Design and Analysis of a Laminated Composite Tube

Jiatai Deng, Jiawei Shuang, Xuanye Hu

Department of Mechanical, Aerospace and Civil Engineering

March 9, 2020

Contents

1 General description and requirement

A cylindrical tube of 4 layers of composite with a layup of $[\alpha\ \beta\ \alpha\ \beta]$is to be designed and by winding tapes cut from a UD carbon-epoxy prepreg sheet on to a cylindrical mandrel of a 25mm radius. For the consideration of practicality, the range of these two winding angles will have to fall in [-75°, -30°] or [+30°, +75°] to the axis of the tube. The thickness of the prepreg is 0.25mm. The tube should be made to a length of 300mm.

Material properties:

Internal pressure: $q = 3 \times 10^6$ Axial force: $P = 25kN$

Tube radius: $R = 25 \times 10^{-3}$ Tube length $L = 0.3$

Elastic constant: $E1 = 236GPa$ $E2 = 5GPa$ $G = 2.6GPa$ $\upsilon_{12} = 0.25GPa$

Strengths: $\sigma^*_{1t} = 3800MPa$ $\sigma^*_{2t} = 41MPa$ $\sigma^*_{1c} = 689MPa$ $\sigma^*_{2c} = 107MPa$ $\tau^*_{12} = 69MPa$

2 Design approach and theory

The thickness of each layers is 0.25mm, thickness in the range is between 0.1 1.0 mm can be seen as lamina. So the developed tube is a laminate with layup: $[\alpha/\beta/\alpha/\beta]$ and the laminate theory can be used for stress analysis.

STEP I: Define the two winding angles:

According to the requirement, the winding angle of 4 layers can be defined:

$$-75^\circ \leq \alpha \leq -30^\circ \text{ or } 75^\circ \leq \alpha \leq 30^\circ$$

$$-75^\circ \leq \beta \leq -30^\circ \text{ or } 75^\circ \leq \beta \leq 30^\circ$$

STEP II: Define the Stress-Strain Relationship $[Q]$:

According to the Laminar Stress-Strain Relationship in Material coordinate system:

$$\{\sigma\} = [Q]\{\epsilon\}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E1}{1-\upsilon^2\frac{E2}{E1}} & \frac{\Upsilon E2}{1-\upsilon^2\frac{E2}{E1}} & 0 \\ \frac{\Upsilon E2}{1-\upsilon^2\frac{E2}{E1}} & \frac{E2}{1-\upsilon^2\frac{E2}{E1}} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{1}$$

STEP III: Define the Coordinate transformation $[T]$

The winding angle of 4 layers of composite: $\theta = \begin{pmatrix} \alpha \\ \beta \\ \alpha \\ \beta \end{pmatrix}$, the coordinate transformation between (x-y) and (1-2) matrix:

$$\{T\} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 2\cos\theta\sin\theta \\ \cos\theta\sin\theta & -\cos\theta\sin\theta & cos^2\theta - sin^2\theta \end{bmatrix} \tag{2}$$

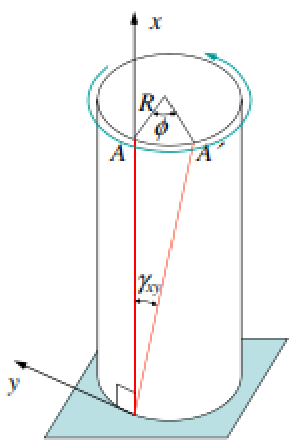
STEP IV: Define maximum stress failare criterion

According to the maximum stress failure criterion, material will fail when any of the following conditions is violated:

$$\begin{aligned} &\frac{\sigma_1}{\sigma^*_{1t}} \leq 1 \quad if \quad \sigma_1 > 0 \qquad &&\frac{|\sigma_1|}{\sigma^*_{1c}} \leq 1 \quad if \quad \sigma_1 < 0 \\ &\frac{\sigma_2}{\sigma^*_{2t}} \leq 1 \quad if \quad \sigma_2 > 0 \qquad &&\frac{|\sigma_2|}{\sigma^*_{2c}} \leq 1 \quad if \quad \sigma_2 < 0 \\ & &&\frac{|\tau_{12}|}{\tau^*_{12}} \leq 1 \end{aligned} \tag{3}$$

STEP V: Define Twist angle

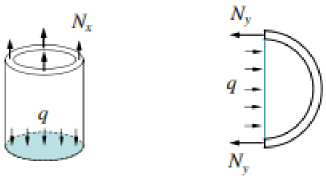
Tube is fixed at the bottom end and axially compressed at the top. Generator on the tube deforms by an angle γ_{xy}. Point A at the top end moves to A' by a distance $\gamma_{xy}L$. We can get the twist angle of the tube formula:



$$\theta = \frac{\gamma_{xy}L}{R} \tag{4}$$

There are two loading conditions, separately analysis:

2.1 Case 1: Internal pressure of 3 MPa (ends closed)



The generalised stresses:

$$N = \begin{Bmatrix} N_x = \frac{1}{2}qR \\ N_y = qR \\ N_{xy} = 0 \end{Bmatrix} \tag{5}$$

Generalised stress-strain Relationship:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \end{Bmatrix}$$
$$[A] = t\,[T]\,[Q]\,[T]^T \tag{6}$$

Strain in x-direction:

$$\epsilon_x = [A]^{-1}\,N \tag{7}$$

Lamina 1: The winding angle is α.
In the STEP II, Stress-Strain Relationship has been defined:

$$\epsilon_\alpha = [T_\alpha]\,[A]^{-1\,T}\epsilon_x \qquad \epsilon_\alpha = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \qquad \sigma_\alpha = [Q]\,\epsilon_\alpha \qquad \sigma_\alpha = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{8}$$

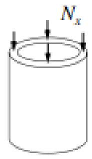
Maximum stress failure criterion is given in step IV, substitute σ into the formula to determine failure situation. And the results can indicate the failure modes, if $\frac{|\sigma_{1/2}|}{\sigma^*_{t/c}} \geq 1$, this means Shear failure occurs, if $\frac{|\tau_{12}|}{\tau^*_{12}} \geq 1$,this means Transverse tensile failure occurs.

Lamina 2: The winding angle is β.
In the STEP II, Stress-Strain Relationship has been defined:

$$\epsilon_\beta = [T_\beta]\,[A]^{-1\,T}\epsilon_x \qquad \epsilon_\beta = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \qquad \sigma_\beta = [Q]\,\epsilon_\beta \qquad \sigma_\beta = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{9}$$

Using the maximum stress failure criterion to indicate the failure modes as Lamina 1.

2.2 Case 2:Axial compression of $P = 25kN$



The generalised stresses:

$$N = \begin{Bmatrix} N_x = \frac{P}{2\pi R} \\ N_y = 0 \\ N_{xy} = 0 \end{Bmatrix} \tag{10}$$

Generalised stress-strain Relationship:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \end{Bmatrix} \tag{11}$$
$$[A] = t\,[T]\,[Q]\,[T]^T$$

Strain in x-direction: $\epsilon_x = [A]^{-1}\, N$
The rest of the calculation method is the same as Case 1.

2.3 The maximum angle of twist without failure

Among all the winding angles which enable the tube to withstand the axial load and the internal pressure, there will be a set of angle values: α and β to make the tube get the maximum twist angle without failure.

