ECE 254 Homework 5

Uncertain Amplitude Signal

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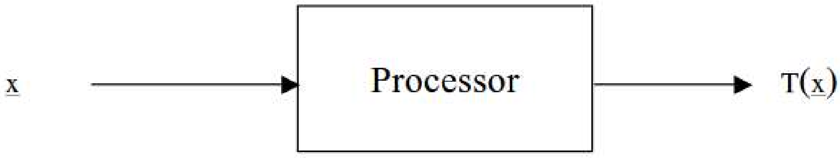
- Title: Uncertain Amplitude Signal
- Objective:

Consider the following processor structure:



Our goal is to decide presence or absence of a signal buried in uncorrelated Gaussian noise

where: H_0: $x(n) = w(n)$, $n = 0,1,\ldots, N-1$

H_1: $x(n) = A\, s(n) + w(n)$, $n = 0,1,\ldots, N-1$

$w(n)$ is an uncorrelated Gaussian noise sequence $\sim N(0,\sigma^2)$

$s(n) = \sin(2\pi f_c n + \phi)$, $f_c = 1/16$

$N = 128$.

1.

Bayesian Approach (SKEA)

A. Assume $A \sim N(0,\sigma_A^2)$. Summarize briefly the analytical derivation of the test statistic and performance for the Bayes optimum SKEA detection receiver.

B. Plot the performance of the SKEP, SKEA, and Rayleigh fading signal optimum detectors:

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}, 10^{-2}$, and 10^{-3} and ENR from 0 to 20 dB.

Note: ENR is the expected energy-to-noise ratio.

2.

Generalized Likelihood Ratio Test Approach (GLRT)

A. Summarize briefly the analytical derivation of the test statistic and performance for the GLRT for uncertain amplitude.

B. Plot the performance of the clairvoyant NP detector and the uncertain amplitude GLRT:

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB.

Note: ENR is the energy-to-noise ratio.

- Approach:

  See handwriting.

- Results(including plots):

