End-Term Project

Note: You should treat this project as a take-home exam. Thus, you should neither give nor receive assistance on completing the project. See the "Academic Integrity" document (Misc_Handouts/Overview folder). Select one of the following three problems to work. Include grid lines on your P_D vs. P_F and P_D vs. ENR plots. Also, include in an appendix your Matlab code.

I. Gaussian Signal in Gaussian Noise

Explore the trade-offs between distributing the signal energy over different numbers of samples for detecting an uncorrelated Gaussian signal ~$N(0, \sigma_s^2)$ buried in uncorrelated Gaussian noise ~$N(0, \sigma^2)$ (see IC, IIC, and III in HW #4, "Rayleigh Fading Signal").

A. Express the functional form of the test statistic $T(\mathbf{x})$.

B. Express the functional form of P_D and P_F in terms of the threshold, SNR = σ_s^2/σ^2, and number of samples N. Indicate how you determine the threshold for a given value of P_F when generating plots of P_D vs. ENR (e.g. show the iterative formula in Prob. 5.1 and comment on how it is used).

C. Compute and plot the performance of the detection receiver for N = 2, 4, 8, 16, 32, and 64.

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB and 15 dB. Plot all cases of N for a given ENR on one plot.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB. Plot all cases of N for a given P_F on one plot.

Note: ENR = (N)(SNR) = (N)(σ_s^2/σ^2) is the expected energy-to-noise ratio.

D. Explore the structure of P_D vs. N (linear axes) for fixed ENR and P_F. Investigate for N = {2, 4, 6, 8, 10, …, 64}, ENR = {10,15} dB, and P_F = {0.001, 0.01, 0.1}. Plot all cases of P_F for a given ENR on one plot. For a given ENR and P_F, is there an optimum N? Provide a table showing the value of N for maximum P_D for each (ENR, P_F) case.

Note: The iterative formula in Problem 5.1 is valid only for N even.

II. Unknown Amplitude, Phase, Frequency, and Arrival Time

As discussed in Sect. 7.6.4 and Example 7.5 in [1], the GLRT processor for unknown amplitude, phase, frequency, and arrival time has considerable practical importance for radar/sonar systems. Assume the detection problem is similar to that defined in HW #6, "Unknown Amplitude, Phase, and Frequency" with the addition of unknown arrival time, n_0.

A. For each of the following processors, express the functional form of the test statistic $T(\mathbf{x})$:

1. Clairvoyant NP detector (i.e. known signal).

2. GLRT unknown amplitude and phase detector.

3. GLRT unknown amplitude, phase, and frequency detector.

4. GLRT unknown amplitude, phase, frequency, and arrival time detector.

B. For each of the four detectors above, express the functional form of P_D in terms of P_F. In the case of unknown frequency, allow the number of frequency bins examined to be variable (K) and not fixed at (N/2 – 1). In the case of unknown arrival time, assume that there are I nonoverlapping data blocks.

C. Compute and plot the performance for each of the four detectors above. Assume K = I = 8.

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB (one plot).

    a. Evaluate the theoretical performance expressions.

    b. Determine the performance via Monte Carlo simulation.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB (three separate plots, one plot for each value of P_F).

    a. Evaluate the theoretical performance expressions.

Note: ENR is the energy-to-noise ratio. For the theoretical P_D vs. P_F curves, compare the exact versus approximate results using Eq. 7.36 versus Eq. 7.37 and Eq. 7.38 in [1]. For the P_D vs. ENR curves, simply use Eq. 7.38 but comment on the region of P_F for which this expression is valid.

III. Signal Amplitude and Noise Variance Unknown

Our goal is to decide presence or absence of a signal buried in uncorrelated Gaussian noise where:

H_0: $x(n) = w(n)$, $\quad n = 0,1,\ldots, N-1$
H_1: $x(n) = As(n) + w(n)$, $\quad n = 0,1,\ldots, N-1$
$w(n)$ is an uncorrelated Gaussian noise sequence $\sim N(0,\sigma^2)$, A is the unknown signal amplitude, and $s(n)$ is the deterministic signal shape.

A. For each of the following processors, express the functional form of the test statistic $T(\mathbf{x})$:

1. Clairvoyant NP detector (i.e. known signal and noise variance).

2. GLRT unknown signal amplitude detector.

3. GLRT unknown signal amplitude and noise variance detector.

B. For each of the three detection receivers above, express the functional form of P_D and P_F in terms of ENR = $(A^2\mathbf{s}^T\mathbf{s})/(\sigma^2)$.

C. Compute and plot the performance of the three detection receivers above for N = 8, 16, and 32.

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB (one plot).

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB (three separate plots, one plot for each value of P_F).

Note: ENR is the energy-to-noise ratio.

D. Compute and plot the performance of the three detection receivers above for 10 log (ENR) = 10 dB.

1. P_D (linear) vs. N ($P_F = 0.01$) for N = {2, 3, 4, …, 30}   (one plot)

2. P_F (linear) vs. N ($P_D = 0.8$) for N = {2, 3, 4, …, 30}   (one plot)

References

[1] S. Kay. *Fundamentals of Statistical Signal Processing*. *Vol. II: Detection Theory*. Prentice-Hall: 1998.

[2] H.L. Van Trees. *Detection, Estimation, and Modulation Theory*. *Part I: Detection, Estimation, and Linear Modulation Theory*. John Wiley and Sons: 1968.