3 Results and analysis to prove the success of the design

A Appendix

```
%%Unit: Pa,m%%
%%Term CASE1FM loading case 1 Failure matrix%%
%%Term CASE2FM loading case 2 Failure matrix%%
%%Term AssemblyFM Assembled Falure matrix [-30,30] has been removed%%
%%Term CASE2TA loading case 2 twist angle%%
%%Term BP Best point%%

format short %%Modify according to the requirement%%

%%Basic parameters%%

resolution=1; %%Modify according to the requirement%%
Delete=[0]; %%leave it alone%%
q=3e6;
P=-2.5e4;
```

```
R=25e-3;
thick=2.5e-4;
L=0.3;
E1=236e9;
E2=5e9;
G=2.6e9;
v=0.25;
Xt=3800e6;
Xc=689e6;
Yt=41e6;
Yc=107e6;
S=69e6;
TAT_1=zeros(150/resolution+1); %%Initial matrix of Twist Angle%%
TAT_1=zeros(150/resolution+1);
for i=-75/resolution:1:75/resolution %%2 for loots control the Alpha
    and Beta%%
    Alpha=i*resolution;
    AlphaF=i+75/resolution+1;
    for i=-75/resolution:1:75/resolution
        Beta=i*resolution;
        BetaF=i+75/resolution+1;
        Theta=[Alpha;Beta;Alpha;Beta];
        t=[thick;thick;thick;thick];
        ctraQ=1-((v^2)*(E2/E1));
        Q=[E1/ctraQ v*E2/ctraQ 0; v*E2/ctraQ E2/ctraQ 0; 0 0 G];
        TT=[0;0;0];
            for i = 1:4 %%Find related equition in sheet%%
                straTAlpha=sind(Theta(i));
                ctraTAlpha=cosd(Theta(i));
                TAlpha=[ctraTAlpha^2 straTAlpha^2 -2*ctraTAlpha*
                    straTAlpha;
                straTAlpha^2 ctraTAlpha^2 2*ctraTAlpha*straTAlpha;
                ctraTAlpha*straTAlpha -ctraTAlpha*straTAlpha
                    ctraTAlpha^2-straTAlpha^2];
                TT=[TT TAlpha];
                eval(['TT',num2str(i),'=TAlpha;'])
            end
        TT=TT(:, 2:13);
        A=zeros(3);
            for i=1:4
                m=(i-1)*3+1;
                n=i*3;
```

```
            TTa=TT(:,m:n);
            A=A+thick*TTa*Q*TTa';
            Delete=Delete';
        end
    a=A^(-1);
    Q0=Q;

    %%Case one%%

    N_1=[0.5*q*R; q*R; 0];
    Epsilonx_1=a*N_1;
    Phi_1=Epsilonx_1(3)*L/R*180/pi;
    TAT1(AlphaF,BetaF)=Phi_1;
        for i=1:4
            m=(i-1)*3+1;
            n=i*3;
            TTa=TT(:,m:n);
            Epsilon_1= TTa'*Epsilonx_1;
            Delete=Delete';
            Sigma=Q0*Epsilon_1;
                if Sigma(1)>=0
                    w1=Sigma(1)/Xt;
                else
                    w1=-Sigma(1)/Xc;
                end
                if Sigma(2)>=0
                    w2=Sigma(2)/Yt;
                else
                    w2=-Sigma(2)/Yc;
                end
            w3=abs(Sigma(3)/S);
            w=[w1; w2; w3];
            eval(['wcone',num2str(i),'=w;'])
        end
    wwassembly_1=[wcone1 wcone2 wcone3 wcone4];
    Fail_case_one_1=all(wwassembly_1(:)<=1);
    TATT_1(AlphaF,BetaF)=Fail_case_one_1;
    TATT_1=TATT_1*1;

    %%Case two%%

    N_2=[P/(2*pi*R); 0; 0];
```

```
        Epsilonx_2=a*N_2;
        Phi_2=Epsilonx_2(3)*L/R*180/pi;
        TAT_2(AlphaF,BetaF)=Phi_2;
            for i=1:4
                m=(i-1)*3+1;
                n=i*3;
                TTa=TT(:,m:n);
                Epsilon_2= TTa'*Epsilonx_2;
                Delete=Delete';
                Sigma=Q0*Epsilon_2;
                    if Sigma(1)>=0
                        w1=Sigma(1)/Xt;
                    else
                        w1=-Sigma(1)/Xc;
                    end
                    if Sigma(2)>=0
                        w2=Sigma(2)/Yt;
                    else
                        w2=-Sigma(2)/Yc;
                    end
                w3=abs(Sigma(3)/S);
                w=[w1; w2; w3];
                eval(['wcone',num2str(i),'=w;'])
            end
        wwassembly_2=[wcone1 wcone2 wcone3 wcone4];
        Fail_case_one_2=all(wwassembly_2(:)<=1);
        TATT_2(AlphaF,BetaF)=Fail_case_one_2;
        TATT_2=TATT_2*1;
    end
end

CASE1FM=TATT_1;
XX=-75:resolution:75;
YY=-75:resolution:75;
CASE2FM=TATT_2;
CASE2TA=TAT_2;
CASE1FM((-30+75)/resolution+1:(30+75)/resolution+1,:)=0;
CASE1FM(:,(-30+75)/resolution+1:(30+75)/resolution+1)=0;

CASE2FM((-30+75)/resolution+1:(30+75)/resolution+1,:)=0;
CASE2FM(:,(-30+75)/resolution+1:(30+75)/resolution+1)=0;

```

```
%%Assemble the Failure matrix%%

AssemblyFM=CASE1FM.*CASE2FM;

%%Find the best point%%

TAFM=AssemblyFM.*CASE2TA;
BPZ=max(max(TAFM));
[BPX BPY]=find(TAFM==max(max(TAFM)));
BPX=(BPX(1)-1-75/resolution)*resolution;
BPY=(BPY(1)-1-75/resolution)*resolution;
BP=[BPX BPY BPZ]

%%0 become NAN for figure%%

ind=find(TAFM==0);
TAFM(ind)=NaN;

%%Figure%%
    %%Colormap transformation%%

colormap(parula);

    %%TA Figure%%

mesh(XX,YY,CASE2TA);
title('Twist Angle','Fontname', 'Times New Roman','FontSize',24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x3=zlabel('Twist angle (degree)','Fontname', 'Times New Roman','
    FontSize',18);
set(x1,'Rotation',18);
set(x2,'Rotation',-25);

saveas(gcf,'Twist Angle.jpg');

    %%TA plane figure%%

contour(XX,YY,CASE2TA,10,'ShowText','on');
```

```
title('Twist Angle (degree)','Fontname', 'Times New Roman','FontSize'
    ,24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);

saveas(gcf,'Twist Angle (degree).jpg');

    %%TA Figure without Failure%%

mesh(XX,YY,TAFM);
title('Twist Angle without Failure','Fontname', 'Times New Roman','
    FontSize',24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x3=zlabel('Twist angle (degree)','Fontname', 'Times New Roman','
    FontSize',18);
set(x1,'Rotation',18);
set(x2,'Rotation',-25);

saveas(gcf,'Twist Angle without Failure.jpg');

    %%Colormap transformation%%

colormap(gray);

    %%Devided failure analysis figure%%

gca=pcolor(XX,YY,CASE1FM);
set(gca, 'LineStyle','none');
title('Failure Analysis of Case One','Fontname', 'Times New Roman','
    FontSize',24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize'
    ,18);

saveas(gcf,'Failure Analysis of Case One.jpg');

```

```
gca=pcolor(XX,YY,CASE2FM);
set(gca, 'LineStyle','none');
title('Failure Analysis of Case Two','Fontname', 'Times New Roman','FontSize',24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize',18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize',18);

saveas(gcf,'Failure Analysis of Case Two.jpg');

    %%Assembled failure analysis figure%%

gca=pcolor(XX,YY,AssemblyFM);
set(gca, 'LineStyle','none');
title('Failure Analysis','Fontname', 'Times New Roman','FontSize',24);
x1=xlabel('Alpha (degree)','Fontname', 'Times New Roman','FontSize',18);
x2=ylabel('Beta (degree)','Fontname', 'Times New Roman','FontSize',18);

saveas(gcf,'Failure Analysis.jpg');
```