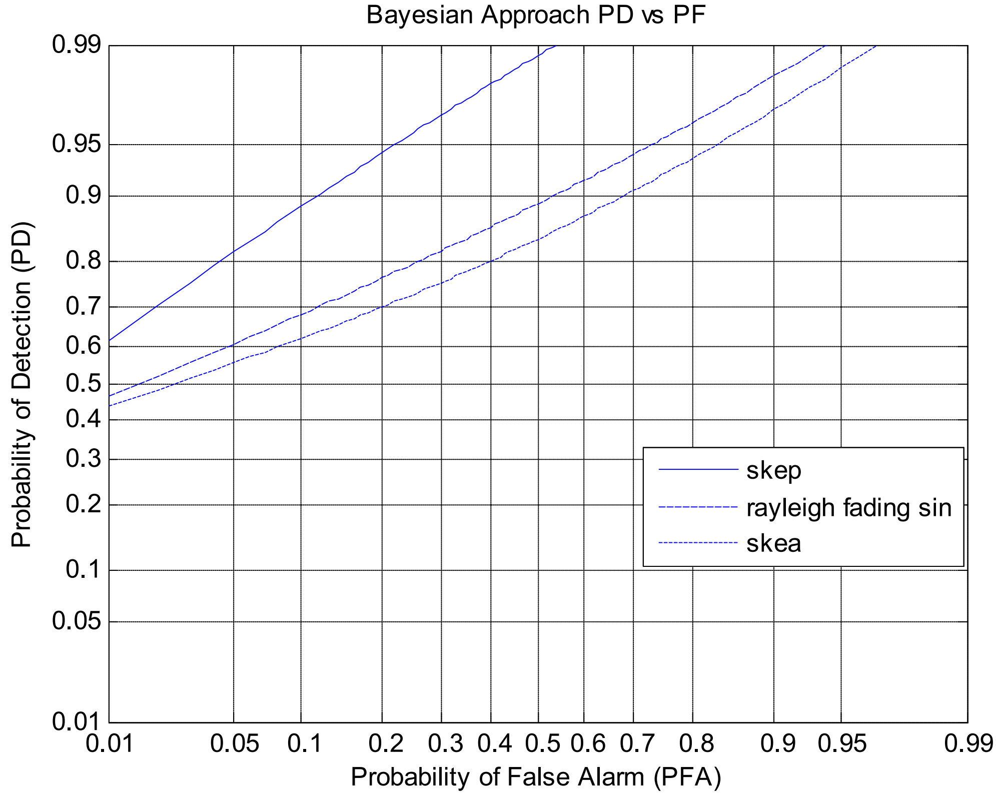


Figure 1 Bayesian Approach PD vs PF on normal probability paper for ENR = 10

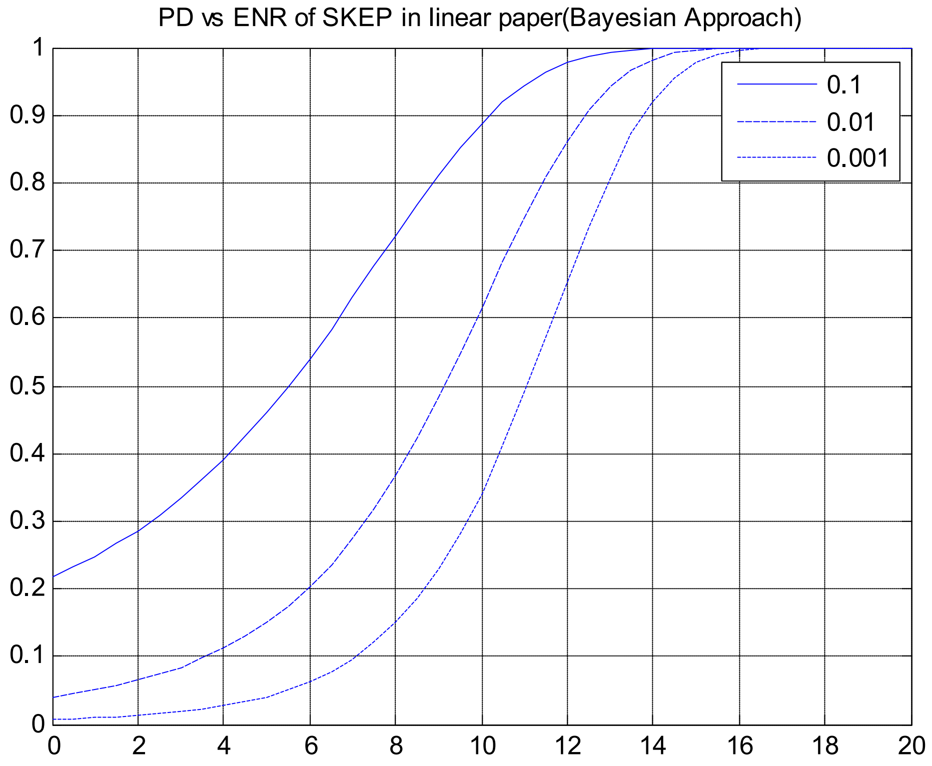


