**Stability Investigation and Stabilization of Nonlinear Switched Mechanical Systems via Decomposition**

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*A nonlinear hybrid mechanical system is studied. It is assumed that on the system homogeneous switched positional forces, linear gyroscopic forces and homogeneous dissipative ones act. Conditions are determined under which the trivial equilibrium position of the considered system is asymptotically stable for any admissible switching law. It is well known that to prove the asymptotic stability uniform with respect to switching law, it is sufficient to construct a common Lyapunov function for the family of subsystems corresponding to the hybrid system. However, till now there are no general constructive methods for finding of common Lyapunov functions, even for families composed of linear subsystems. The problem is especially difficult for mechanical systems with switched force fields, since such systems are described by differential equations of the second order. This results in the appearance of certain special properties of motions and essentially complicates the analysis of system dynamics. In particular, well known results obtained for switched systems of general form might be inefficient or even inapplicable for mechanical systems. In the present paper, a new approach to the stability analysis is proposed. The approach is based on the decomposition of the original system consisting of differential equations of the second order into two isolated first order subsystems of the same dimension. It should be noted that one of the isolated subsystem is switched, whereas another one is nonswitched. Thus, the decomposition method permits us to reduce the problem of a common Lyapunov function constructing for the original family of nonlinear systems of dimension with a special structure to that for an auxiliary family of subsystems of dimension which, generally, do not possess a special structure. The problem of stabilization of the equilibrium position of a system for any switching mode scaling the potential with the aid of small forces of radial correction is considered. For a model of the magnetic bearing of a rotor with nonlinear switched circular forces, the stabilizing feedback control law is constructed by the use of linear gyroscopic and nonlinear dissipative forces.*

***Keywords:*** *hybrid mechanical systems, asymptotic stability, control, Lyapunov functions, stabilization, decomposition, switching law*

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