Lecture 18:
Monte Carlo methods

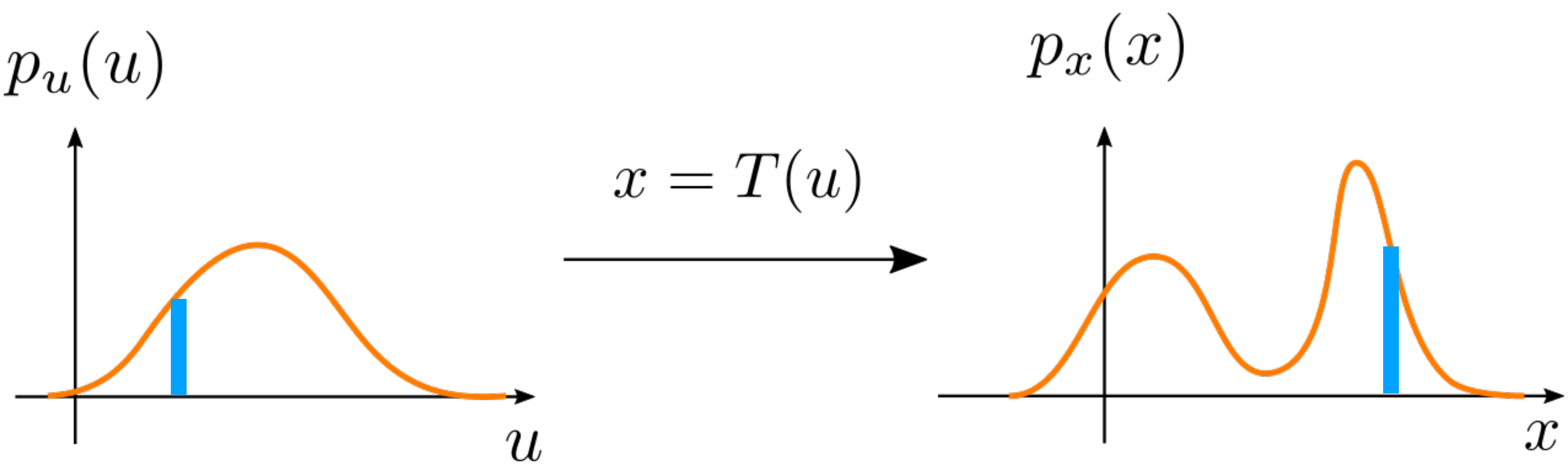
Monaco

Monte Carlo(MC)

- Have been seeing Monte Carlo methods throughout class
  - Any time we randomly sample thats an MC method
  - Effectively we are just rolling the die

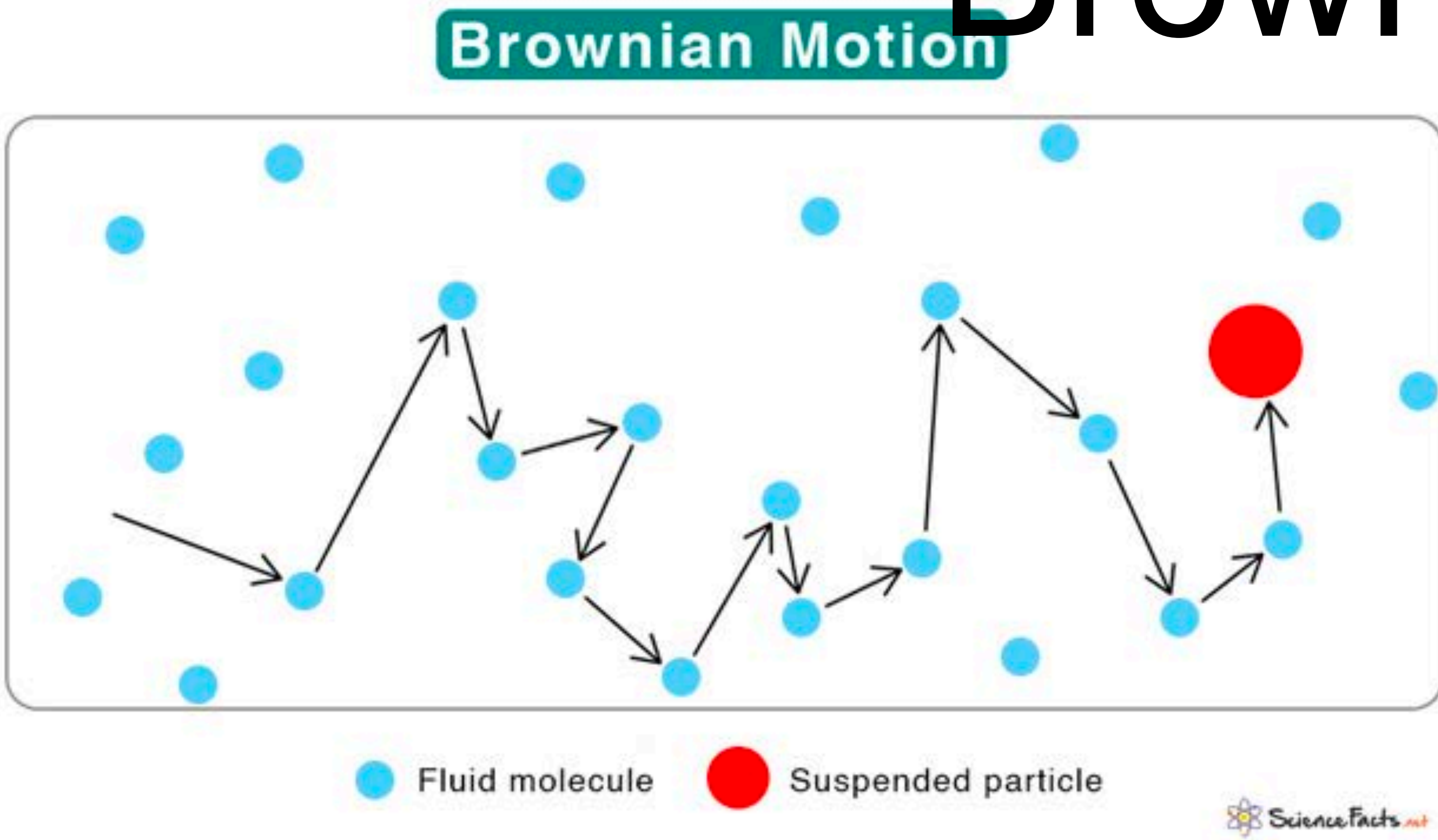
Monte Carlo vs Integration

- Monte Carlo is a form of integrator
    - However non-deterministic and varies over distribution



- Monte Carlo typically used when
    - we can’t model things analytically any more
    - Replace a whole distribution with just an event (small region

Brownian Motion



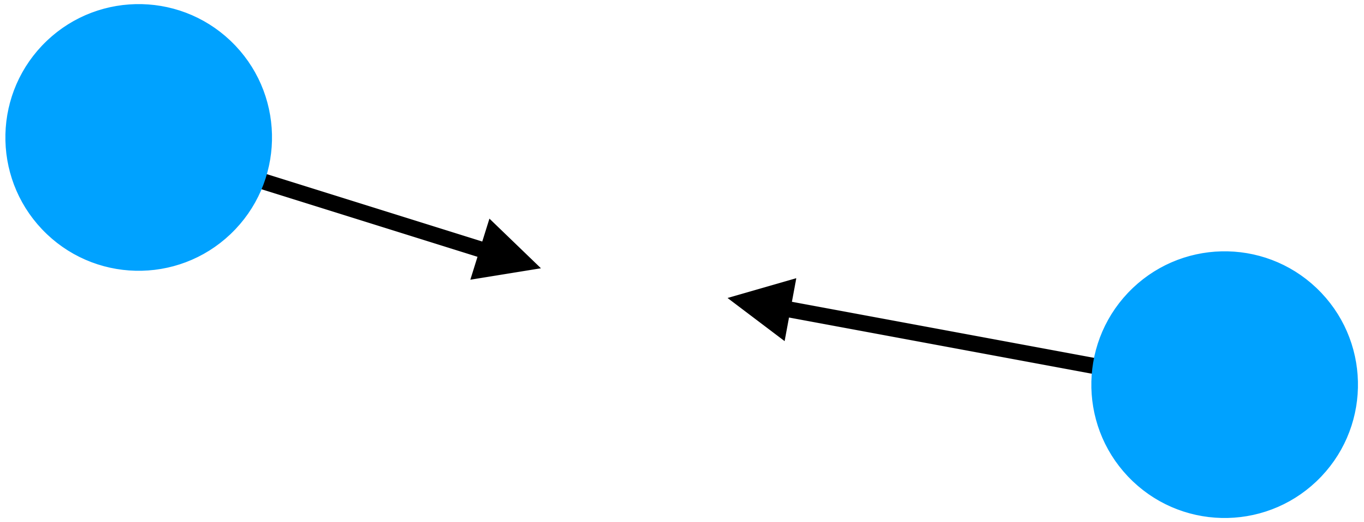
$$f(v_x, v_y, v_z) = \left[\frac{m}{2\pi kT}\right]^{3/2} e^{-m(v_x^2+v_y^2+v_z^2)/2kT}$$

