Trust Region

ll z

December 2021

1 Dogleg

Algorithm 1 *Dogleg*$(f(x), x_0)$

Require: $\hat{\Delta} > 0$, $\Delta_0 \in (0, \hat{\Delta})$, $\eta = 0$, $K > 0$
Ensure: minimizer x^*
  for $k := 0 \to K$ **do**
    $p_k^N = -B^{-1}g$
    $p_k^C = -\frac{g^T g}{g^T B g} g$
    if $p_k^N \leq \Delta_k$ **then**
      $p_k = p_k^N$
    else if $p_k^C \geq \Delta_k$ **then**
      $p_k = \Delta_k \frac{p_k^C}{\|p_k^C\|}$
    else
      solve $\tau \in (1, 2)$ where $\|p_k^C + (\tau - 1)(p_k^N - p_k^C)\|^2 = \Delta^2$
      $p_k = p_k^C + (\tau - 1)(p_k^N - p_k^C)$
    end if
    $\rho_k = \frac{f(x_k) - f(x_k + p_k)}{m_k(0) - m_k(p_k)}$
    if $\rho_k < \frac{1}{4}$ **then**
      $\Delta_{k+1} = \frac{1}{4}\Delta_k$
    else if $\rho_k > \frac{3}{4}$ and $\|p_k\| = \Delta_k$ **then**
      $\Delta_{k+1} = \min(2\Delta_k, \hat{\Delta})$
    else
      $\Delta_{k+1} = \Delta_k$
    end if
    if $\rho_k > \eta$ **then**
      $x_{k+1} = x_k + p_k$
    else
      $x_{k+1} = x_k$
    end if
  end for

2 Double Dogleg

Algorithm 2 *Double_Dogleg*$(f(x), x_0)$

Require: $\hat{\Delta} > 0$, $\Delta_0 \in (0, \hat{\Delta})$, $\eta = 0$, $K > 0$
Ensure: minimizer x^*
  for $k := 0 \to K$ **do**
    $p_k^N = -B^{-1}g$
    $p_k^C = -\frac{g^T g}{g^T B g} g$
    if $p_k^N \leq \Delta_k$ **then**
      $p_k = p_k^N$
    else if $p_k^C \geq \Delta_k$ **then**
      $p_k = \Delta_k \frac{p_k^C}{\|p_k^C\|}$
    else
      $\gamma = \frac{\|g\|^4}{(g^T B g)(g^T B^{-1} g)}$
      $\mu = 0.8\gamma + 0.2$
      $p_k^{\hat{N}} = \mu p_k^N$
      solve $\lambda \in (0, 1)$ where $\|p_k^C + \lambda(p_k^{\hat{N}} - p_k^C)\|^2 = \Delta_k^2$
      $p_k = p_k^C + \lambda(p_k^{\hat{N}} - p_k^C)$
    end if
    $\rho_k = \frac{f(x_k) - f(x_k + p_k)}{m_k(0) - m_k(p_k)}$
    if $\rho_k < \frac{1}{4}$ **then**
      $\Delta_{k+1} = \frac{1}{4}\Delta_k$
    else if $\rho_k > \frac{3}{4}$ and $\|p_k\| = \Delta_k$ **then**
      $\Delta_{k+1} = \min(2\Delta_k, \hat{\Delta})$
    else
      $\Delta_{k+1} = \Delta_k$
    end if
    if $\rho_k > \eta$ **then**
      $x_{k+1} = x_k + p_k$
    else
      $x_{k+1} = x_k$
    end if
  end for