Composites Coursework (MathCAD work sheet)

Internal pressure: $q := 3 \cdot 10^{6}$

Axial force: $P := -25000$

Tube radius: $R := 0.0255$

Tube length: $L := 0.300$

Elastic constants: $E1 := 236 \cdot 10^{9}$ $\quad E2 := 5 \cdot 10^{9}$

$G := 2.6 \cdot 10^{9}$ $\quad \nu := .25$

Display all Toolbars for quick use

Strengths: $Xt := 3800 \cdot 10^{6}$ $\quad Yt := 41 \cdot 10^{6}$

$Xc := 689 \cdot 10^{6}$ $\quad Yc := 107 \cdot 10^{6}$

$S := 69 \cdot 10^{6}$

MathCAD worksheet is sequential.
It read from top to bottom and left to right.
Make sure a value has been defined before you make use of the value.

Go to tools,
Select Worksheet Options,
Change Array Origin to 1.

Part 1: Failure analysis for a given layup.

Layups: $\alpha := 35$ $\quad \beta := 55$ $\quad \theta := \begin{pmatrix} \alpha \\ \beta \\ \alpha \\ \beta \end{pmatrix}$

You should change these angles for your failure analysis.

Layer thickness $\quad t := \begin{pmatrix} 0.00025 \\ 0.00025 \\ 0.00025 \\ 0.00025 \end{pmatrix}$

Be aware of the difference between different equal signs:
- := assigning a value to a variable
- = evaluating an expression

There is another equal sign from the palette on top or by "ctrl =" which is for equations

Laminate analysis:

$$Q(E1, E2, G, \nu) := \left| \begin{array}{l} c \leftarrow 1 - \nu^{2} \cdot \dfrac{E2}{E1} \\ \begin{pmatrix} \dfrac{E1}{c} & \dfrac{\nu \cdot E2}{c} & 0 \\ \dfrac{\nu \cdot E2}{c} & \dfrac{E2}{c} & 0 \\ 0 & 0 & G \end{pmatrix} \end{array} \right.$$

$$T(\theta) := \left| \begin{array}{l} s \leftarrow \sin(\theta \cdot deg) \\ c \leftarrow \cos(\theta \cdot deg) \\ \begin{pmatrix} c^{2} & s^{2} & -2 \cdot c \cdot s \\ s^{2} & c^{2} & 2 \cdot c \cdot s \\ c \cdot s & -c \cdot s & c^{2} - s^{2} \end{pmatrix} \end{array} \right.$$

$$A(n, \theta, t) := \left| \begin{array}{l} A \leftarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ Q \leftarrow Q(E1, E2, G, \nu) \\ \text{for } i \in 1..n \\ \quad \left| \begin{array}{l} TT \leftarrow T(\theta_i) \\ A \leftarrow A + t_i \cdot TT \cdot Q \cdot TT^{T} \end{array} \right. \end{array} \right.$$

Matrix transposition and inversion signs must be inserted by using the matrix palette.

Use matrix palette to insert subsript for matrix component

$Q0 := Q(E1, E2, G, \nu)$ $\quad A := A(4, \theta, t)$ $\quad a := A^{-1}$

Maximum stress failure criterion:

$$\mathrm{FMS}(\sigma, Xt, Xc, Yt, Yc, S) := \left| \begin{array}{l} w_1 \leftarrow \dfrac{\sigma_1}{Xt} \ \text{ if } \sigma_1 \ge 0 \\ w_1 \leftarrow -\dfrac{\sigma_1}{Xc} \ \text{ if } \sigma_1 < 0 \\ w_2 \leftarrow \dfrac{\sigma_2}{Yt} \ \text{ if } \sigma_2 \ge 0 \\ w_2 \leftarrow -\dfrac{\sigma_2}{Yc} \ \text{ if } \sigma_2 < 0 \\ w_3 \leftarrow \left| \dfrac{\sigma_3}{S} \right| \\ w \end{array} \right.$$

The subscripts for σ and w here are subscript for matrix component

Be aware of the difference between absolute value sign and determinant sign for a matrix. They look identical on the screen

This one is absolute value sign

Load Case 1: Internal pressure:

$$N := \begin{pmatrix} \frac{1}{2} \cdot q \cdot R \\ q \cdot R \\ 0 \end{pmatrix} \qquad N = \begin{pmatrix} 3.825 \times 10^{4} \\ 7.65 \times 10^{4} \\ 0 \end{pmatrix} \qquad \varepsilon x := a \cdot N \qquad \varepsilon x = \begin{pmatrix} 4.414 \times 10^{-3} \\ 6.475 \times 10^{-3} \\ -0.011 \end{pmatrix}$$