$$= \left[\frac{m}{2\pi kT}\right]^{3/2} e^{-mv^2/2kT}$$

using $v^2 = v_x^2 + v_y^2 + v_z^2$

- At each step
  - We just randomly sample the velocity from a Gaussian
  - We can do this many times to look at overall motion

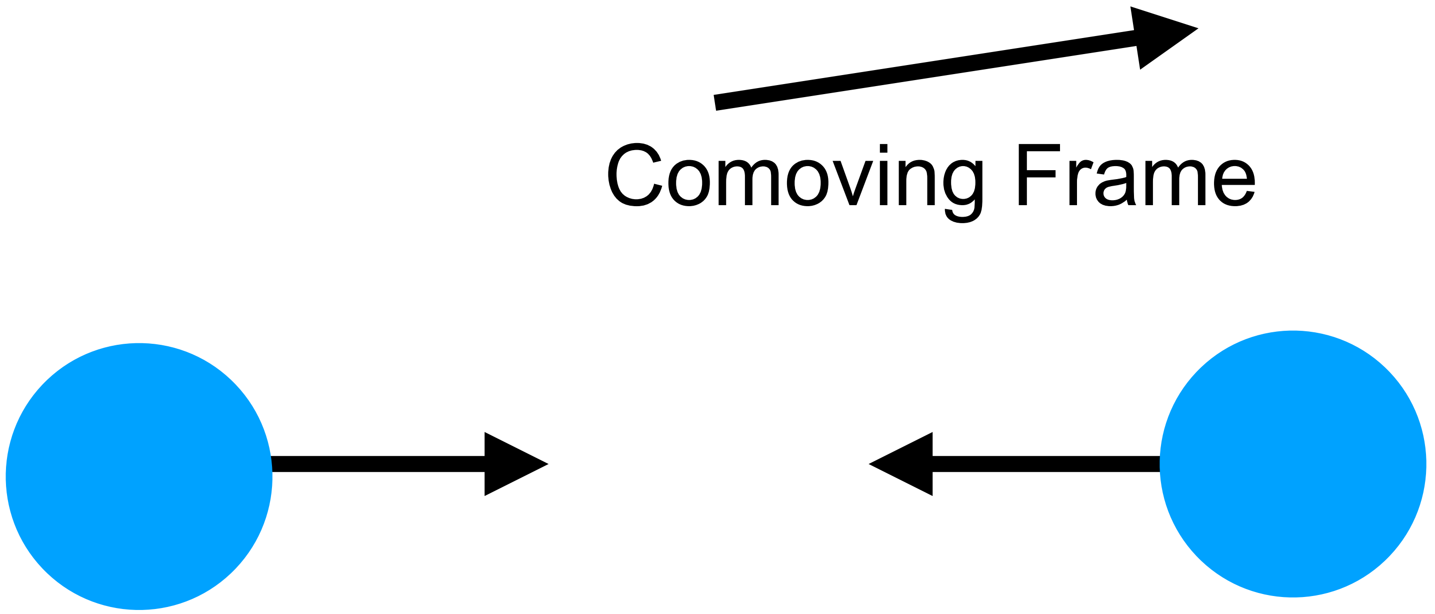
The motion at each step



Elastic Collision

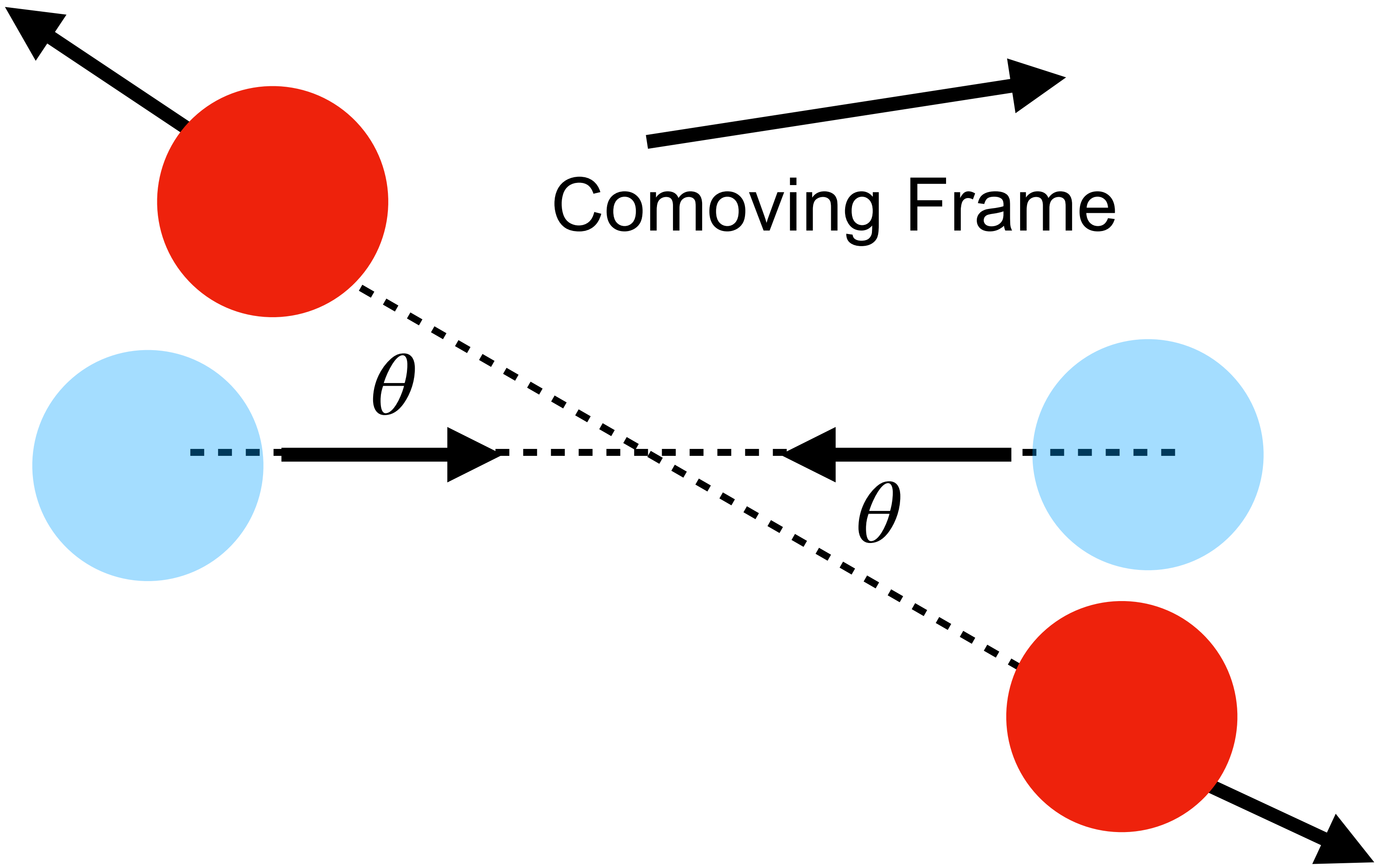
- Just sample particle collisions at each step

