Rayleigh Fading Signal

Consider the following processor structure:

$$\underline{x} \longrightarrow \boxed{\text{Processor}} \longrightarrow T(\underline{x})$$

Our goal is to decide presence or absence of a signal buried in uncorrelated Gaussian noise

where: H_1: $x(n) = s(n) + w(n)$, $n = 0,1,\ldots, N-1$

H_0: $x(n) = w(n)$, $n = 0,1,\ldots, N-1$

$w(n)$ is an uncorrelated Gaussian noise sequence $\sim N(0,\sigma^2)$

I. Consider three different classes of signals:

A. $s(n) = A \sin(2\pi f_c n + \phi)$, $f_c = 1/16$

A known and ϕ uniformly distributed.

B. $s(n) = A \sin(2\pi f_c n + \phi)$, $f_c = 1/16$

A Rayleigh distributed and ϕ uniformly distributed.

C. $s(n) = w_s(n)$

Uncorrelated Gaussian signal $\sim N(0,\sigma_s^2)$

II. Summarize briefly the analytical derivation of the test statistic and performance for the following optimum detection receivers:

A. SKEP (N = 128)

B. Rayleigh fading sinusoid (N = 128)

C. Energy detector (N = 128 and N = 16).

Express P_D in terms of P_F for the SKEP and Rayleigh fading sinusoid processors.

III. Plot the performance of the processors in II above as:

A. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB.

B. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 30 dB.

ENR is the expected energy-to-noise ratio.

Notes:

1. Include grid lines on your performance plots in III above.

2. For the energy detector, see Prob. 5.1 in [1] (pp. 176-177) for an iterative formula to calculate the threshold for a given P_F. Note that γ'' defined on p. 144 and used in the expression for P_D differs from that defined on p. 176 (the iterative formula) by a factor of 2.

Reference

[1] S. Kay. Fundamentals of Statistical Signal Processing. Vol. II: Detection Theory. Prentice-Hall (1998).