Design and Analysis of a Laminated Composite Tube

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1 General description and requirement

A cylindrical tube of 4 layers of composite with a layup of $[\alpha\ \beta\ \alpha\ \beta]$is to be designed and by winding tapes cut from a UD carbon-epoxy prepreg sheet on to a cylindrical mandrel of a 25mm radius. For the consideration of practicality, the range of these two winding angles will have to fall in [-75°, -30°] or [+30°, +75°] to the axis of the tube. The thickness of the prepreg is 0.25mm. The tube should be made to a length of 300mm.

Material properties:

Internal pressure: $q = 3 \times 10^6$ Axial force: $P = 25kN$

Tube radius: $R = 25 \times 10^{-3}$ Tube length $L = 0.3$

Elastic constant: $E1 = 236GPa$ $E2 = 5GPa$ $G = 2.6GPa$ $\upsilon_{12} = 0.25GPa$

Strengths: $\sigma^*_{1t} = 3800MPa$ $\sigma^*_{2t} = 41MPa$ $\sigma^*_{1c} = 689MPa$ $\sigma^*_{2c} = 107MPa$ $\tau^*_{12} = 69MPa$

2 Design approach and theory

The thickness of each layers is 0.25mm, thickness in the range is between 0.1 1.0 mm can be seen as lamina. So the developed tube is a laminate with layup: $[\alpha/\beta/\alpha/\beta]$ and the laminate theory can be used for stress analysis.

STEP I: Define the two winding angles:

According to the requirement, the winding angle of 4 layers can be defined:

$$-75^\circ \leq \alpha \leq -30^\circ \text{ or } 75^\circ \leq \alpha \leq 30^\circ$$

$$-75^\circ \leq \beta \leq -30^\circ \text{ or } 75^\circ \leq \beta \leq 30^\circ$$

STEP II: Define the Stress-Strain Relationship $[Q]$ **:**

According to the Laminar Stress-Strain Relationship in Material coordinate system:

$$\{\sigma\} = [Q]\{\epsilon\}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E1}{1-\upsilon^2\frac{E2}{E1}} & \frac{\Upsilon E2}{1-\upsilon^2\frac{E2}{E1}} & 0 \\ \frac{\Upsilon E2}{1-\upsilon^2\frac{E2}{E1}} & \frac{E2}{1-\upsilon^2\frac{E2}{E1}} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

STEP III: Define the Coordinate transformation $[T]$

The winding angle of 4 layers of composite: $\theta = \begin{pmatrix} \alpha \\ \beta \\ \alpha \\ \beta \end{pmatrix}$, the coordinate transformation between (x-y) and (1-2) matrix:

$$\{T\} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 2\cos\theta\sin\theta \\ \cos\theta\sin\theta & -\cos\theta\sin\theta & cos^2\theta - sin^2\theta \end{bmatrix}$$

STEP IV: Define maximum stress failare criterion

According to the maximum stress failure criterion, material will fail when any of the following conditions is violated:

$$\frac{\sigma_1}{\sigma_{1t}^*} \leq 1 \quad if \quad \sigma_1 > 0 \qquad\qquad \frac{|\sigma_1|}{\sigma_{1c}^*} \leq 1 \quad if \quad \sigma_1 < 0$$

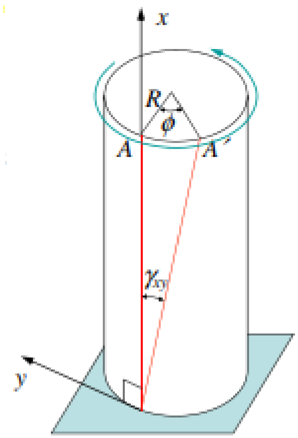
$$\frac{\sigma_2}{\sigma_{2t}^*} \leq 1 \quad if \quad \sigma_2 > 0 \qquad\qquad \frac{|\sigma_2|}{\sigma_{2c}^*} \leq 1 \quad if \quad \sigma_2 < 0$$

$$\frac{|\tau_{12}|}{\tau_{12}^*} \leq 1$$

STEP V: Define Twist angle

Tube is fixed at the bottom end and axially compressed at the top. Generator on the tube deforms by an angle γ_{xy}. Point A at the top end moves to A' by a distance $\gamma_{xy}L$. We can get the twist angle of the tube formula:

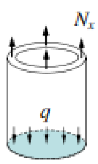
$$\theta = \frac{\gamma_{xy}L}{R}$$

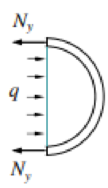


There are two loading conditions, separately analysis:

2.1 Case 1: Internal pressure of 3 MPa (ends closed)

The generalised stresses: $N = \begin{Bmatrix} N_x = \frac{1}{2}qR \\ N_y = qR \\ N_{xy} = 0 \end{Bmatrix}$





Generalised stress-strain Relationship: $\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \end{Bmatrix}$, $[A] = t\,[T]\,[Q]\,[T]^T$

Strain in x-direction: $\epsilon_x = [A]^{-1}\,N$

Lamina 1: The winding angle is α.

$\epsilon = [T_\alpha]\,[A]^{-1\,T}\epsilon_x$

$\sigma = [Q]\,\epsilon$

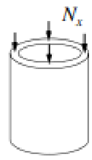
Lamina 2: The winding angle is β.

$\epsilon = \left[T_\beta\right]\,[A]^{-1\,T}\epsilon_x$

$\sigma = [Q]\,\epsilon$

2.2 Case 2:Axial compression of $P = 25kN$

The generalised stresses: $\begin{Bmatrix} N_x = \frac{P}{2\pi R} \\ N_y = 0 \\ N_{xy} = 0 \end{Bmatrix}$



3 Results and analysis to prove the success of the design