Figure 2 Bayesian Approach PD vs ENR for PF = 10^-1, 10^-2, 10^-3 with SKEP

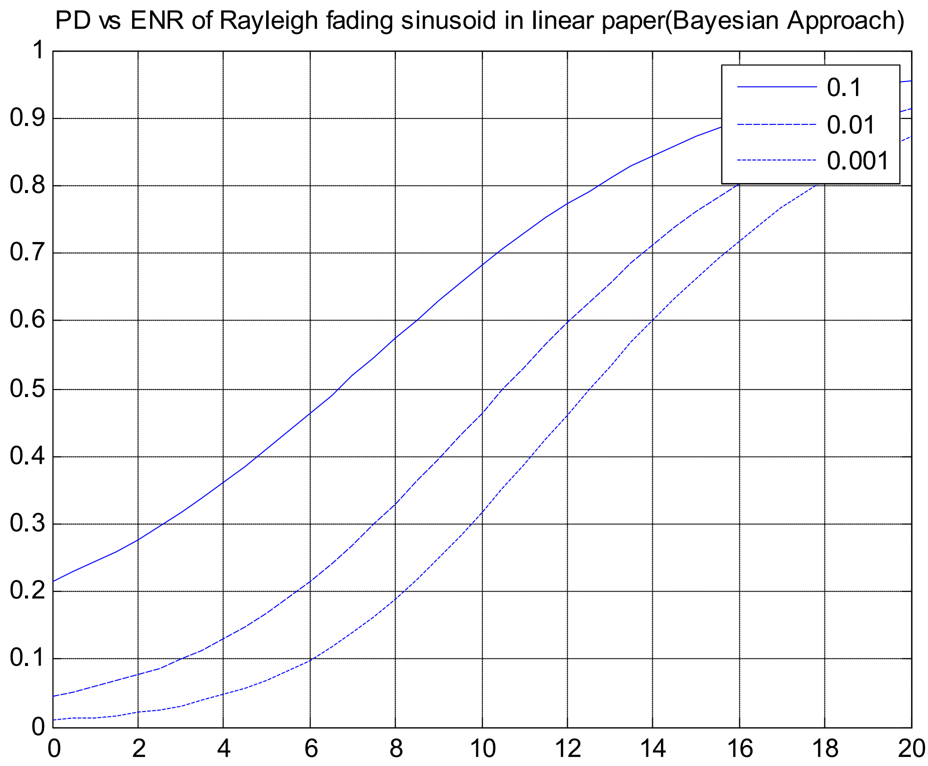


Figure 3 Bayesian Approach PD vs ENR for PF = 10^-1, 10^-2, 10^-3 with Rayleigh fading sinusoid

