ECE 254 Homework 6

Unknown Amplitude, Phase, and Frequency

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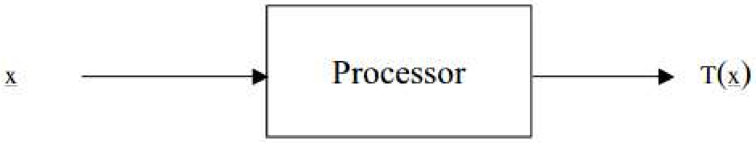
- Title: Unknown Amplitude, Phase, and Frequency
- Objective:

Consider the following processor structure:



Our goal is to decide presence or absence of a signal buried in uncorrelated Gaussian noise

where: H_0: $x(n) = w(n)$, $n = 0,1,\ldots, N\text{-}1$
H_1: $x(n) = s(n) + w(n)$, $n = 0,1,\ldots, N\text{-}1$
$w(n)$ is an uncorrelated Gaussian noise sequence $\sim N(0,\sigma^2)$
$s(n) = A \sin(2\pi f_c n + \phi)$, $f_c = 1/16$
$N = 128$.

I. Generalized Likelihood Ratio Test for Unknown Amplitude, Phase, and Frequency

A. For each of the following problems, express the functional form of the test statistic T(x):

1. Clairvoyant NP detector (i.e. known signal).
2. GLRT unknown amplitude detector.
3. GLRT unknown amplitude and phase detector.
4. GLRT unknown amplitude, phase, and frequency detector.

B. For each of the four detectors in IA, express the functional form of P_D in terms of P_F.

Note: In the case of unknown frequency, allow the number of frequency bins examined (K) to be variable and not fixed at (N/2 – 1).

II. Performance

A. Plot the performance of the clairvoyant NP detector and the three GLRT detectors:

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB

Note: ENR is the energy-to-noise ratio.

B. In the case of unknown frequency, assume that the number of bins examined is K = 8 and K = 64.

- Approach:

  See handwriting.

- Results(including plots):

