Tarea: Resolucion de sistemas de ecuaciones

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1 Método de Gauss

1.1 Ejercicio 1

$$16x + 16y + 17z = 10$$
$$-14x + 17y - 3z = 75$$
$$-5x - 11y - 18z = 43$$

$$\begin{bmatrix} 16 & 16 & 17 & | & 10 \\ -14 & 17 & -3 & | & 75 \\ -5 & -11 & -18 & | & 43 \end{bmatrix} \quad R1\left(\frac{1}{16}\right)$$

$$\begin{bmatrix} 1 & 1 & \frac{17}{16} & | & \frac{5}{8} \\ -14 & 17 & -3 & | & 75 \\ -5 & -11 & -18 & | & 43 \end{bmatrix} \quad R1\,(14) + R2 \quad y \quad R1\,(5) + R3$$

$$\begin{bmatrix} 1 & 1 & \frac{17}{16} & | & \frac{5}{8} \\ 0 & 31 & 11\frac{7}{8} & | & 83\frac{3}{4} \\ 0 & -6 & -12\frac{11}{16} & | & 46\frac{1}{8} \end{bmatrix} \quad R2\left(\frac{1}{31}\right)$$

$$\begin{bmatrix} 1 & 1 & \frac{17}{16} & | & \frac{5}{8} \\ 0 & 1 & \frac{95}{248} & | & 2\frac{87}{124} \\ 0 & -6 & -12\frac{11}{16} & | & 46\frac{1}{8} \end{bmatrix} \quad R2\,(6) + R3$$

$$\begin{bmatrix} 1 & 1 & \frac{17}{16} & | & \frac{5}{8} \\ 0 & 1 & \frac{95}{248} & | & 2\frac{87}{124} \\ 0 & 0 & -10\frac{193}{496} & | & 62\frac{83}{248} \end{bmatrix}$$

Despejando "z":

$$-10\frac{193}{496}z = 62\frac{83}{248}$$
$$z = \frac{62\frac{83}{248}}{-10\frac{193}{496}}$$
$$z = -6$$

Sustituyendo "z" para despejar "y":

$$y + \frac{95}{248}z = 2\frac{87}{124}$$
$$y + \frac{95}{248}(-6) = 2\frac{87}{124}$$
$$y + -2\frac{37}{124} = 2\frac{87}{124}$$
$$y = 2\frac{87}{124} + 2\frac{37}{124}$$
$$y = 5$$

Sustituyendo "z" y "y" para despejar "x":

$$x + y + \frac{17}{16}z = \frac{5}{8}$$
$$x + 5 + \frac{17}{16}(-6) = \frac{5}{8}$$
$$x + 5 + (-6\frac{3}{8}) = \frac{5}{8}$$
$$x = \frac{5}{8} - 5 + 6\frac{3}{8}$$
$$x = 2$$

Comprobando: 16x+16y+17z=10

$$16x + 16y + 17z = 10$$
$$16(2) + 16(5) + 17(-6) = 10$$
$$32 + 80 - 102 = 10$$
$$10 = 10$$

Comprobando: -14x+17y-3z=75

$$-14x + 17y - 3z = 75$$
$$-14(2) + 17(5) - 3(-6) = 75$$
$$-28 + 85 + 18 = 75$$
$$75 = 75$$

Comprobando: -5x-11y-18z=43

$$-5x - 11y - 18z = 43$$
$$-5(2) - 11(5) - 18(-6) = 43$$
$$-10 - 55 + 108 = 43$$
$$43 = 43$$

1.2   Ejercicio 2

$$x + 2y - z = 1$$
$$2x + 3y + z = 2$$
$$x + 3y - 2z = 1$$

$$\begin{bmatrix} 1 & 2 & -1 & | & 1 \\ 2 & 3 & 1 & | & 2 \\ 1 & 3 & -2 & | & 1 \end{bmatrix} \quad R1\,(-2) + R2 \quad y \quad R1\,(-1) + R3$$

$$\begin{bmatrix} 1 & 2 & -1 & | & 1 \\ 0 & -1 & 3 & | & 0 \\ 0 & 1 & -1 & | & 0 \end{bmatrix} \quad R2\,(-1)$$

$$\begin{bmatrix} 1 & 2 & -1 & \vdots & 1 \\ 0 & 1 & -3 & \vdots & 0 \\ 0 & 1 & -1 & \vdots & 0 \end{bmatrix} \quad R2\,(-1) + R3$$

$$\begin{bmatrix} 1 & 2 & -1 & \vdots & 1 \\ 0 & 1 & -3 & \vdots & 0 \\ 0 & 0 & 2 & \vdots & 0 \end{bmatrix} \quad R3\left(\frac{1}{2}\right)$$

$$\begin{bmatrix} 1 & 2 & -1 & \vdots & 1 \\ 0 & 1 & -3 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$$