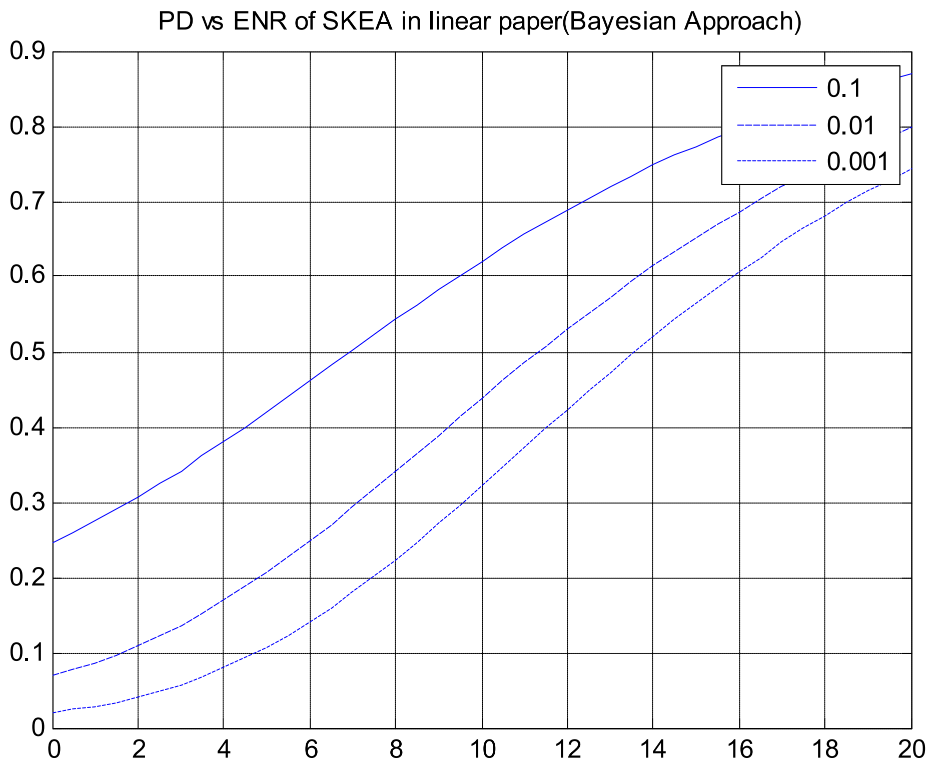


Figure 4 Bayesian Approach PD vs ENR for PF = 10^-1, 10^-2, 10^-3 with SKEA

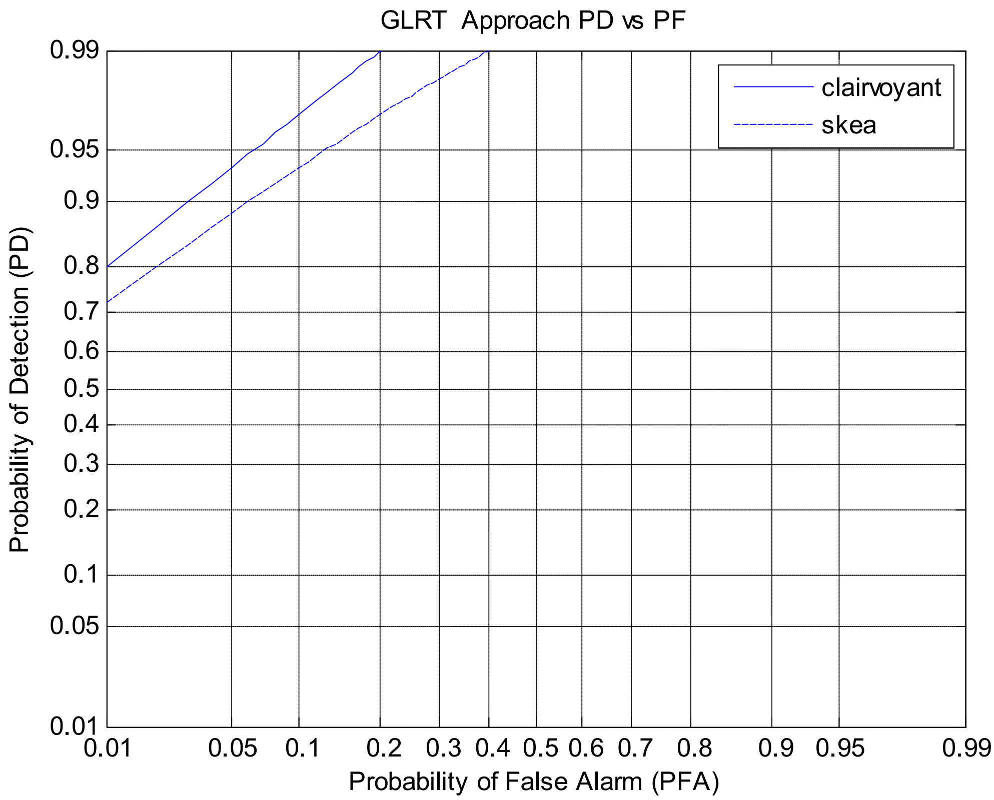


Figure 5 GLRT Approach PD vs PF on normal probability paper for ENR = 10

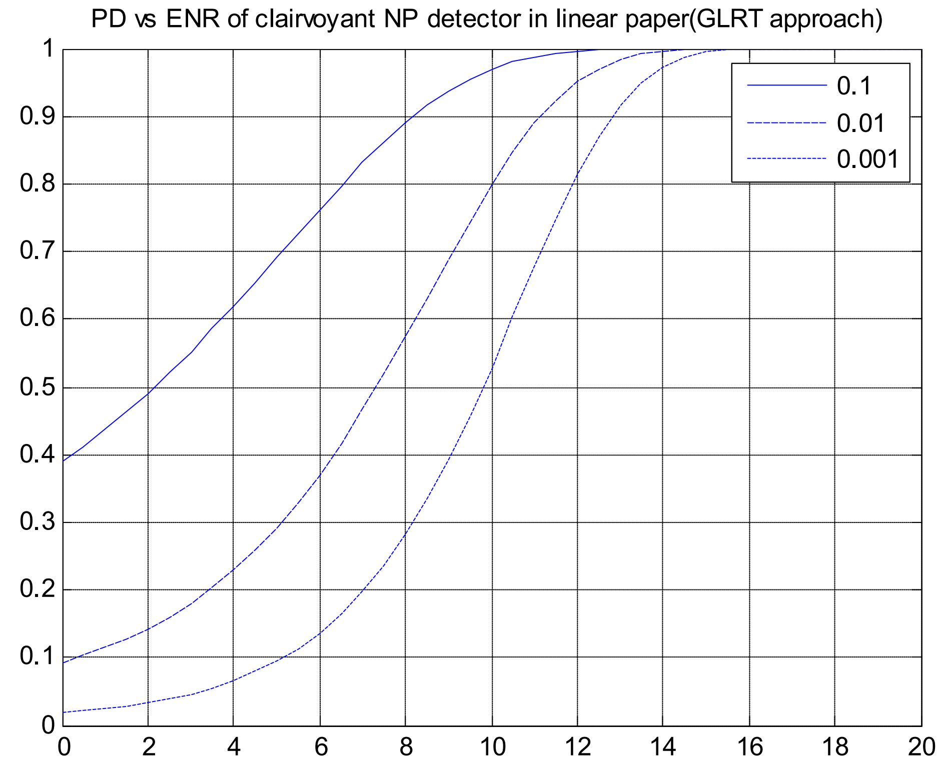


Figure 6 GLRT Approach PD vs ENR for PF = 10^-1, 10^-2, 10^-3 with clarivoyant NP

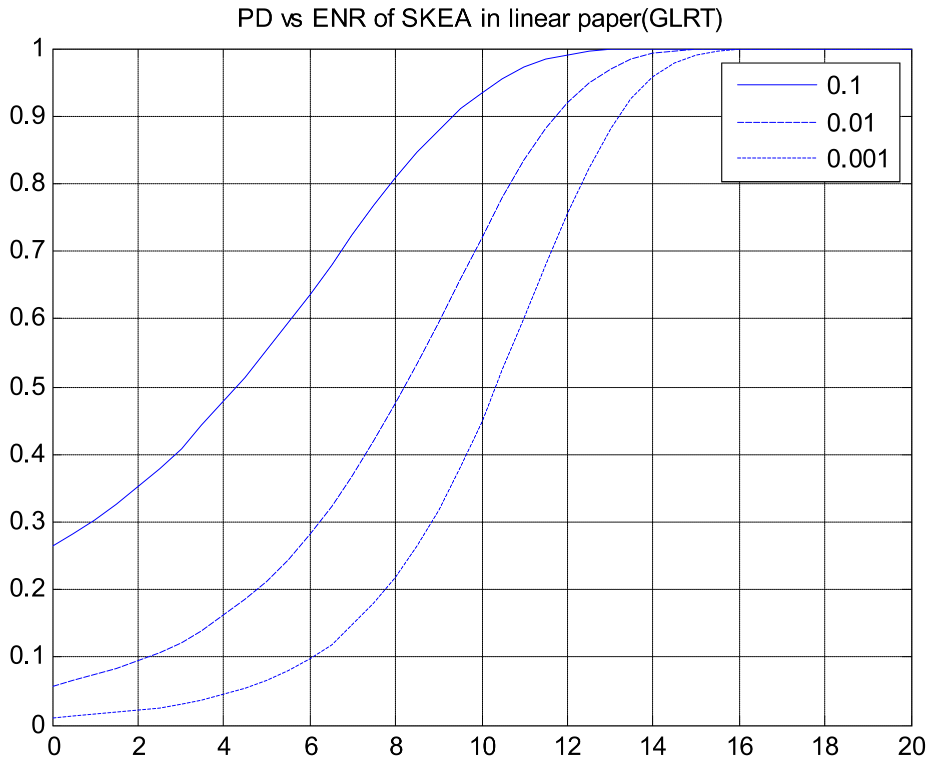


Figure 7 GLRT Approach PD vs ENR for PF = 10^-1, 10^-2, 10^-3 with SKEA

Discussion:

1. For figure 1, we can see that, for Bayesian Approach:
   a) SKEP performs the best, Rayleigh fading sinusoid is the second and the SKEA is the worst. This may be because that Rayleigh fading sinusoid is using two cross-correlation(one sin and one cos) while the SKEA is only using one.
2. For figure2 – figure4, for Bayesian Approach:
   a) With the decrease of pf, the curve (pd vs ENR) turn to right a little bit each time.
   b) With the decrease of pf, the slope of the curve becomes steeper.
   c) With the increase of ENR, the performance becomes better.
   d) The slope of the curve in SKEP is steeper than others. The next is Rayleigh fading sinusoid and the last one is SKEA.
3. For figure5 – figure7, for GLRT approach:
   a) Clairvoyant NP detector performs better than SKEA.
   b) With the decrease of pf, the curve (pd vs ENR) turn to right a little bit each time.
   c) The slope of the curve in GLRT approach with SKEA is steeper than that of Bayesian Approach.

