In [1]:

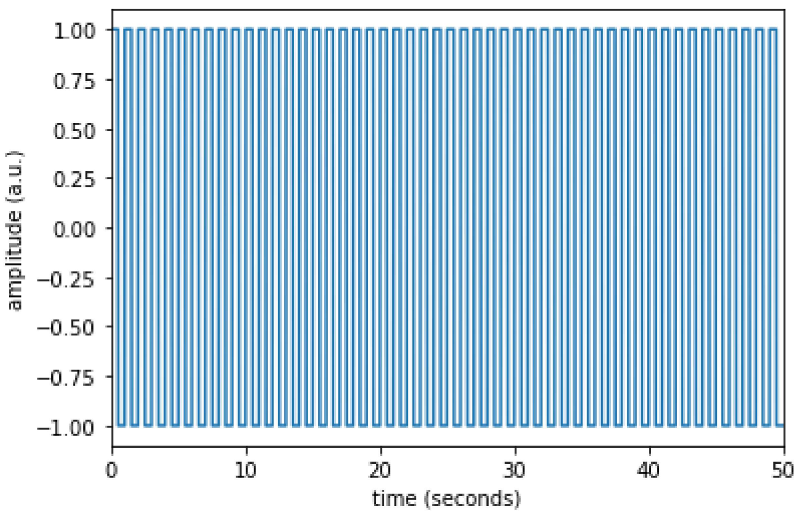
```
# In this Python tutorial we show how to compute the Fourier transform (and
# inverse Fourier transform) of a set of discrete data using 'fft()' ('ifft()')).
# FFT stands for Fast Fourier Transform.  We will first demonstrate the use
# of 'fft()' using some artificial data which shows a square wave of amplitude
# 1 as a function of time.  The period of the square wave is 1 second.  The
# data is stored in a file called "squareWave.dat".
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:

```
# Use the SciPy module to generate a square wave between +/-1 with a period of 1 s.
from scipy import signal
time = np.linspace(0, 50, 10000, endpoint=False)
amplitude = signal.square(2 * np.pi * 1 * time)
```

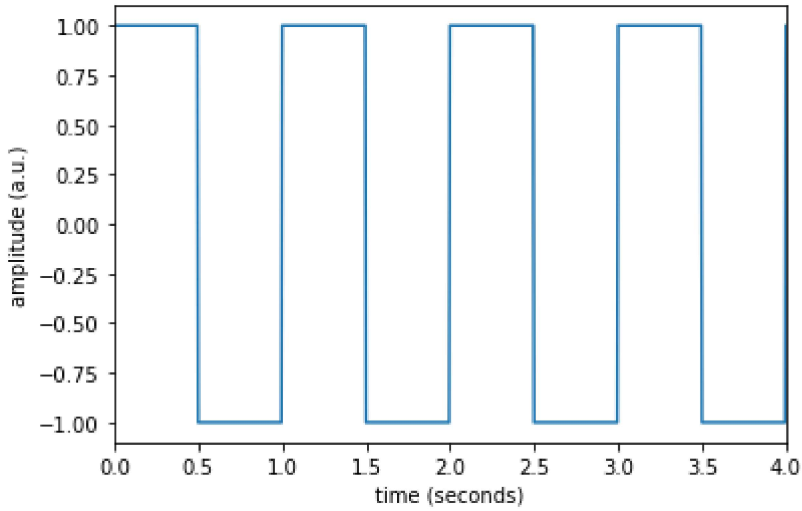
In [6]:

```
# Plot the square wave
sq_wave = plt.figure()
plt.plot(time, amplitude)
plt.axis((0, 50, -1.1, 1.1))
plt.xlabel('time (seconds)')
plt.ylabel('amplitude (a.u.)');
```



In [8]:

```
# Here's a zoomed-in view of the square wave showing the first few periods.
plt.plot(time, amplitude)
plt.axis((0, 4, -1.1, 1.1))
plt.xlabel('time (seconds)')
plt.ylabel('amplitude (a.u.)');
```



In [9]:
```
# Compute the discrete Fourier Transform of the amplitude data using 'fft()'.
# The fft function outputs a vector of complex numbers.
from scipy.fft import fft
y = fft(amplitude)
```

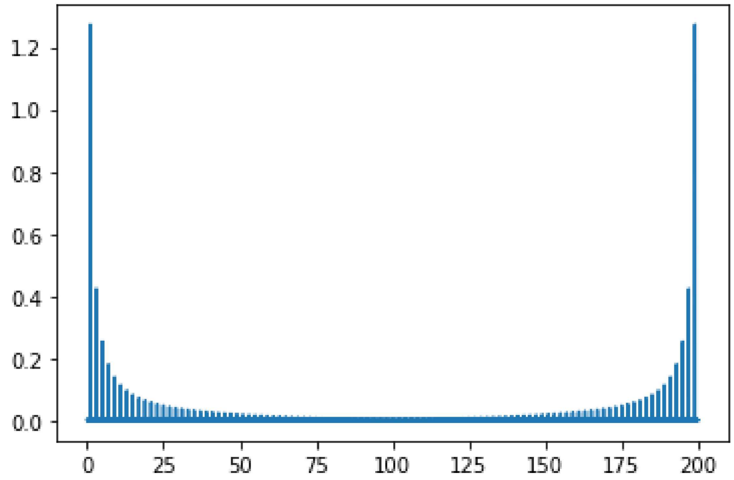
In [10]:
```
# The magnitude of the fft can be calculated by adding the squares of the
# real and imaginary components (and then taking the square root).  The
# tangent of the phase is determined by the ratio of the imaginary and real parts.
# NumPy has built-in functions 'np.abs()' and 'np.angle()' that can be used
# to calculate these quantities directly.
N = len(y)
mag = np.abs(y)
phase = np.angle(y)
```

In [11]:
```
# We have to determine the appropriate frequency scale for the x-axis.  The
# maximum frequency is set by the spacing between adjacent times.  The
# frequency step, is set by fmax/(number of points - 1).
fmax = 1/(time[1]-time[0])
fstep = fmax/(len(time) - 1)
fmax, fstep
```

Out[11]:
```
(200.0, 0.020002000200020003)
```

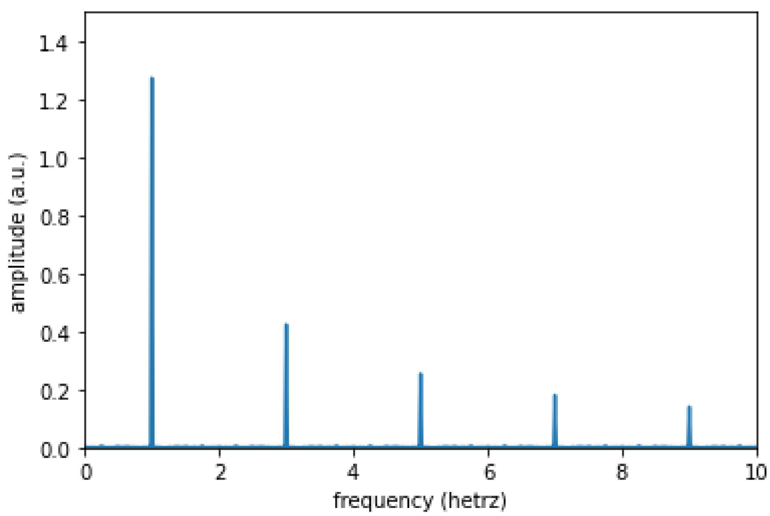
In [12]:
```
# Therefore the frequency axis is:
freq = np.arange(0, fmax + fstep, fstep)
```

In [14]:
```
# Below, we plot the magnitude of the fourier transform.  I don't know
# enough about fft to know why, but the magnitude that is output
# is the amplitude of the time signal multiplied by one half the number of
# points.  To get just the amplitude, I will divided the magnitude by N/2
# before plotting.
fft_mag = plt.figure()
plt.plot(freq, mag/(len(time)/2));
```



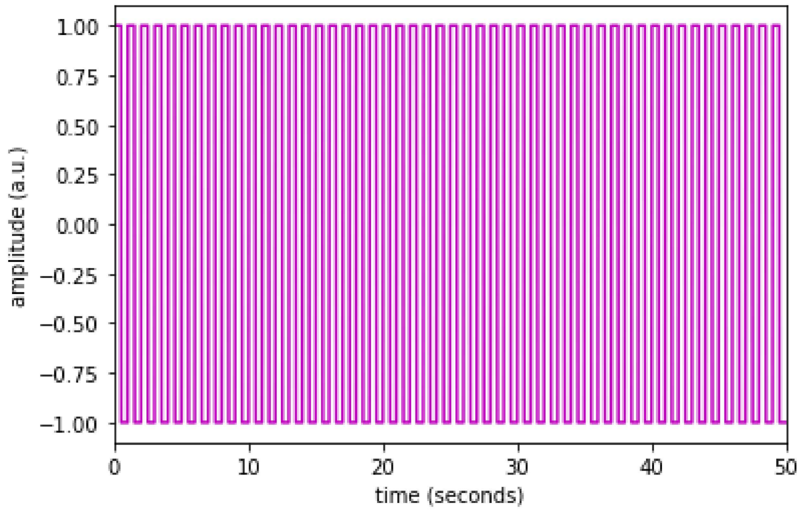
In [16]:

```
# Here's a zoomed-in view of the fft magnitude.
plt.plot(freq, mag/(len(time)/2));
plt.axis((0, 10, 0, 1.5))
plt.xlabel('frequency (hetrz)')
plt.ylabel('amplitude (a.u.)');
# Notice that the fft shows nonzero frequency components only at the odd
# harmonics of the fundamental frequency (1 Hz).  Also, the amplitudes
# (after removing the N/2 contribution) are correct.  One expects the
# height of the fundamental frequency component of a square wave of amplitude
# 1 to be 4/pi = 1.27.  The higher order harmonics should have heights equal
# 4/(pi*n) where n = 3, 5, 7, ...
```



In [17]:

```
# We next verify that the inverse fourier transform 'ifft()' works as expected.
#  If we apply 'ifft()' to y, we should recover the original square wave.
from scipy.fft import ifft
g = ifft(y)
plt.figure()
plt.plot(time, np.real(g), 'm-')
plt.axis((0, 50, -1.1, 1.1))
plt.xlabel('time (seconds)')
plt.ylabel('amplitude (a.u.)');
```

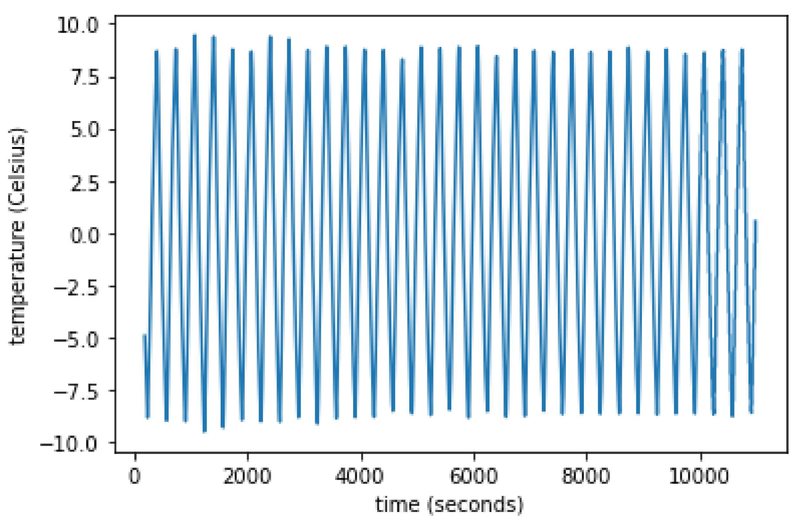


In [18]:

```
# All of this works well when using artificial (i.e. noiseless) data.  In
# this last example, we'll import some data from the PHYS 232 thermal waves
# experiment and apply the fft routine.  The data file that we'll work with
# is called "fftdata.dat".
# Import data.
PHYS232 = np.loadtxt('fftdata.dat')
time = PHYS232[:, 0]
amplitude = PHYS232[:, 1]
```

In [19]:

```
# Plot the imported data
plt.plot(time,amplitude)
plt.xlabel('time (seconds)')
plt.ylabel('temperature (Celsius)');
```



In [21]:

```
# Compute the discrete Fourier Transform of the amplitude data using 'fft()'.
# The fft function outputs a vector of complex numbers.
y = fft(amplitude)
m = abs(y)
```

In [22]:

```
# We have to determine the appropriate frequency scale for the x-axis.  The
# maximum frequency is set by the spacing between adjacent times.  The
```

```
# frequency step, is set by fmax/(number of points - 1).
fmax = 1/(time[1]-time[0])
fstep = fmax/len(time)
fmax, fstep
```

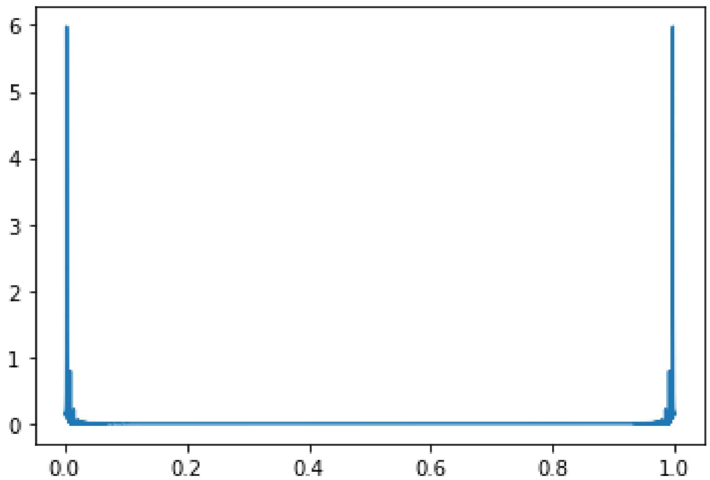
Out[22]: (1.0, 9.264406151565684e-05)

In [23]:

```
# Therefore the frequency axis is:
freq = np.arange(0, fmax, fstep)
```

In [24]:

```
# Below, we plot the magnitude of the fourier transform.
fft_magExp = plt.figure()
plt.plot(freq, m/(len(time)/2));
```



In [25]:

```
# Here's a zoomed-in view of the fft magnitude.
plt.plot(freq, m/(len(time)/2))
plt.axis((0, 0.02, 0, 7))
plt.xlabel('frequency (hetrz)')
plt.ylabel('amplitude (a.u.)');
# The fundamental frequency of this data is 0.003 Hz with peaks at the odd
# harmonics (0.009 and 0.015 Hz).
```