ECE 253 Homework 1

Fall 2015

Question 1. *MATLAB basics*

(1) a

(2) Give a Θ *expression for* $T(n)$*. Hint: compare its value to that at nearby powers of* 2.

(3) Consider the following purported proof that $T(n) = O(n)$ *by induction:*

If $n = 1$*, then* $T(1) = 1 = O(1)$.

If $T(m) = O(m)$ *for* $m < n$*, then*

$$T(n) = 2T(\lfloor n/2 \rfloor) + n = O(n) + O(n) = O(n).$$

Thus, $T(n) = O(n)$.

What is wrong with this proof? Hint: consider the implied constants in the big-Os.

For each of these algorithms, compute the asymptotic runtime in the form $\Theta(-)$.

Question 2 (Big-O Computations, 20 points)**.** *For each of the following functions, determine whether or not the expression in question is* $\Theta(n^c)$ *for some constant* c*, and if so determine the value of such a* c*. Remember to justify your answer.*

- $a(n) = \frac{n^2}{7} + 21 + \log(n)$
- $b(n) = n + (n-1) + (n-2) + \ldots + 1$
- $c(n) = 3^{\lceil log_2(n) \rceil}$
- $d(n) = \lfloor \log_2(n) \rfloor !$
- $e(n) = 2^n$

Question 3 (Cycle Finding, 30 points)**.** *Recall that a* 4*-cycle in a graph* G *is a collection of four vertices* v_1, v_2, v_3, v_4 *so that* $(v_1, v_2), (v_2, v_3), (v_3, v_4)$ *and* (v_4, v_1) *are all edges of* G.

(a) Show that if G *is a graph with* $|E| \geq 2|V|^{3/2}$*, that* G *must contain a* 4*-cycle. Hint: For each vertex* $v \in V$ *consider all the pairs* (u, w) *of vertices so that* u *and* w *are both adjacent to* v*. If the same pair* (u, w) *shows up for two different* v*'s, show that there is a* 4*-cycle.*

(b) Find an efficient algorithm to determine whether or not a given graph G *contains a* 4*-cycle. What is the asymptotic runtime of this algorithm? You should attempt to do better than the trivial algorithm of simply checking all quadruples* v_1, v_2, v_3, v_4 *of vertices.*

Question 4 (Recurrence Relations, 30 points)**.** *Consider the recurrence relation*

$$T(1) = 1, \quad T(n) = 2T(\lfloor n/2 \rfloor) + n.$$

(a) What is the exact value of $T(2^n)$*?*

(b) *Give a Θ expression for $T(n)$. Hint: compare its value to that at nearby powers of 2.*

(c) *Consider the following purported proof that $T(n) = O(n)$ by induction:*

If $n = 1$, then $T(1) = 1 = O(1)$.

If $T(m) = O(m)$ for $m < n$, then

$$T(n) = 2T(\lfloor n/2 \rfloor) + n = O(n) + O(n) = O(n).$$

Thus, $T(n) = O(n)$.

What is wrong with this proof? Hint: consider the implied constants in the big-Os.

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*