The motion at each step



Elastic Collision
In COM Frame

The motion at each step



Elastic Collision
In COM Frame

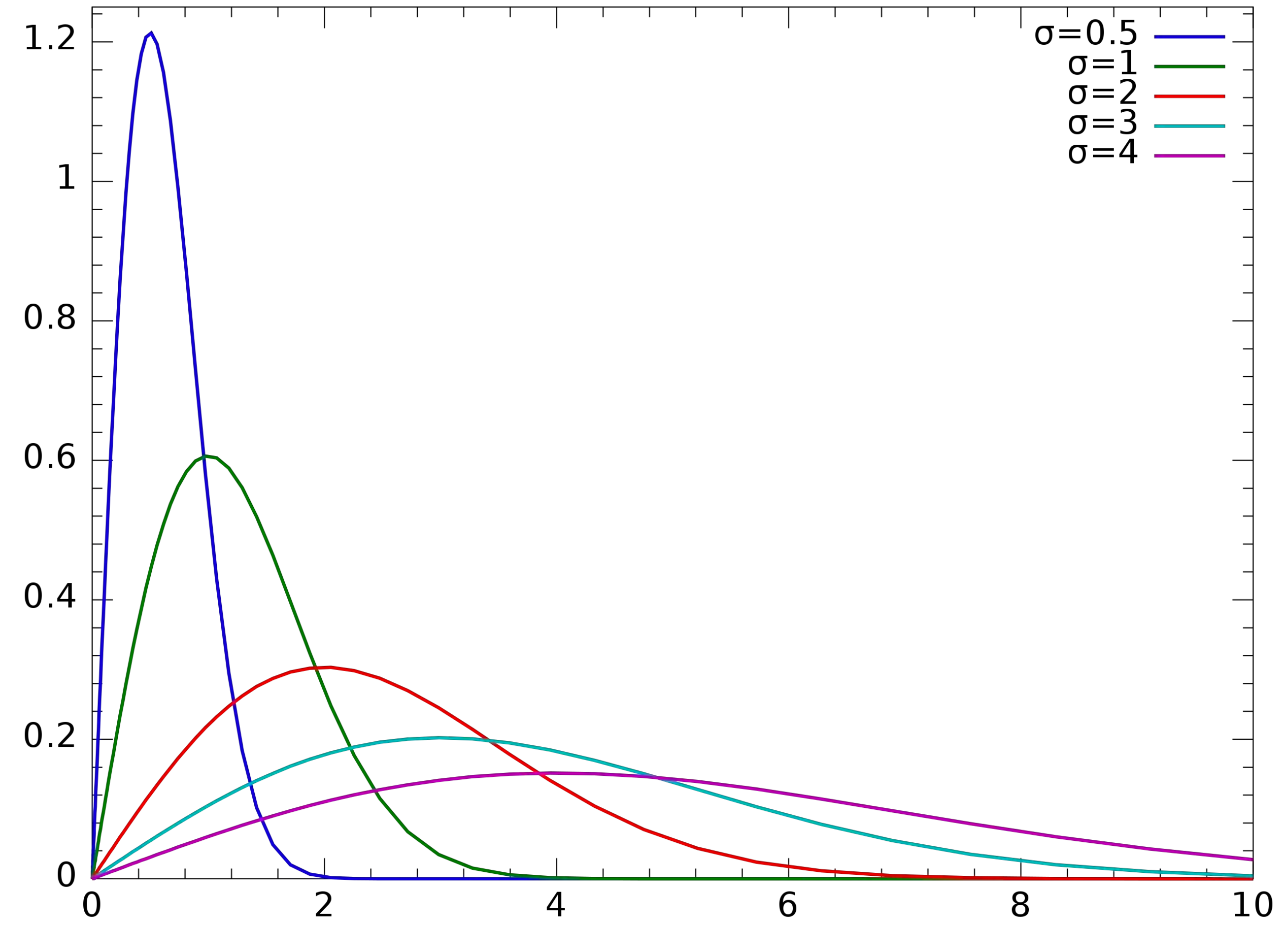
Rayleigh Distribution

Rayleigh is a distribution of the radius in a 2D Gaussian

$$f_U(x;\sigma) = f_V(x;\sigma) = \frac{e^{-x^2/(2\sigma^2)}}{\sqrt{2\pi\sigma^2}}.$$

$$f(x;\sigma) = \frac{x}{\sigma^2}e^{-x^2/(2\sigma^2)}, \quad x \geq 0,$$

$$F_X(x;\sigma) = \frac{1}{2\pi\sigma^2}\int_0^{2\pi}\int_0^x re^{-r^2/(2\sigma^2)}\,dr\,d\theta = \frac{1}{\sigma^2}\int_0^x re^{-r^2/(2\sigma^2)}\,dr.$$