Despejando "z":

$$z = 0$$

Sustituyendo "z" para despejar "y":

$$y - 3z = 0$$
$$y - 3(0) = 0$$
$$y - 0 = 0$$
$$y = 0$$

Sustituyendo "z" y "y" para despejar "x":

$$x + 2y - 1z = 1$$
$$x + 2(0) - 1(0) = 1$$
$$x + 0 + 0 = 1$$
$$x = 1$$

Comprobando: x+2y-z=1

$$x + 2y - z = 1$$
$$1 + 2(0) - 0 = 1$$
$$1 + 0 - 0 = 1$$
$$1 = 1$$

Comprobando: 2x+3y+z=2

$$2x + 3y + z = 2$$
$$2(1) + 3(0) + 0 = 2$$
$$2 + 0 + 0 = 2$$
$$2 = 2$$

Comprobando: x+3y-2z=1

$$x + 3y - 2z = 1$$
$$1 + 3(0) - 2(0) = 1$$
$$1 + 0 + 0 = 1$$
$$1 = 1$$

2   Método de Gauss-Jordán

2.1   Ejercicio 3

$$1x + 2y + 3z = 14$$
$$3x + 2y + 1z = 10$$
$$3x + 1y + 2z = 11$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 3 & 2 & 1 & 10 \\ 3 & 1 & 2 & 11 \end{array}\right] \quad R1\,(-3) + R2 \quad y \quad R1\,(-3) + R3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -4 & -8 & -32 \\ 0 & -5 & -7 & -31 \end{array}\right] \quad R2\left(-\frac{1}{4}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 1 & 2 & 8 \\ 0 & -5 & -7 & -31 \end{array}\right] \quad R2\,(-2) + R1 \quad y \quad R2\,(5) + R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 3 & 9 \end{array}\right] \quad R3\left(\frac{1}{3}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R3\,(1) + R1 \quad y \quad R3\,(-2) + R2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Comprobando: 1x+2y+3z=14

$$1x + 2y + 3z = 14$$
$$1(1) + 2(2) + 3(3) = 14$$
$$1 + 4 + 9 = 14$$
$$14 = 14$$

Comprobando: 3x+2y+1z=10

$$3x + 2y + 1z = 10$$
$$3(1) + 2(2) + 1(3) = 10$$
$$3 + 4 + 3 = 10$$

Comprobando: 3x+1y+2z=11

$$3x + 1y + 2z = 11$$
$$3(1) + 1(2) + 2(3) = 11$$
$$3 + 2 + 6 = 11$$
$$11 = 11$$

2.2 Ejercicio 4

$$2a + 2b - c + d = 4$$
$$4a + 3b - c + 2d = 6$$
$$8a + 5b - 3c + 4d = 12$$
$$3a + 3b - 2c + 2d = 6$$

$$\left[\begin{array}{cccc|c}
2 & 2 & -1 & 1 & 4 \\
4 & 3 & -1 & 2 & 6 \\
8 & 5 & -3 & 4 & 12 \\
3 & 3 & -2 & 2 & 6
\end{array}\right] \qquad R1\left(\frac{1}{2}\right)$$

$$\left[\begin{array}{cccc|c}
1 & 1 & -\frac{1}{2} & \frac{1}{2} & 2 \\
4 & 3 & -1 & 2 & 6 \\
8 & 5 & -3 & 4 & 12 \\
3 & 3 & -2 & 2 & 6
\end{array}\right] \qquad R1\left(-4\right)+R2 \qquad y \qquad R1\left(-8\right)+R3 \qquad y \qquad R1\left(-3\right)+R4$$

$$\left[\begin{array}{cccc|c}
1 & 1 & -\frac{1}{2} & \frac{1}{2} & 2 \\
0 & -1 & 1 & 0 & -2 \\
0 & -3 & 1 & 0 & -4 \\
0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0
\end{array}\right] \qquad R2\left(-1\right)$$

$$\left[\begin{array}{cccc|c}
1 & 1 & -\frac{1}{2} & \frac{1}{2} & 2 \\
0 & 1 & -1 & 0 & 2 \\
0 & -3 & 1 & 0 & -4 \\
0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0
\end{array}\right] \qquad R2\left(-1\right)+R1 \qquad y \qquad R2\left(3\right)+R3$$

$$\left[\begin{array}{cccc|c}
1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 1 & -1 & 0 & 2 \\
0 & 0 & -2 & 0 & 2 \\
0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0
\end{array}\right] \qquad R3\left(-\frac{1}{2}\right)$$

$$\left[\begin{array}{cccc|c}
1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 1 & -1 & 0 & 2 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0
\end{array}\right] \qquad R3\left(-\frac{1}{2}\right)+R1 \qquad y \qquad R3\left(1\right)+R2 \qquad y \qquad R3\left(\frac{1}{2}\right)+R4$$

$$\left[\begin{array}{cccc|c}
1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}
\end{array}\right] \qquad R4\left(2\right)$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & \vdots & \frac{1}{2} \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 0 & 1 & \vdots & -1 \end{bmatrix} \quad R4\left(-\frac{1}{2}\right) + R1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$$

Comprobando: 2a+2b-c+d=4

$$\begin{aligned} 2a + 2b - c + d &= 4 \\ 2(1) + 2(1) - (-1) + (-1) &= 4 \\ 2 + 2 + 1 - 1 &= 4 \\ 4 &= 4 \end{aligned}$$

Comprobando: 4a+3b-c+2d=6

$$\begin{aligned} 4a + 3b - c + 2d &= 6 \\ 4(1) + 3(1) - (-1) + 2(-1) &= 6 \\ 4 + 3 + 1 - 2 &= 6 \\ 6 &= 6 \end{aligned}$$

Comprobando: 8a+5b-3c+4d=12

$$\begin{aligned} 8a + 5b - 3c + 4d &= 12 \\ 8(1) + 5(1) - 3(-1) + 4(-1) &= 12 \\ 8 + 5 + 3 - 4 &= 12 \\ 12 &= 12 \end{aligned}$$

Comprobando: 3a+3b-2c+2d=6

$$\begin{aligned} 3a + 3b - 2c + 2d &= 6 \\ 3(1) + 3(1) - 2(-1) + 2(-1) &= 6 \\ 3 + 3 + 2 - 2 &= 6 \\ 6 &= 6 \end{aligned}$$