STAT370 Lecture 1 Notes

Additive Counting Principle

Sum Rule - k **distinct/disjoint experiments** with n1, n2, …, nk outcomes.
outcomes = n1 + n2 + … + nk

Generalized Sequential Counting Principle

Generalized Product Rule - k **successive** experiments n1, n2, n3
outcomes = n1 * n2 * … * nk

Seating Arrangement problems
3 People, Moe, Larry and Curly
How many seating arrangements? 3 * 2 * 1 = 3! = 6

Why?

**3 Successive Experiments*

1) Choose someone to sit - 3 outcomes
2) Choose someone to sit - 2 outcomes
3) Choose someone to sit - 1 outcomes

Another,

3 guys and 3 girls in six seats
How many seating arrangements? (3! * 3!) + (3! * 3!)

1) Seating order based on gender - 2 outcomes
   BBBGGG
   GGGBBB
2) For each outcome of step 1, 6 successive experiments
   BBBGGG:
   1. *Choose a guy to sit - 3*
   2. Choose a guy to sit - 2
   3. Choose a guy … - 1
   4. *Choose a girl to sit - 3*
   5. Choose a girl … - 2
   6. Choose a girl… - 1
      -> BBBGGG = 3! * 3! outcomes

   GGGBBB:
   1. *Choose a girl to sit - 3*
   2. Choose a girl to sit - 2
   3. Choose a girl … - 1
   4. *Choose a guy to sit - 3*
   5. Choose a guy … - 2
   6. Choose a guy - 1
      -> GGGBBB = 3! * 3! outcomes

Solution: 2(3! * 3!) outcomes

Permutation

- Arrangement of items in a certain order
- n!

*There are n! Permutations of n distinct items

Generalized Permutation Principle (GPP)

4 digit PIn numbers, distinct digits

= Choose 4 people from 10 people to sit in a row with 4 seats
= Choose 4 items from a set of 10 items where order matters

4 successive experiments
1) Choose a digit - 10
2) Choose a digit - 9
3) Choose a digit - 8
4) Choose a digit - 7

GPP = $\frac{n!}{(n-r)!\,r!}$

r = # of distinct objects (4)
n = total # of objects (10)
= 10! / (10 - 4)! = 10!/6!

Practice:

1. # Arrangements of A A A B C

   # Arrangements of A1 A2 A3 B C? 5!

   * Overcounted A A A B C
     - Duplicates with each arrangement of A1 A2 A3
     -

   # Arrangements of A1 A2 A3 = 3!

   Solution = 5! / 3!

2. # Arrangements of A A A B B

   # Arrangements of A1 A2 A3 B1 B2 = 5!

   Overcounted:
   A1 A2 A3 = 3!
   B1 B2 = 2!
   = 3! * 2!

   Solution: 5! / (3! * 2!)