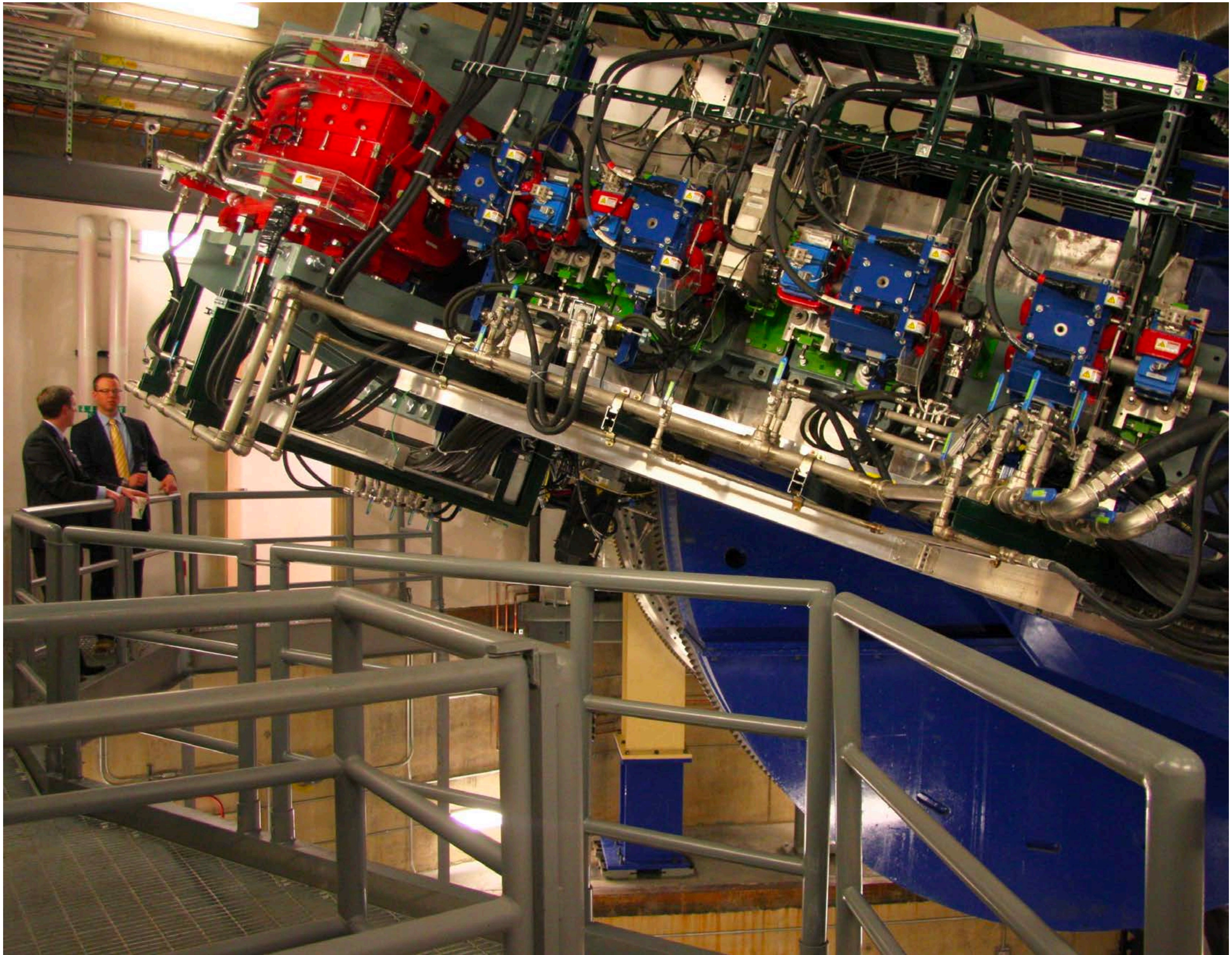
Proton Therapy

Proton Therapy Center at MGH

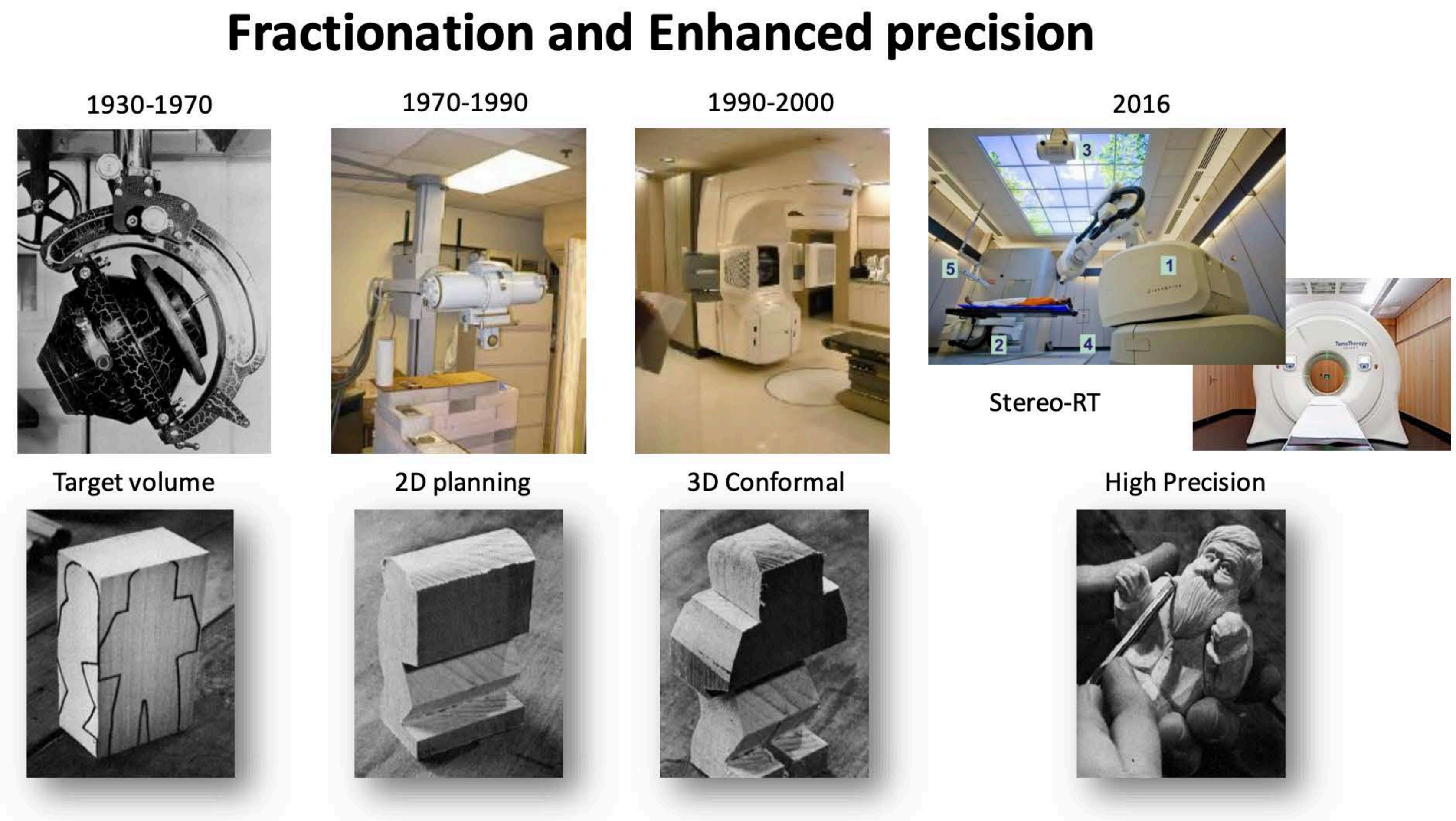
Typical Device

Particle Therapy Centre

Mayo Clinic



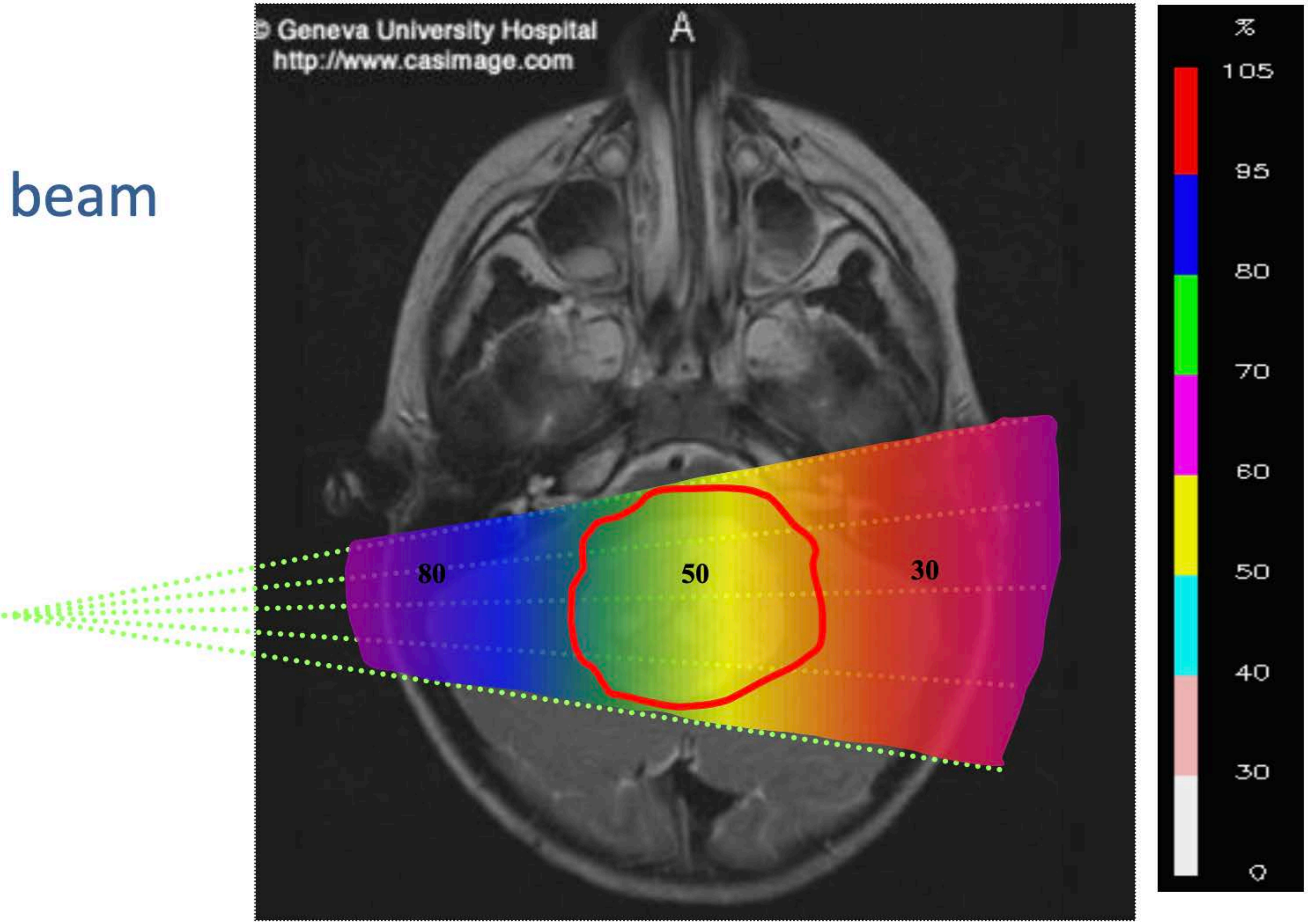
Radiation Therapy



- To fight Cancer
  - Radiation therapy has had a long history of usage
  - Radiation is sent to a tumor to kill it
  - Critical when you can't cut the tumor out

