1. Summary

This paper studies Hilbert geometries: discrete subgroups of projective general linear groups acting on properly convex sets in real projective space. This type of geometry, which also goes by the names convex real projective geometry, or convex real projective structures, includes both hyperbolic geometry and (some) higher rank symmetric spaces as examples, and in general can feature behavior that is somehow in between these two extremes. It is of great interest to the geometric structures community, and also to those studying higher Teichmuller-Thurston theory, discrete subgroups of Lie groups, and other subjects. The subject has classical roots in work of Benzecri, Koszul, Vey, Vinberg, Goldman etc, and was launched into a modern context by a series of works by Benoist.

The main result of this paper is, in some sense, an expansion of a result of Benoist, which gave the first indication of the close connection between the geometry of the convex set and abstract group-theoretic properties of the group acting. While Benoist's result concerned strictly convex sets, the author here studies what seems to be a much more general type of Hilbert geometry, that he calls *rank one* by analogy to a similar notion studied by Ballman in CAT(0) geometry. The main result is that for these geometries, the discrete group acting must be *acylindarically hyperbolic*, a notion from geometric group theory introduced recently by Osin. This leads to some nice applications about strong group theoretic properties for the discrete groups acting in rank one Hilbert geometries.

The theme guiding this paper is that while Hilbert geometries are almost never CAT(0), there should nonetheless be a strong analogy to CAT(0) geometry. In my opinion, one of the major contributions here is that the author figures out the right analogies to make. That the arguments are, for the most part, elegant and not overly technical reinforces that the new notions presented here are good ones worthy of deeper exploration.

While the main result concerns acylindrical hyperbolicity, the author does not work directly with this notion. Rather, following Sisto in the CAT(0) setting, the author shows that rank one isometries, in the sense introduced in the paper, are contracting elements. From there, results of Osin and Sisto give the acylindrical hyperbolicity property. The main work of the paper then is understanding in fine detail how individual elements act on a given convex set, what the structure of the boundary of the convex set can look like, and how the rank-one condition ensures nice enough behavior to get the contracting condition. Many of the arguments are dynamical in nature.

2. Evaluation

This is an excellent paper deserving of publication in a top journal such as Geometry and Topology. The results are compelling and suggestive, the

arguments are mostly elegant, and the paper is laid out in a clear and inviting way.

3. Some comments/corrections for the author