- Appendix:

Hw5.m

```
%% Bayesian PD vs PF
% SKEP
ENR=10.^(10/10);
lambda=ENR;
PFA1=0.01:0.01:1;
x1=-2*log(PFA1);
PD1=Qchipr2(2,lambda,x1,1e-5);
figure(1)
probpaper(PFA1,PD1, 'r');
% Rayleigh
PFA2=0.01:0.01:1;
PD2=PFA2.^(1/(1+ENR/2));
figure(1)
hold on
probpaper(PFA2,PD2, 'g')
% SKEA
PFA3=0.01:0.01:1;
PF=PFA3/2;
PD3=2*Q(1/(ENR+1)^(1/2)*Qinv(PF));
figure(1)
probpaper(PFA3,PD3, 'b')

%% Bayesian PD vs ENR
PFA1=10^-1;
PFA2=10^-2;
PFA3=10^-3;
ENR=0:0.5:20;
% SKEP
lambda=10.^(ENR/10);
x1=2*log(1/PFA1);
x2=2*log(1/PFA2);
x3=2*log(1/PFA3);
PD1=zeros(1,41);
PD2=zeros(1,41);
PD3=zeros(1,41);
for i=1:41
    PD1(i)=Qchipr2(2,lambda(i),x1,1e-5);
    PD2(i)=Qchipr2(2,lambda(i),x2,1e-5);
    PD3(i)=Qchipr2(2,lambda(i),x3,1e-5);
end
figure(2)
```

```
plot(ENR,PD1,'r')
hold on
plot(ENR,PD2,'g')
plot(ENR,PD3,'b')
grid;

% Rayleigh
x=10.^(ENR/10);
y=1./(x/2+1);
PD4=PFA1.^y;
PD5=PFA2.^y;
PD6=PFA3.^y;
figure(3)
plot(ENR,PD4,'r')
hold on
plot(ENR,PD5,'g')
plot(ENR,PD6,'b')
grid;

%SKEA
avgENR=10.^(ENR/10);
x7=Qinv(PFA1/2);
x8=Qinv(PFA2/2);
x9=Qinv(PFA3/2);
PD7=2*Q(1./(avgENR+1).^(1/2)*x7);
PD8=2*Q(1./(avgENR+1).^(1/2)*x8);
PD9=2*Q(1./(avgENR+1).^(1/2)*x9);
figure(4)
plot(ENR,PD7,'r')
hold on
plot(ENR,PD8,'g')
plot(ENR,PD9,'b')
grid;

%% GLRT PD vs PF
% clairvoyant NP detector
ENR=10.^(10/10);
d=(ENR)^(1/2);
PFcl=0.01:0.01:1;
PDcl=Q(Qinv(PFcl)-d);
figure(5)
probpaper(PFcl,PDcl,'r')

% SKEA
```

```
PFskea=0.01:0.01:1;
PDskea=Q(Qinv(PFskea/2)-d)+Q(Qinv(PFskea/2)+d);
figure(5)
hold on
probpaper(PFskea,PDskea,'g')
grid;

%% GLRT PD vs ENR
% clairvoyant NP detector
ENR=0:0.5:20;
d=(10.^(ENR/10)).^(1/2);
PFA1=10^-1;
PFA2=10^-2;
PFA3=10^-3;
PDske1=Q(Qinv(PFA1)-d);
PDske2=Q(Qinv(PFA2)-d);
PDske3=Q(Qinv(PFA3)-d);
figure(6)
plot(ENR,PDske1,'r')
hold on
plot(ENR,PDske2,'g')
plot(ENR,PDske3,'b')
grid;

%SKEA
PDskea1=Q(Qinv(PFA1/2)-d)+Q(Qinv(PFA1/2)+d);
PDskea2=Q(Qinv(PFA2/2)-d)+Q(Qinv(PFA2/2)+d);
PDskea3=Q(Qinv(PFA3/2)-d)+Q(Qinv(PFA3/2)+d);
figure(7)
plot(ENR,PDskea1,'r')
hold on
plot(ENR,PDskea2,'g')
plot(ENR,PDskea3,'b')
grid;
```