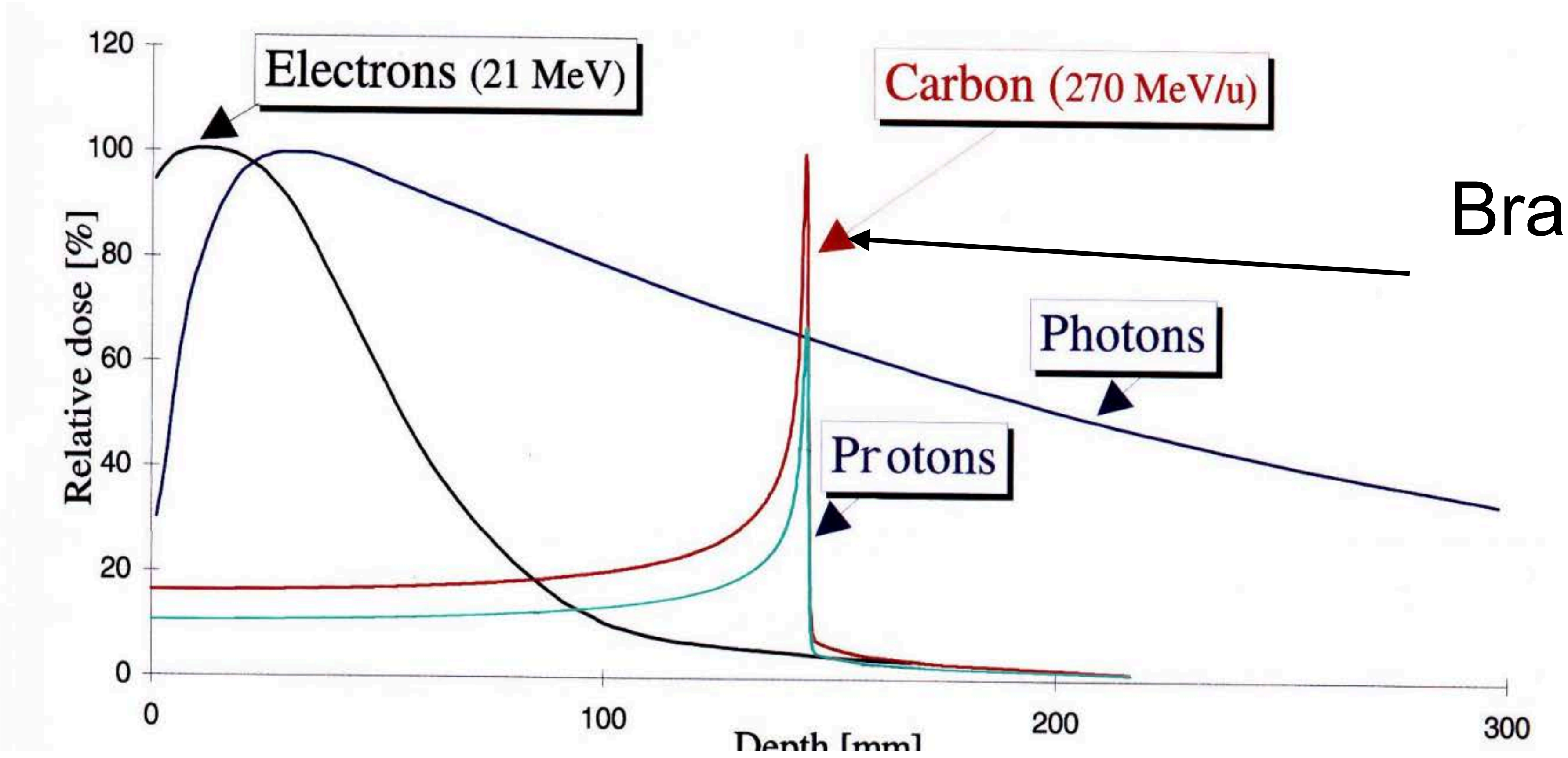
Classical Radiotherapy with X-rays

single beam



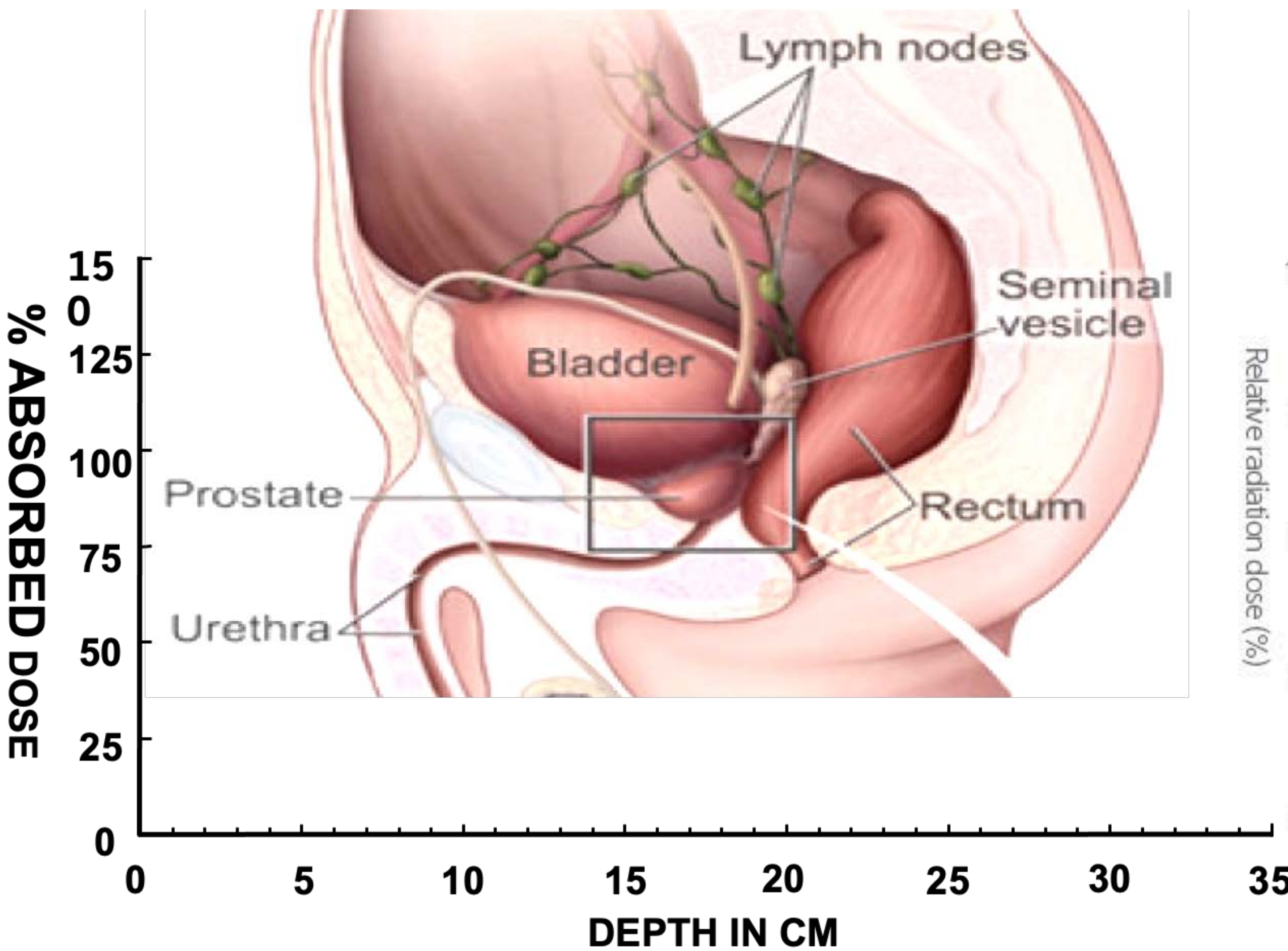
Hadron Therapy

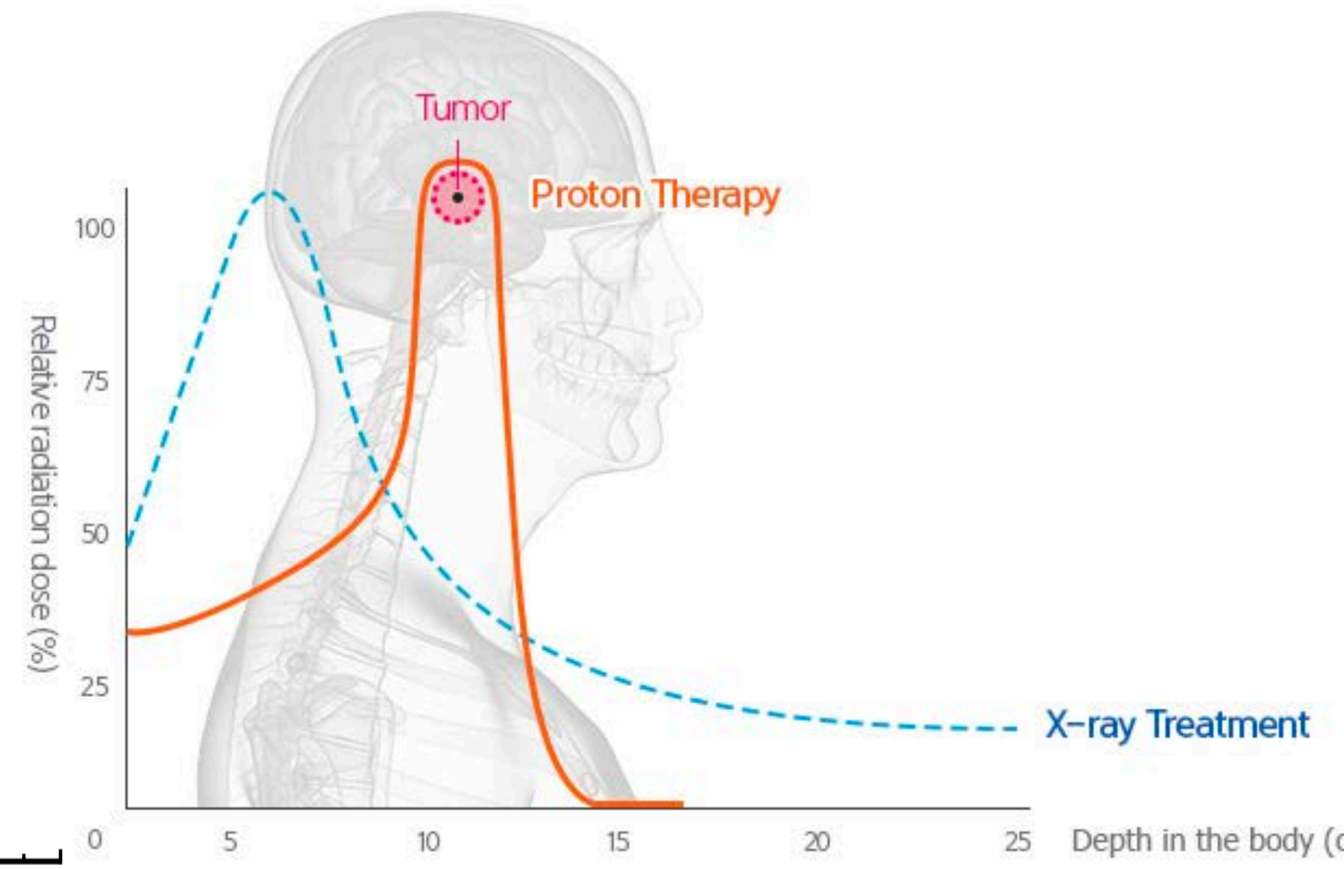
- Therapy
  - Hadrons allow you to control deposit
  - Can vary the depth of the hadrons through Bragg scatter



Bragg Scatter Peak

