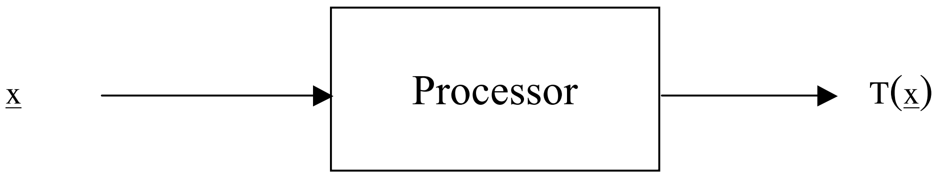
Unknown Amplitude, Phase, and Frequency

Consider the following processor structure:



Our goal is to decide presence or absence of a signal buried in uncorrelated Gaussian noise

where: H_0: $x(n) = w(n)$, $n = 0,1,\ldots, N-1$
H_1: $x(n) = s(n) + w(n)$, $n = 0,1,\ldots, N-1$
$w(n)$ is an uncorrelated Gaussian noise sequence $\sim N(0,\sigma^2)$
$s(n) = A \sin(2\pi f_c n + \phi)$, $f_c = 1/16$
$N = 128$.

I. Generalized Likelihood Ratio Test for Unknown Amplitude, Phase, and Frequency

A. For each of the following problems, express the functional form of the test statistic $T(\underline{x})$:

1. Clairvoyant NP detector (i.e. known signal).

2. GLRT unknown amplitude detector.

3. GLRT unknown amplitude and phase detector.

4. GLRT unknown amplitude, phase, and frequency detector.

B. For each of the four detectors in IA, express the functional form of P_D in terms of P_F.

Note: In the case of unknown frequency, allow the number of frequency bins examined (K) to be variable and not fixed at (N/2 – 1).

II. Performance

A. Plot the performance of the clairvoyant NP detector and the three GLRT detectors:

1. P_D vs. P_F on normal probability paper for 10 log (ENR) = 10 dB.

2. P_D (linear) vs. ENR (dB) for $P_F = 10^{-1}$, 10^{-2}, and 10^{-3} and ENR from 0 to 20 dB

   Note: ENR is the energy-to-noise ratio.

B. In the case of unknown frequency, assume that the number of bins examined is K = 8 and K = 64.

Note:

1. Include grid lines on your performance plots in Part IIA above.