities $x_1(1) = 0.0$, $x_2(1) = 0.2$, $x_3(1) = 0.6$, $x_4(1) = 0.2$ are the probabilities that indicate that a lower high or a downward trend the previous day will follow the same trend with the highest probability of 60% during the 1st day. In other words, on the 1st day the end state is 'Lower Highs' with the highest probability of 60%.

Similarly using the same equation the state probabilities for the second day, the third day can be computed as follows:

For Day 2:

x(2) = x(1) * P

$$x(2) = [0 \quad 0.2 \quad 0.6 \quad 0.\,2] \begin{bmatrix} 0.941 & 0.059 & 0.0 & 0.0 \\ 0.020 & 0.946 & 0.034 & 0.0 \\ 0.0 & 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.5 & 0.5 \end{bmatrix}$$

x(2) = [0.004 0.3092 0.4668 0.22]

From x(2) it can be noticed that:

1)The probability of starting at state 'Lower Highs' and ending at state 'Higher Highs' at the end of day 2= 0.4%
2) The probability of starting at state 'Lower Highs' and ending at state 'Higher Lows' at the end of day 2= 30.92%
3) The probability of starting at state 'Lower Highs' and ending at state 'Lower Highs' at the end of day 2= 46.68%
4) The probability of starting at state 'Lower Highs' and ending at state 'Lower Lows' at the end of day 2= 22%

For Day 3:

x(3)=x(2) * P

x(3) =

$$[0.004 \quad 0.3092 \quad 0.4668 \quad 0.22] \begin{bmatrix} 0.941 & 0.059 & 0.0 & 0.0 \\ 0.020 & 0.946 & 0.034 & 0.0 \\ 0.0 & 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.5 & 0.5 \end{bmatrix}$$

x(3) = [0.009 0.386 0.40 0.20]

From x(3) it can be noticed that:

1)The probability of starting at state 'Lower Highs' and ending at state 'Higher Highs' at the end of day 3= 0.9%
2) The probability of starting at state 'Lower Highs' and ending at state 'Higher Lows' at the end of day 3= 38.6%
3) The probability of starting at state 'Lower Highs' and ending at state 'Lower Highs' at the end of day 3= 40%
4) The probability of starting at state 'Lower Highs' and ending at state 'Lower Lows' at the end of day 3= 20%

From the calculation, it is noticed that the state 'Lower Highs' (LH) has the highest probability for both the days. This implies that the stock value will not have any major change in these days i.e. there is 46.68% and 40% chance that the stock value will remain unchanged for day 2 and day3 respectively.

The same process is followed and step state vector of prediction for 15 days is observed as shown below.

```
x( 0 ) [0 0 1 0]
x( 1 ) [0.  0.2 0.6 0.2]
x( 2 ) [0.004  0.3092 0.4668 0.22  ]
x( 3 ) [0.009948  0.3860992 0.4005928 0.20336  ]
x( 4 ) [0.01708305 0.44595534 0.35516305 0.18179856]
x( 5 ) [0.02499426 0.49391426 0.31915959 0.16193189]
x( 6 ) [0.03339788 0.53254947 0.28925479 0.14479786]
x( 7 ) [0.0420784  0.56361323 0.26405849 0.13024989]
x( 8 ) [0.05086804 0.58847244 0.24272289 0.11793664]
x( 9 ) [0.05963627 0.60824072 0.22461012 0.1075129 ]
x( 10 ) [0.06828254 0.62383628 0.2092027  0.09867847]
x( 11 ) [0.0767306  0.63601833 0.19607129 0.09117978]
x( 12 ) [0.08492386 0.64541471 0.18485729 0.08480415]
x( 13 ) [0.09282165 0.65254428 0.17526054 0.07937353]
x( 14 ) [0.10039606 0.65783547 0.1670296  0.07473887]
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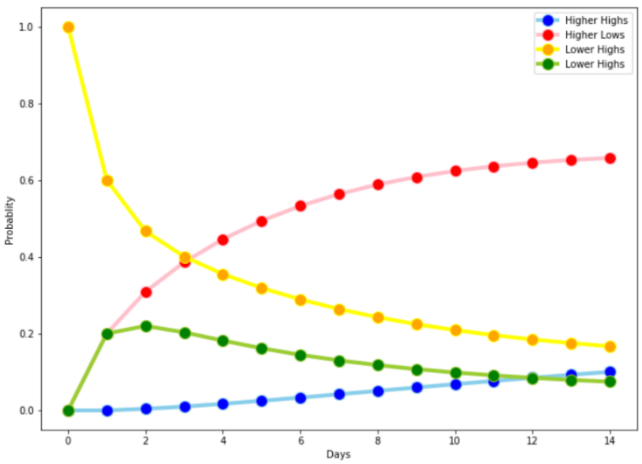

The above table and graph make it clear that when the power of the matrix (or days) increases the row elements of the matrix approach to some constant or steady-state probabilities. After a period of 15 trading days, the matrix attains the state of equilibrium or a steady-state position where each row of the matrix is almost identical. From this calculation, it can be concluded that if the closing value is initially in any one of the states: HH, HL, LH, LL then it will attain the state HH with probability 0.10, state HL with probability 0.65, and similarly irrespective of the initial state the stock value will reach the state LH with probability 0.16 and will reach state LL with a probability of 0.07. Thus, it can be said that there is approximately 75% chance of stock values increasing and a 25% chance of stock values decreasing. This forecast though is only based on the analysis done for the short period of consecutive 15 days.

VII. ANALYSIS AND INTERPRETATION

The Markov Chain model is further applied to 14 different stock indices and the prediction is carried out separately for data collected for 7 years, 3 years, and 1 year. To check for the efficiency of the Markov model in predicting the stock index trend, it is compared with the existing forecasting models like 90 Day Moving average and Trend prediction. Using the above forecasting methods, the stock index values of the selected indices are forecasted for the same period and the estimated values are converted into probability. These probabilities are then compared with the results of the Markov Chain Model.

Table 1, Table 2, and Table 3 show the accuracy obtained for each model using 7 years,3 years and 1-year data respectively.

Table 1: Stock Market Trend Prediction using First Order Markov Chain Model using 7 years data

S.No	Model	No of Trails	Accurate Predictions	Accuracy of Prediction
1	Markov Chain Model	14	8	57.14%
2	Moving Average	14	7	50%
3	Trend Projection	14	7	50%

Table 2: Stock Market Trend Prediction using First Order Markov Chain Model using 3 years data

S.No	Model	No of Trails	Accurate Predictions	Accuracy of Prediction
1	Markov Chain Model	14	11	78.57%
2	Moving Average	14	11	78.57%
3	Trend Projection	14	11	78.57%

Table 3: Stock Market Trend Prediction using First Order Markov Chain Model using 1-year data

S.No	Model	No of Trails	Accurate Predictions	Accuracy of Prediction
1	Markov Chain Model	14	12	85.71%
2	Moving Average	14	9	64.28%
3	Trend Projection	14	9	64.28%

The above analysis shows that in most cases the Markov Model outperforms other traditional models. In particular, using 1-year data or short-term data for prediction proved to produce the most accurate prediction.

VIII. CONCLUSION

Markov Chains are an essential mathematical tool that simplifies the prediction of the future state of complex stochastic processes. It depends only on the current state of the process and regards the future to be independent of the past. Utilising the Markov Property, stock prediction can be done more accurately when compared to the traditional methods.

Highest accuracy of prediction is obtained when the Markov predictions are carried on 1-year data. The superiority of the method could be because the Markov model calculates the day-to-day changes in the index values and categorizes them into the four states. Further, the transitions from one state to the other are computed using the transition probability matrix.

In the real situation, the fluctuation of stock value is influenced by many factors, therefore, the forecasting made for the future value by any one method may not be adequate but the results obtained by Markov Chain model is quite encouraging.

This model will help researchers in identifying future trends in the stock markets. It would be a useful indicator for investors to make better investment decisions. However, the present study is conducted only with the First Order Markov Chain assuming only four possible states (Higher Highs, Higher Lows, Lower Highs, Lower Lows), and further studies could be conducted using higher-order Markov chains to gain better insight into the behaviour of the market.

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