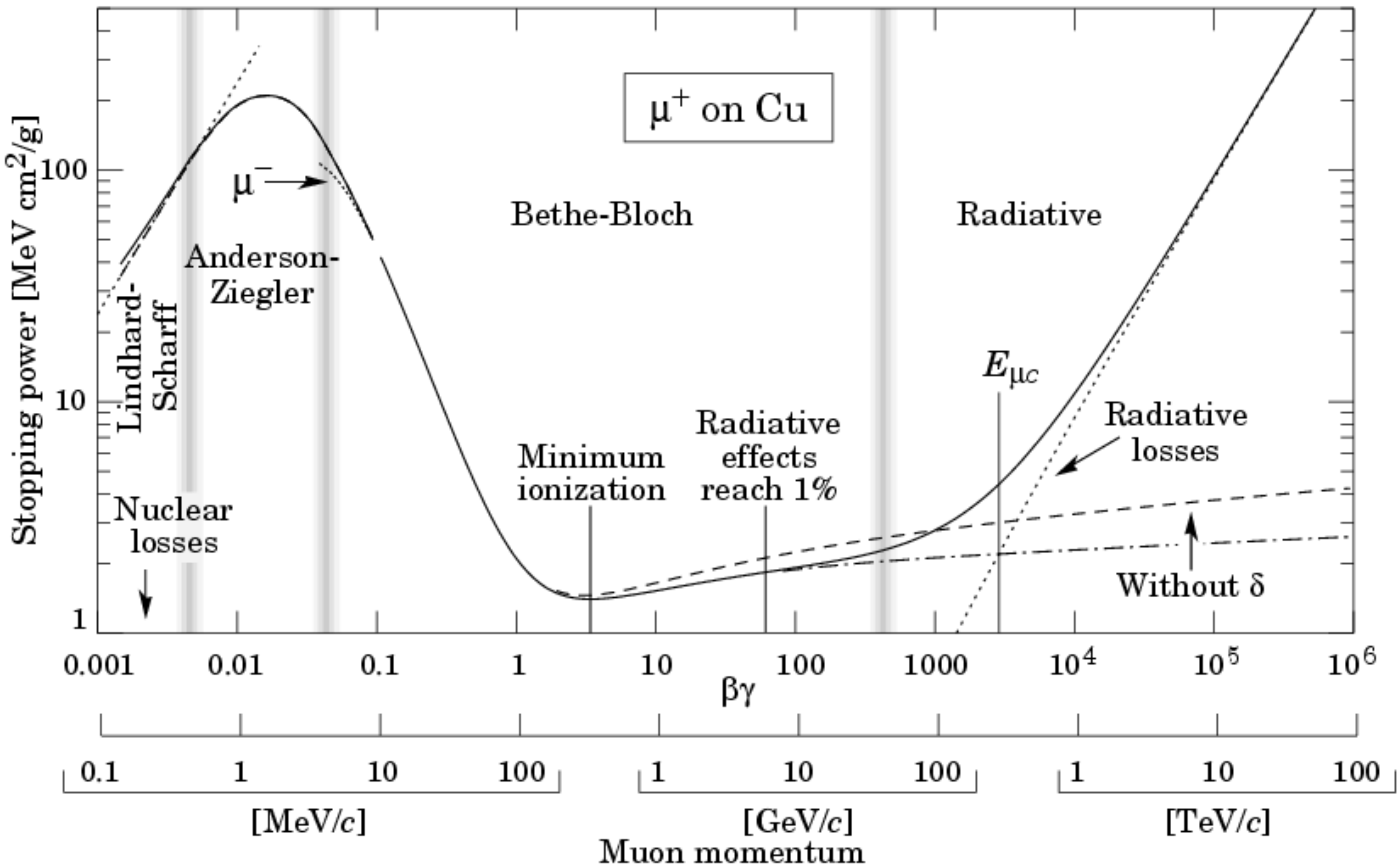
Proton Therapy





Bethe-Bloch Equation

- Charged Particles in matter are goverened by this equation



Protons Governed

$$\boxed{-\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]} \; [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma\, m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

