

04 - part 3 - Filtering in the frequency domain

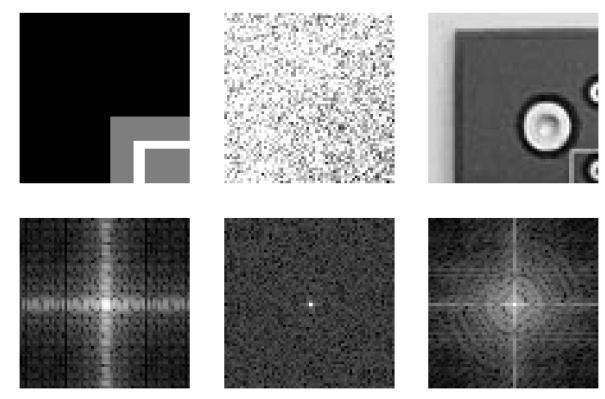
```
In [1]:
         import numpy as np
         import imageio
         import matplotlib.pyplot as plt
         import time
In [2]:
         img1 = imageio.imread("images/pattern.png")
         img2 = imageio.imread("images/gradient_noise.png")
         img3 = imageio.imread("images/board.jpg")
In [3]:
         # implementing a 2D version of the DFT
         def DFT2D(f):
             # create empty array of complex coefficients
             F = np.zeros(f.shape, dtype=np.complex64)
             n,m = f.shape[0:2]
             # creating indices for x, to compute multiplication using numpy (f^*exp)
             x = np.arange(n)
             # for each frequency 'u,v'
             for u in np.arange(n):
                 for v in np.arange(m):
                     for y in np.arange(m):
                          F[u,v] += np.sum(f[:,y] * np.exp((-1j*2*np.pi) * (((u*x)/n)+((
             return F/np.sqrt(n*m)
In [4]:
         # implementing a 2D version of the DFT
         def IDFT2D(F):
             # create empty array of complex coefficients
             f = np.zeros(F.shape, dtype=np.int32)
             n,m = F.shape[0:2]
             # creating indices for x, to compute multiplication using numpy (f^*exp)
             u = np.arange(n)
             # for each frequency 'u,v'
             for x in np.arange(n):
                 for y in np.arange(m):
                     for v in np.arange(m):
                          f[x,y] += np.real(np.sum(F[:,v] * np.exp( (1j*2*np.pi) * (((u*x))))))
             return np.real(f/np.sqrt(n*m))
In [5]:
         # extracting subimages with 64x64=4096 pixels
         sizes = 64
         img1 s = img1[:sizes, :sizes]
         img2_s = img2[:sizes, :sizes]
         img3 s = img3[5:sizes+5, 5:sizes+5]
         F1s = DFT2D(img1 s)
         F2s = DFT2D(img2 s)
```

```
F3s = DFT2D(img3\_s)
```

Visualizing the **Power Spectrum** $|F(u,v)|^2$

```
In [6]:
         plt.figure(figsize=(12,8))
         plt.subplot(231)
         plt.imshow(img1_s, cmap="gray"); plt.axis('off')
         plt.subplot(232)
         plt.imshow(img2_s, cmap="gray"); plt.axis('off')
         plt.subplot(233)
         plt.imshow(img3_s, cmap="gray"); plt.axis('off')
         # the log of the magnitudes
         plt.subplot(234)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F1s))), cmap="gray")
         plt.axis('off')
         plt.subplot(235)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F2s))), cmap="gray")
         plt.axis('off')
         plt.subplot(236)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F3s))), cmap="gray")
         plt.axis('off')
```

Out[6]: (-0.5, 63.5, 63.5, -0.5)



It is possible to notice the different frequency patterns that arise from different images.

- 1. the first one (leftmost) contains mainly horizontal and vertical oscilations,
- 2. the second (in the middle) is only noise so the spectrum is aproximately uniform,
- 3. the last (right-most) has horizontal, vertical but also circular patterns, which shows on the spectrum.

Let us now filter frequencies in those images in order to see the effects of filters:

- high passs
- low pass
- band stop

```
In [7]:
         n2 = F1s.shape[0]//2
         m2 = F1s.shape[1]//2
         F1p = np.fft.fftshift(F1s.copy())
         F1p[n2-9:n2+9], m2-9:m2+9] = 0 # square high pass filter, removes first frequence
         F1p = np.fft.ifftshift(F1p)
         F2p = np.fft.fftshift(F2s).copy()
         F2p[:n2-9, :] = 0 \# square low pass filter, removes higher frequencies
         F2p[:, :m2-9] = 0 # square low pass filter, removes higher frequencies
         F2p[n2+9:, :] = 0 \# square low pass filter, removes higher frequencies
         F2p[:, m2+9:] = 0 # square low pass filter, removes higher frequencies
         F2p = np.fft.ifftshift(F2p)
         F3p = F3s \cdot copy()
         F3p[5:-5,5:-5] = 0 \# band stop filter
         #F3p = np.fft.ifftshift(F3p)
         i1p = IDFT2D(F1p)
         i2p = IDFT2D(F2p)
         i3p = IDFT2D(F3p)
```

```
In [8]:
         plt.figure(figsize=(9,9))
         plt.subplot(331)
         plt.imshow(i1p, cmap="gray"); plt.axis('off'); plt.title('filtered')
         plt.subplot(332)
         plt.imshow(i2p, cmap="gray"); plt.axis('off'); plt.title('filtered')
         plt.subplot(333)
         plt.imshow(i3p, cmap="gray"); plt.axis('off'); plt.title('filtered')
         # the log of the magnitudes
         plt.subplot(334)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F1p))), cmap="gray")
         plt.axis('off'); plt.title('high pass')
         plt.subplot(335)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F2p))), cmap="gray")
         plt.axis('off'); plt.title('low pass')
         plt.subplot(336)
         plt.imshow(np.log(1 + np.fft.fftshift(np.abs(F3p))), cmap="gray")
         plt.axis('off'); plt.title('per frequency band pass')
         plt.subplot(337)
         plt.imshow(img1 s, cmap="gray"); plt.axis('off'); plt.title('original')
         plt.subplot(338)
         plt.imshow(img2_s, cmap="gray"); plt.axis('off'); plt.title('original')
         plt.subplot(339)
         plt.imshow(img3_s, cmap="gray"); plt.axis('off'); plt.title('original')
```

Out[0]. Tovt(0 5 1 0 'original')