Lamina 1: $T\alpha := T(\theta_1)$

$$\varepsilon := T\alpha^{T} \cdot \varepsilon x \qquad \varepsilon = \begin{pmatrix} -1.51 \times 10^{-4} \\ 0.011 \\ -1.88 \times 10^{-3} \end{pmatrix} \qquad \sigma := Q0 \cdot \varepsilon \qquad \sigma = \begin{pmatrix} -2.186 \times 10^{7} \\ 5.509 \times 10^{7} \\ -4.888 \times 10^{6} \end{pmatrix}$$

Maximum stress failure criterion: $\mathrm{FMS}(\sigma, Xt, Xc, Yt, Yc, S) = \begin{pmatrix} 0.032 \\ 1.344 \\ 0.071 \end{pmatrix}$

Lamina 2: $T\beta := T(\theta_2)$

$$\varepsilon := T\beta^{T} \cdot \varepsilon x \qquad \varepsilon = \begin{pmatrix} 5.539 \times 10^{-4} \\ 0.01 \\ 5.753 \times 10^{-3} \end{pmatrix} \qquad \sigma := Q0 \cdot \varepsilon \qquad \sigma = \begin{pmatrix} 1.438 \times 10^{8} \\ 5.244 \times 10^{7} \\ 1.496 \times 10^{7} \end{pmatrix}$$

Maximum stress failure criterion: $\mathrm{FMS}(\sigma, Xt, Xc, Yt, Yc, S) = \begin{pmatrix} 0.038 \\ 1.279 \\ 0.217 \end{pmatrix}$

Load Case 2: Axial compression:

$$N := \begin{pmatrix} \frac{P}{2\pi \cdot R} \\ 0 \\ 0 \end{pmatrix} \qquad N = \begin{pmatrix} -1.56 \times 10^{5} \\ 0 \\ 0 \end{pmatrix} \qquad \varepsilon x := a \cdot N \qquad \varepsilon x = \begin{pmatrix} -0.012 \\ -3.2 \times 10^{-3} \\ 0.015 \end{pmatrix}$$

Lamina 1:

$$\varepsilon := T\alpha^{T} \cdot \varepsilon x \qquad \varepsilon = \begin{pmatrix} -1.712 \times 10^{-3} \\ -0.013 \\ 0.013 \end{pmatrix} \qquad \sigma := Q0 \cdot \varepsilon \qquad \sigma = \begin{pmatrix} -4.209 \times 10^{8} \\ -6.771 \times 10^{7} \\ 3.403 \times 10^{7} \end{pmatrix}$$

Maximum stress failure criterion:
$$FMS(\sigma, Xt, Xc, Yt, Yc, S) = \begin{pmatrix} 0.611 \\ 0.633 \\ 0.493 \end{pmatrix}$$

Lamina 2:

$$\varepsilon := T\beta^{T} \cdot \varepsilon x \qquad \varepsilon = \begin{pmatrix} 1.164 \times 10^{-3} \\ -0.016 \\ 2.71 \times 10^{-3} \end{pmatrix} \qquad \sigma := Q0 \cdot \varepsilon \qquad \sigma = \begin{pmatrix} 2.55 \times 10^{8} \\ -7.85 \times 10^{7} \\ 7.046 \times 10^{6} \end{pmatrix}$$

Maximum stress failure criterion:
$$FMS(\sigma, Xt, Xc, Yt, Yc, S) = \begin{pmatrix} 0.067 \\ 0.734 \\ 0.102 \end{pmatrix}$$

Angle of twist:
$$\phi := \frac{\varepsilon x_3 \cdot L}{R} \cdot deg^{-1} \qquad \phi = 10.229$$

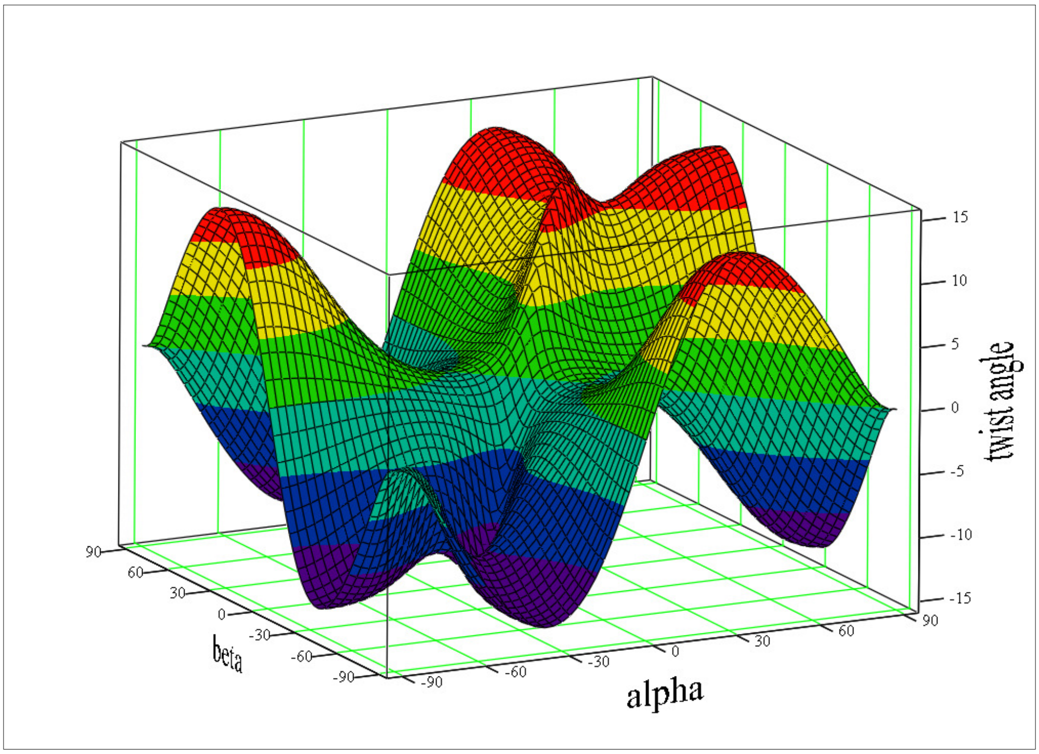
Part 2: Calculate the tube twist angle Φ as function of fibre winding angles (α & β) under axial compression. TA is the Twist Anlge

$$TA(\alpha, \beta) := \left| \begin{array}{l} A \leftarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ Q \leftarrow Q(E1, E2, G, \nu) \\ \theta_1 \leftarrow \alpha \\ \theta_2 \leftarrow \beta \\ \theta_3 \leftarrow \alpha \\ \theta_4 \leftarrow \beta \\ \text{for } i \in 1..4 \\ \quad \left| \begin{array}{l} TT \leftarrow T(\theta_i) \\ A \leftarrow A + t_i \cdot TT \cdot Q \cdot TT^{T} \end{array} \right. \\ a \leftarrow A^{-1} \\ N_1 \cdot a_{1,3} \cdot \frac{L}{R} \cdot deg^{-1} \end{array} \right.$$

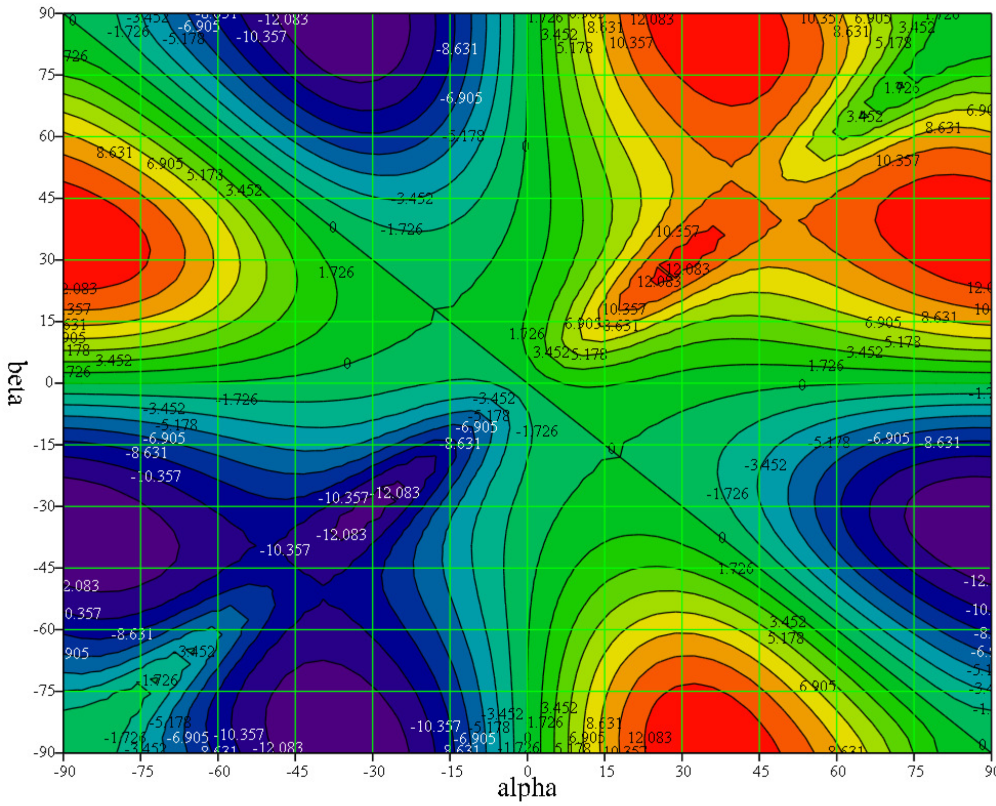
Matrix component subscripts for θ, t, N and a.

1,3 are matrix component subscripts for a.

The plots below shows the graphic representation of how the twist angle changes when the two fibre winding angles (α, β) change. This helps you to find the maximum twist angle.



TA



TA