$N_A = 6.022 \cdot 10^{23}$

[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm

[Classical electron radius]

$m_e = 511$ keV

[Electron mass]

$\beta = v/c$

[Velocity]

$\gamma = (1-\beta^2)^{-2}$

[Lorentz factor]

z : Charge of incident particle

M : Mass of incident particle

Z : Charge number of medium

A : Atomic mass of medium

I : Mean excitation energy of medium

δ : Density correction [transv. extension of electric field]

Validity:

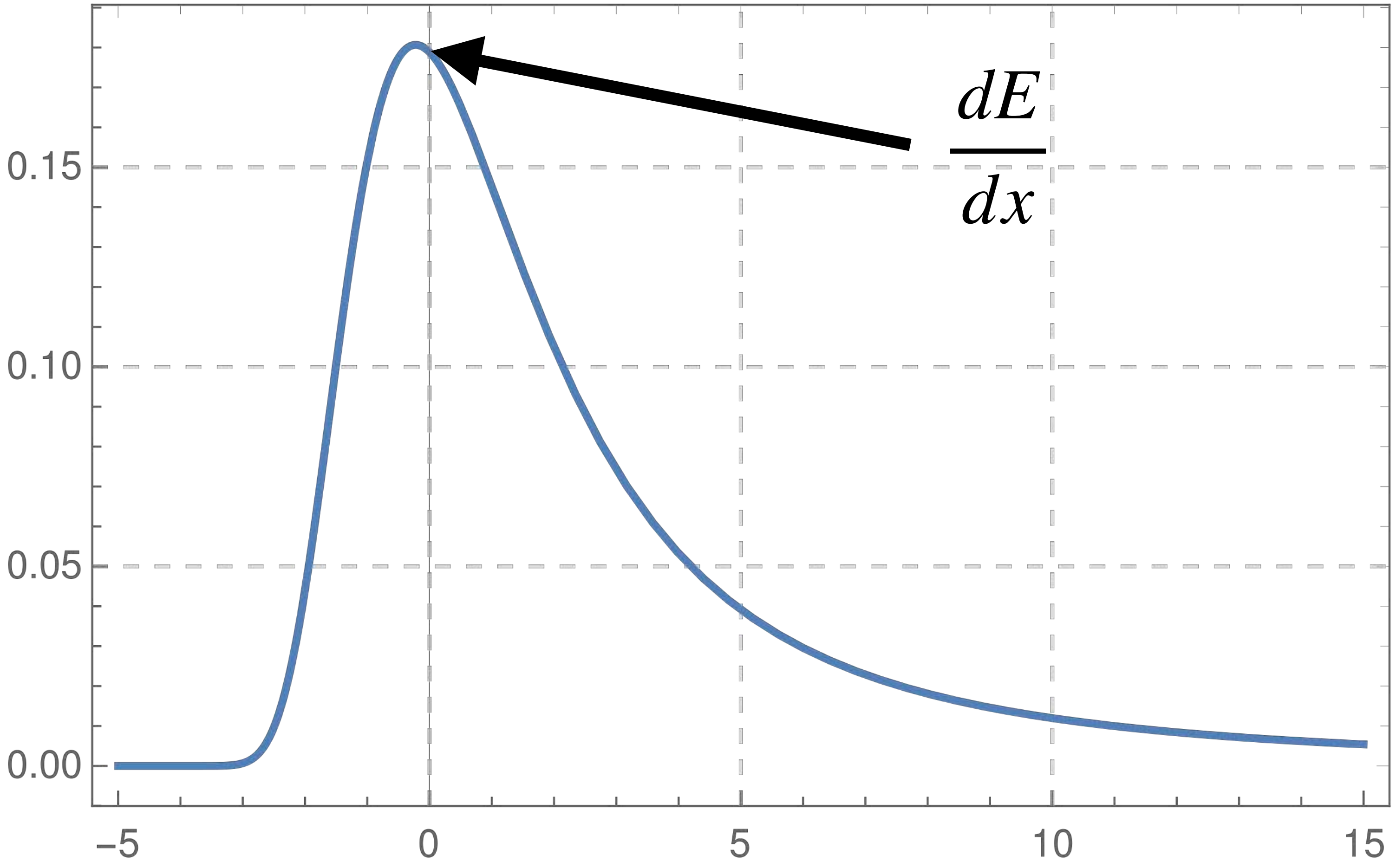
$.05 < \beta\gamma < 500$

$M > m_\mu$

Actual Energy Loss

- As we step along we lose energy by the Landau distribution

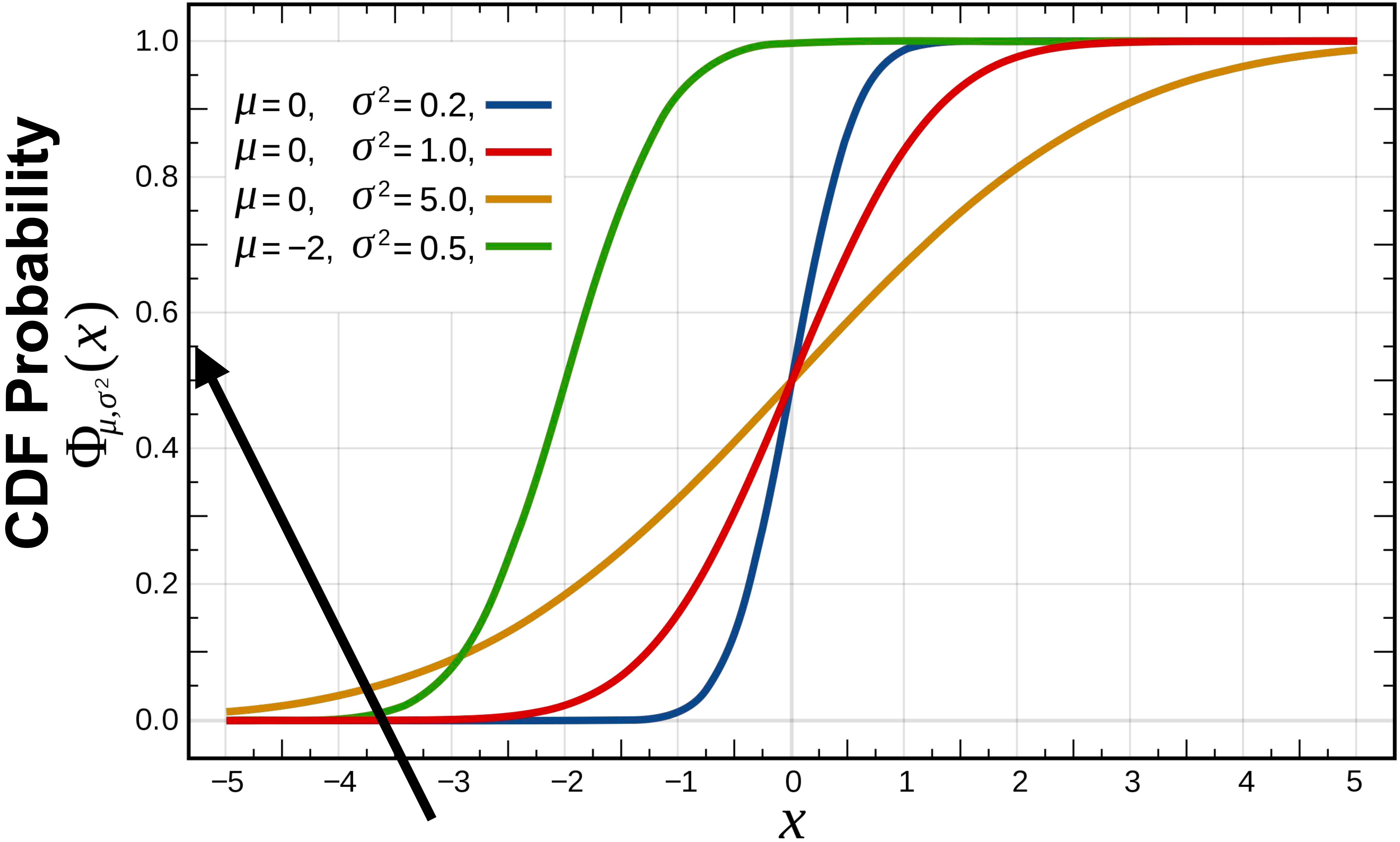
$$p(x) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{s\log(s)+xs}\, ds,$$