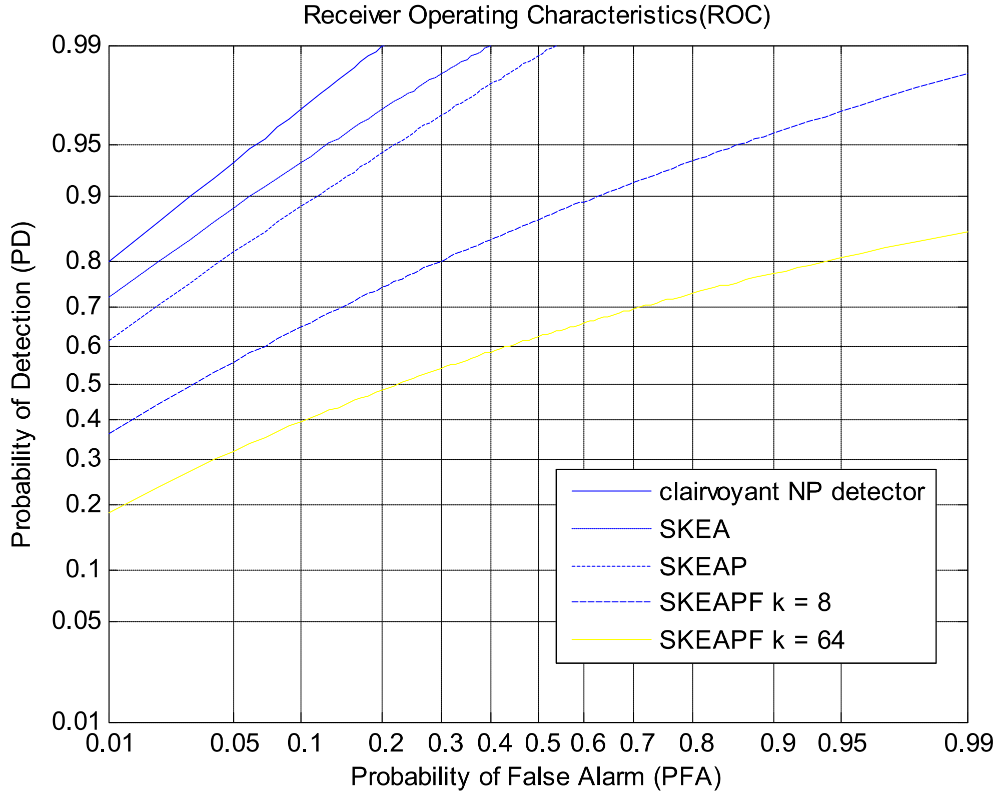


Figure 1 PD vs PF clairvoyant NP detector and the three GLRT detectors

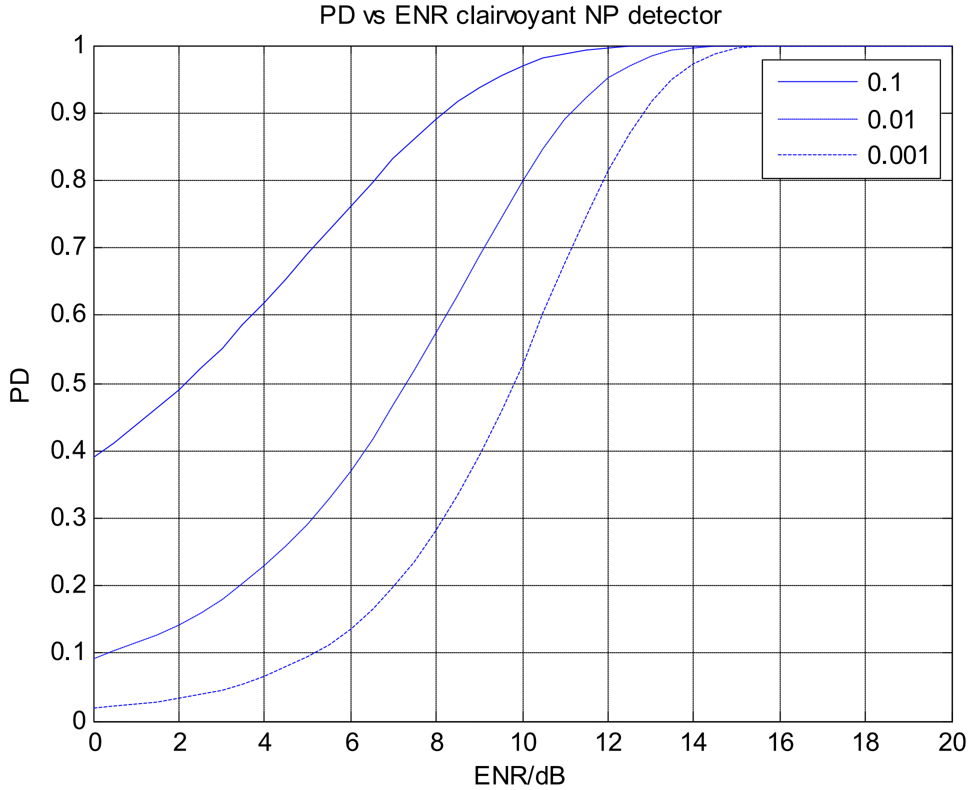


Figure 2 PD vs ENR clairvoyant NP detector

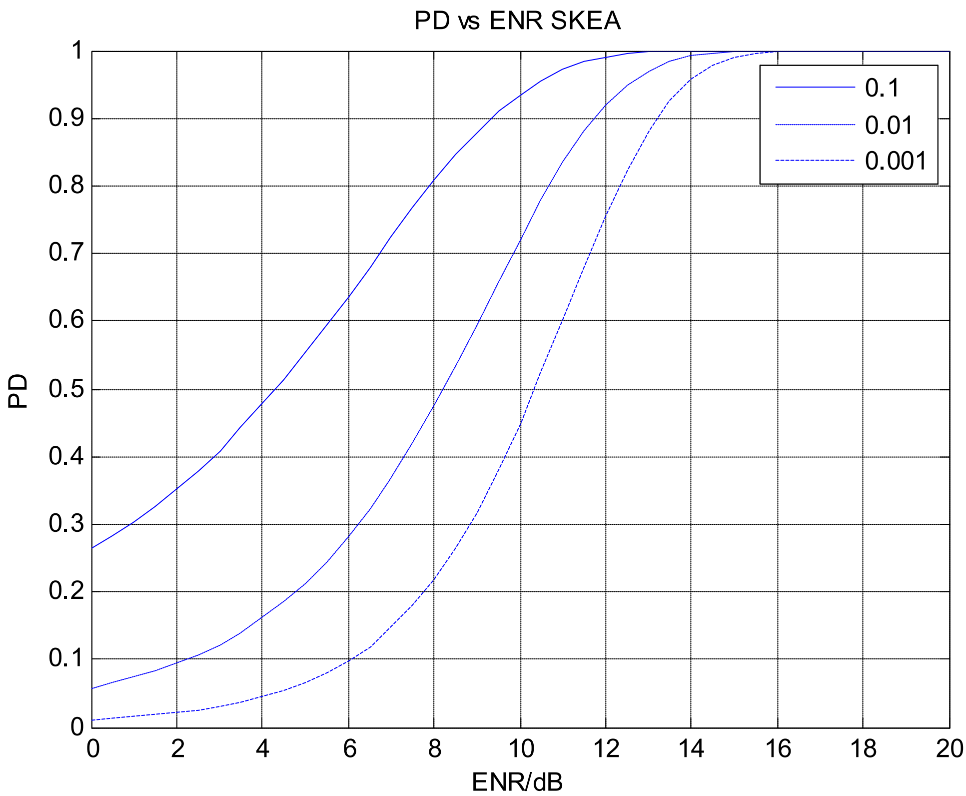


Figure 3 PD vs ENR SKEA

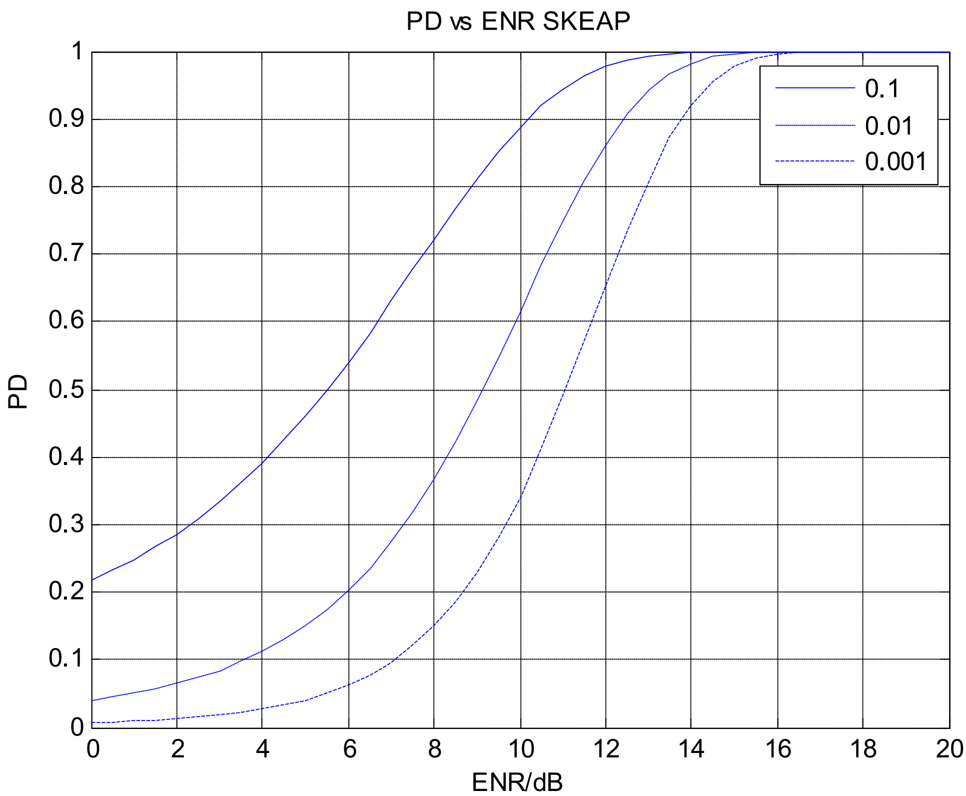


Figure 4 PD vs ENR SKEAP

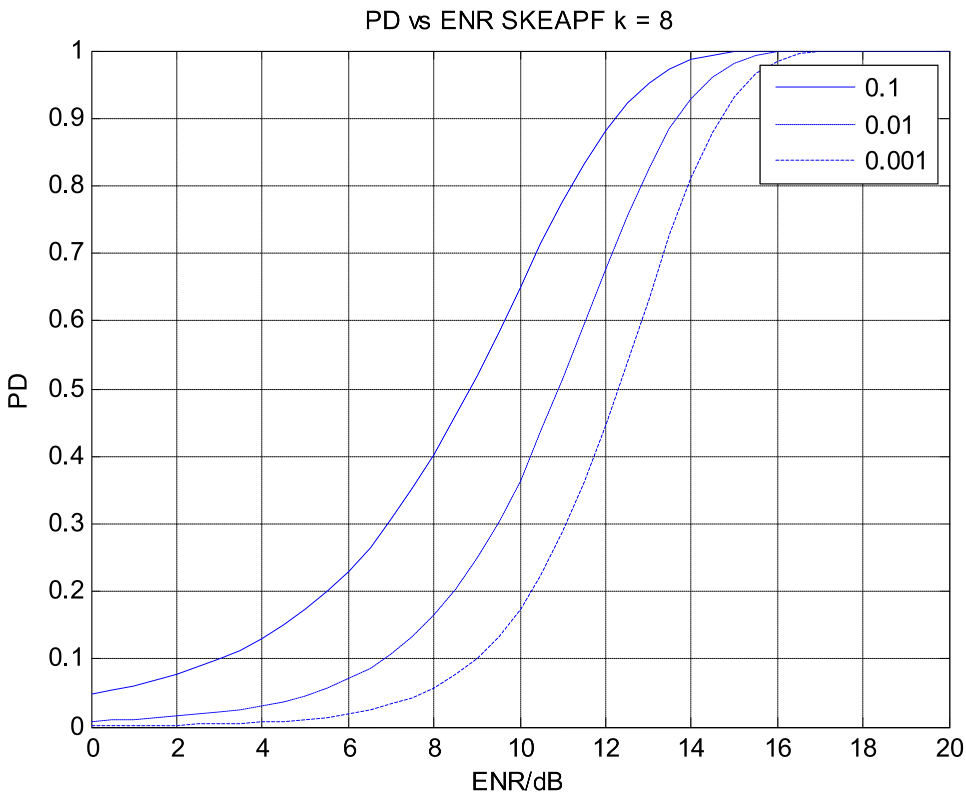


Figure 5 PD vs ENR SKEAPF k=8

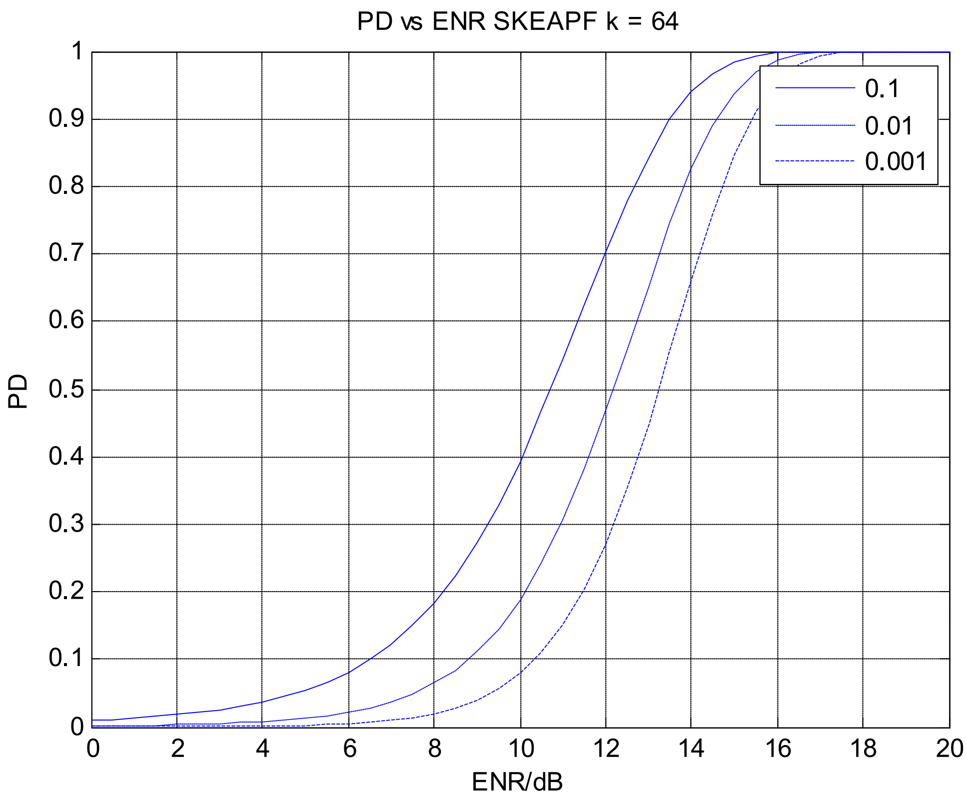


Figure 6 PD vs ENR SKEAPF k = 64

Discussion:

1. For figure 1, we can see that, for GLRT Approach:
    a) From figure 1, we know that the rank of the performance is: clairvoyant NP detector > SKEA > SKEAP > SKEAPF.
    b) We know that clairvoyant NP detector is just SKE problem and we know everything of the signal. And after this, SKEA, SKEAP and SKEAPF have missing properties and each one has more than the previous one. So we can have the conclusion that the more unknown parameters we have, the larger the error is.
    c) This conclusion is not hard to understand. GLRT approach uses Maximum Likelihood Estimation for those unknown parameters and the error is larger when there is more parameters needed to be estimated.
    d) For SKEAPF problem, with the increasing of the PD, the error is larger and larger. And the performance is better when k = 8 compared to k = 64.
2. For figure 2 – 6, we know that:
    a) With the decrease of pf, the curve (pd vs ENR) turn to right a little bit each time.
    b) With the decrease of pf, the slope of the curve becomes steeper.
    c) With the increase of ENR, the performance becomes better.

d) With different PF, the difference of PD when ENR is fixed in clairvoyant, SKEA, SKEAP is larger than SKEAPF, especially in low ENR.

e) On a fixed PF in order to obtain PD=1 we need least ENR for clairvoyant NP or SKEA detector and the uncertain amplitude, phase and frequency needs more or larger ENR than others.

f) We can still find that when k is large, the performance is worse. From the Approach part, we know that test statistic T(x) is related to k and In the case of unknown frequency, allow the number of frequency bins examined (K) to be variable and not fixed at (N/2 – 1).

Appendix:

Hw6.m

```
%% PD vs PF
% clairvoyant NP detector
ENR=10.^(10/10);
d=(ENR)^(1/2);
PFcnd=0.01:0.01:1;
PDcnd=Q(Qinv(PFcnd)-d);
figure(1)
probpaper(PFcnd,PDcnd, '-')

% SKEA
PFskea=0.01:0.01:1;
PDskea = Q(Qinv(PFskea/2)-d)+Q(Qinv(PFskea/2)+d);
figure(1)
hold on
probpaper(PFskea,PDskea, ':')

% SKEAP
PFskeap=0.01:0.01:1;
PDskeap=zeros(1,100);
for i=1:100
    PDskeap(i)=Qchipr2(2,ENR,2*log(1./PFskeap(i)),1e-5);
end
figure(1)
probpaper(PFskeap,PDskeap, '-.')

% SKEAPF k = 8
PFskeapf=0.01:0.01:1;
K1=8;
x1=-2*log(1-(1-PFskeapf).^(1/K1));
PDskeapf8=zeros(1,100);
for i=1:100
    PDskeapf8(i)=Qchipr2(2,ENR,x1(i),1e-5);
end
figure(1)
probpaper(PFskeapf,PDskeapf8, '--')

% SKEAPF k = 64
PFskeapf=0.01:0.01:1;
K2=64;
x2=-2*log(1-(1-PFskeapf).^(1/K2));
```

```
PDskeapf64=zeros(1,100);
for i=1:100
        PDskeapf64(i)=Qchipr2(2,ENR,x2(i),1e-5);
end
figure(1)
probpaper(PFskeapf,PDskeapf64,'y')

legend('clairvoyant NP detector','SKEA','SKEAP','SKEAPF k = 8','SKEAPF k = 64');

%% PD vs ENR
PFA1=10^-1;
PFA2=10^-2;
PFA3=10^-3;
ENR=0:0.5:20;
lamda=10.^(ENR./10);

% clairvoyant NP detector
d=(10.^(ENR/10)).^(1/2);
PDske1=Q(Qinv(PFA1)-d);
PDske2=Q(Qinv(PFA2)-d);
PDske3=Q(Qinv(PFA3)-d);
figure(2)
plot(ENR,PDske1,'-')
hold on
plot(ENR,PDske2,':')
plot(ENR,PDske3,'-.')
grid;
title('PD vs ENR clairvoyant NP detector');
xlabel('ENR/dB');ylabel('PD');
legend('0.1','0.01','0.001');

% SKEA
PDskea1=Q(Qinv(PFA1/2)-d)+Q(Qinv(PFA1/2)+d);
PDskea2=Q(Qinv(PFA2/2)-d)+Q(Qinv(PFA2/2)+d);
PDskea3=Q(Qinv(PFA3/2)-d)+Q(Qinv(PFA3/2)+d);
figure(3)
plot(ENR,PDskea1,'-')
hold on
plot(ENR,PDskea2,':')
plot(ENR,PDskea3,'-.')
grid;
title('PD vs ENR SKEA');
xlabel('ENR/dB');ylabel('PD');
legend('0.1','0.01','0.001');
```

```
% SKEAP
PD1skeap=zeros(1,41);
PD2skeap=zeros(1,41);
PD3skeap=zeros(1,41);
for i=1:41
PD1skeap(i)=Qchipr2(2,lamda(i),2*log(1./PFA1),1e-5);
PD2skeap(i)=Qchipr2(2,lamda(i),2*log(1./PFA2),1e-5);
PD3skeap(i)=Qchipr2(2,lamda(i),2*log(1./PFA3),1e-5);
end
figure(4)
plot(ENR,PD1skeap,'-')
hold on
plot(ENR,PD2skeap,':')
plot(ENR,PD3skeap,'-.')
grid;
title('PD vs ENR SKEAP');
xlabel('ENR/dB');ylabel('PD');
legend('0.1','0.01','0.001');

% SKEAPF k = 8
K1=8;
x1=-2*log(1-(1-PFA1).^(1/K1));
x2=-2*log(1-(1-PFA2).^(1/K1));
x3=-2*log(1-(1-PFA3).^(1/K1));
PD1skeapf=zeros(1,41);
PD2skeapf=zeros(1,41);
PD3skeapf=zeros(1,41);
for i=1:41
    PD1skeapf(i)=Qchipr2(2,lamda(i),x1,1e-5);
    PD2skeapf(i)=Qchipr2(2,lamda(i),x2,1e-5);
    PD3skeapf(i)=Qchipr2(2,lamda(i),x3,1e-5);
end
figure(5)
plot(ENR,PD1skeapf,'-')
hold on
plot(ENR,PD2skeapf,':')
plot(ENR,PD3skeapf,'-.')
grid;
title('PD vs ENR SKEAPF k = 8');
xlabel('ENR/dB');ylabel('PD');
legend('0.1','0.01','0.001');

% SKEAPF k = 64
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```
K2=64;
x4=-2*log(1-(1-PFA1).^(1/K2));
x5=-2*log(1-(1-PFA2).^(1/K2));
x6=-2*log(1-(1-PFA3).^(1/K2));
PD4skeapf=zeros(1,41);
PD5skeapf=zeros(1,41);
PD6skeapf=zeros(1,41);
for i=1:41
PD4skeapf(i)=Qchipr2(2,lamda(i),x4,1e-5);
PD5skeapf(i)=Qchipr2(2,lamda(i),x5,1e-5);
PD6skeapf(i)=Qchipr2(2,lamda(i),x6,1e-5);
end
figure(6)
plot(ENR,PD4skeapf,'-')
hold on
plot(ENR,PD5skeapf,':')
plot(ENR,PD6skeapf,'-.')
grid;
title('PD vs ENR SKEAPF k = 64');
xlabel('ENR/dB');ylabel('PD');
legend('0.1','0.01','0.001');
```