Average of this distribution gives Bethe-Bloch

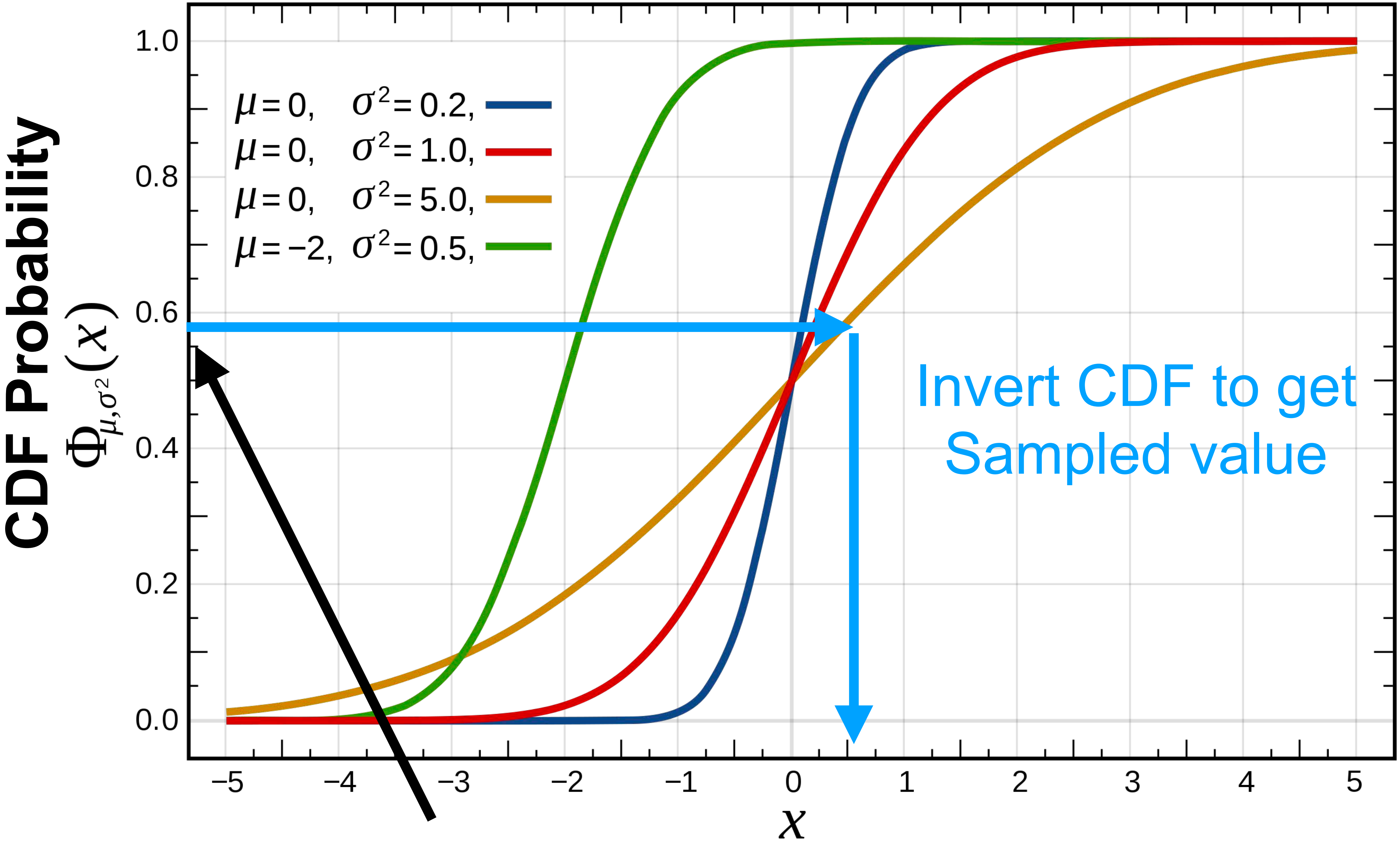
We can sample this At each step

Sampling a Distribution



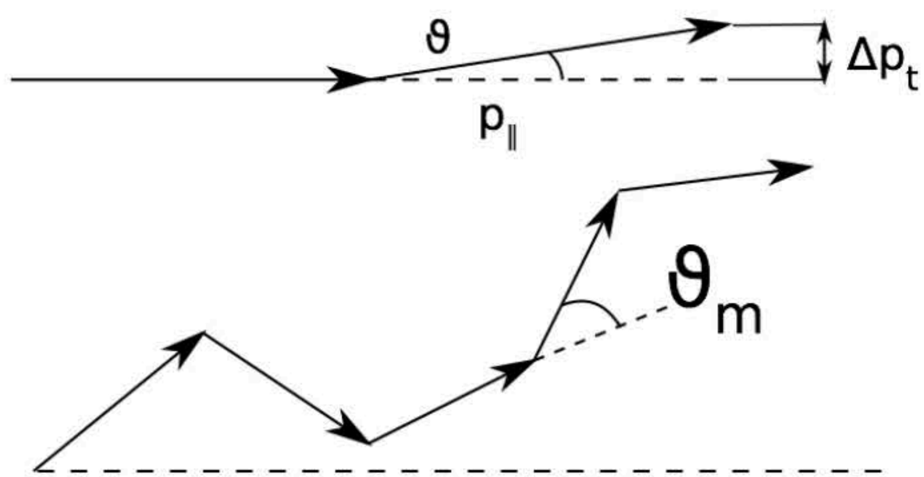
Sample from a p-value from 0 to 1 (flat 0 to 1)

Sampling a Distribution



Sample from a p-value from 0 to 1 (flat 0 to 1)

Multiple Scatter Particles



$$\theta \simeq \frac{\Delta p_\perp}{p_{||}} \simeq \frac{\Delta p_\perp}{p}$$
$$= \frac{2Zze^2}{b}\frac{1}{pv}$$

after k collisions

$$\langle\theta_k^2\rangle = \sum_{m=1}^{k}\theta_m^2 = k\langle\theta^2\rangle$$

- Single collision (thin absorber): Rutherford scattering $d\sigma/d\Omega \propto \sin^{-4}\theta/2$
- Few collisions: difficult problem
- Many (>20) collisions: statistical treatment "Molière theory"

Multiple Scatter Particles

$$\theta \simeq \frac{\Delta p_\perp}{\quad} \simeq \frac{\Delta p_\perp}{\quad}$$

Obtain the mean deflection angle in a plane by averaging over many collisions and integrating over b:

$$\sqrt{\langle \theta^2(x) \rangle} = \theta_{\text{rms}}^{\text{plane}} = \frac{13.6 \text{ MeV}}{\beta pc} z \sqrt{\frac{x}{X_0}} (1 + 0.038 \ln \frac{x}{X_0})$$

- Material constant X_0: radiation length
- $\propto \sqrt{x} \rightarrow$ use thin detectors
- $\propto 1/\sqrt{X_0} \rightarrow$ use light detectors
- $\propto 1/\beta p \rightarrow$ serious problem at low momenta

In 3 dimensions: $\theta_{\text{rms}}^{\text{space}} = \sqrt{2}\, \theta_{\text{rms}}^{\text{plane}}$ $\quad 13.6 \rightarrow 19.2$

Image Sources

image

link:

attribution: