

Mid-Term Project

Note: You should treat this project as a take-home exam. Thus, you should neither give nor receive assistance on completing the project. See the “Academic Integrity” document (Misc_Handouts/Overview folder). Select one of the following five problems to work. Include in an appendix your Matlab code.

I. Matched Filters

A. Ambiguity Functions

For all four signal types in HW #1 “Matched Filters,” plot $s(n)$ and compute/plot $|S(f)|^2$ (dB). In addition, compute/plot (gray level or color but not pseudo-3D) the magnitude-squared (dB) output of the mismatched matched filter where $x(n) = s(n-\tau')$ (noise-free), $s(n)$ as specified, and the impulse response of the matched filter purposely is mismatched in f_c (i.e. matched to different candidate Doppler shifts). By incrementing the mismatch in f_c , generate a surface for each signal type and compare their structures. For your “ambiguity surface” plots, you should normalize the entire surface so that the largest value is unity (0 dB) then use dynamic ranges as requested below.

Explore values of mismatch in f_c from 0 to 0.125 cycles/sample. Note that the intent is that this be the entire range of f_c to explore for the mismatched filters which is ± 0.0625 cycles/sample from the carrier frequency of the reference signal $f_c = 0.0625$ cycles/sample. Implement a parallel bank of at least 256 mismatched filters across this band.

Note also that here we are just exploring the impact of mismatched f_c . Thus, you should keep the lengths of the impulse responses of the mismatched filters the same as in the original definitions of the signals (i.e. $L = 16$ in the case of the short tone ping and $L = 128$ for the long tone, FM, and PRN pings).

Your “ambiguity surface” plots should have their peak value in the center of the surface (at $\tau = 128$ and 0 Doppler shift). Rather than plotting just a single dynamic range from the normalized peak of 0 dB, generate plots with 10, 20, 30, and 40 dB dynamic range to better explore the structure of the surfaces.

In addition, you also should plot 2D cross-sections (slices) through the surfaces at: (1) $\tau = 128$ (i.e. 0 time offset) and (2) 0 Doppler shift. These should be normalized to 0 dB peak and plotted with just a single dynamic range of 30 dB. Note that the peak in the short tone ping Doppler cross-section will be offset slightly (lower in frequency) from 0 Doppler shift. This is due to the calculation not normalizing out the energy of the mismatched filter impulse response which changes as Doppler is varied.

When you display your ambiguity surfaces, you can use a gray-scale or color plot. In Matlab, this likely is most easily generated using “imagesc.” Note that if you generate a gray-scale plot, dark should represent high levels and light as low levels. You also should include a “colorbar” so that it is clear what level each color/gray corresponds to. Also, make sure to annotate your axes so that it is clear how to interpret your display.

B. Transponder Reply Data

Data (8192 points) is available from a $f = 12$ kHz acoustic transponder reply ($f_s = 50$ kHz). Design a detector for these 10 ms duration pings and illustrate its use in the processing of this data. Note that the signal arriving at the ship-deployed transducer will contain more than one arrival due to the multipath propagation characteristics of the channel.

Processing of the data should include the following: (1) Plot the time series, the corresponding power spectrum (dB), and a spectrogram (include a “colorbar” to quantify the dynamic range being displayed). (2) Design a simple matched filter for the transponder reply assuming the transmitted signal is a sine wave, pass the time series through it, and plot the output of the matched filter. Zoom in on and plot the 64 points in the immediate vicinity of the peak output. (3) Implement an envelope detector by first taking the magnitude of the matched filter output then low pass filtering the result with a FIR LPF (the bandwidth of the low pass filter should be 1 kHz or less – plot the impulse response and frequency response (dB) of the low pass filter along with a blow up of the 0-2 kHz region). What are the delay characteristics of the LPF? Plot the envelope detector output time series. Zoom in on and plot the 64 points in the immediate vicinity of the peak output. (4) Repeat (2) and (3) above assuming the transmitted signal is a cosine wave. (5) Implement the equivalent of the SKEP processor by adding together the square of the outputs of the “cosine” and “sine” matched filters and taking the square root of the result. Plot the resulting time series. Zoom in on and plot the 64 points in the immediate vicinity of the peak output.

C. Channel Impulse Response Estimation

Shallow water, vertical line array (VLA) acoustic data was collected north of Elba Island, Italy, in July 2004 [1]. The data available consists of ambient noise followed by the multipath arrival of a single LFM chirp transmitted by a moving source 4.2 km away.

The data consists of a 500 ms time series observed from 32 hydrophones spaced 2 m apart and sampled at $f_s = 12$ kHz (see figure below). The signal conditioning for each array element includes a high pass filter to attenuate low frequency shipping noise and a low pass (anti-aliasing) filter prior to A/D conversion. The source transducer was at a range of 4.2 km, depth of 70 m, and towed at 4 knots. The transmission was a 2-4 kHz LFM chirp of duration 100 ms. The chirp arrival at the array begins approximately 200 ms into the time series with the beginning and end of the observation consisting of ambient noise.

Provide the following analyses of this data:

A. Time-Evolving Power Spectral Analysis

For both the deepest and most shallow hydrophones (Els #1 and #32), plot the time-evolving power spectrum (dB) (e.g. “spectrogram” in Matlab) as a color or gray scale plot (e.g. “imagesc” and “colorbar” in Matlab) and discuss the major features of the results. In addition, plot the averaged power spectrum (dB) (e.g. “pwelch” in Matlab) before and during the chirp arrival.

B. Channel Impulse Response Estimation

Assuming a channel impulse response of the medium $h_i(t)$ from the source to the i th element of the array, express analytically the output of the filter matched to the source waveform $s(t)$ with the i th array element time series $r_i(t)$ as input in terms of the autocorrelation of the LFM chirp and $h_i(t)$.

Plot the time series of the LFM chirp, a spectrogram (dB) showing its time-evolving frequency content, and its spectrum (dB). Plot the time series at the output of the filter matched to $s(t)$ (optionally, the time series envelope computed via a Hilbert transform).

Using the known LFM chirp waveform, match filter (pulse compress) the multipath arrival structure observed on Els #1 and #32. Display the matched filter input and output time series (optionally, the matched filter output time series envelope computed via a Hilbert transform).

Similarly, match filter (pulse compress) the multipath arrival structure observed on all array elements to estimate the channel impulse response from the source to each element of the VLA. Display the matched filter output (or envelope) in a vertical waterfall display (see figure below) or as a color or gray scale plot.

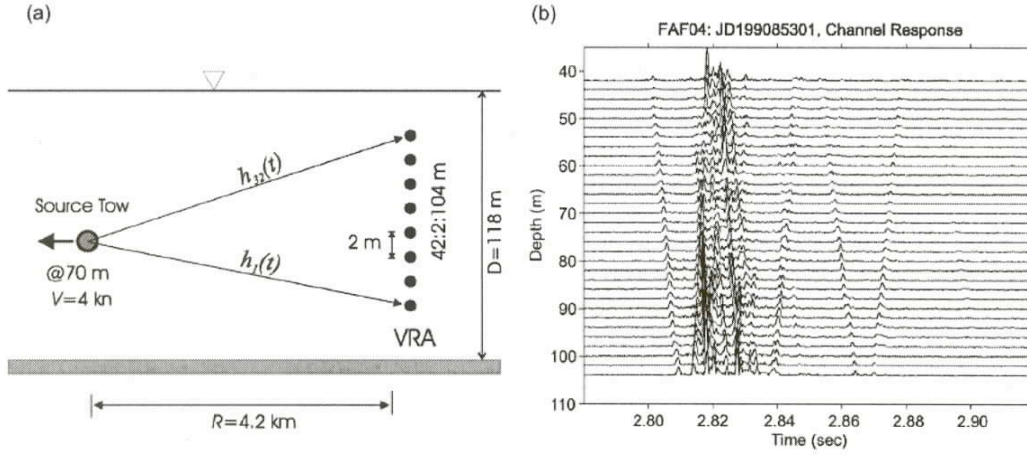


FIG. 8. (Color online) (a) Schematic of passive time reversal communications with a moving source at about 4 knots. (b) The channel responses (envelope) received by the vertical receiver array from a probe source at 70 m depth and 4.2 km range.

Reference

[1] H. Song et. al., “Spatial diversity in passive time reversal communications,” J. Acoust. Soc. Am. 120(4): 2067-2076 (2006).

II. Processor Performance

Similar to HW #2 (“SKE and SKEP Processor Performance”), determine the ROC performance of the following processors both theoretically and via Monte Carlo simulation: (1) Signal Known Except for Phase Independent Increments (SKEPII) ($L = 1, 2, 4$, and 8) and (2) Gaussian Signal in Gaussian Noise (see pp. 142-144 in [1]).

In (1), assume $LN = 128$ (total length of observation) where L = number of independent increments and N = segment or block length over which the signal phase is constant, $(LN)(\text{SNR}) = 9$ (i.e. $2E/N_0$ value in SKE case), $\text{SNR} = A^2/2\sigma^2$, and $\sigma^2 = 1$.

In (2), assume $N = 128$ (total length of observation), $(N)(\text{SNR}) = 9$, $\text{SNR} = \sigma_s^2/\sigma^2$, and $\sigma^2 = 1$.

Compare your results to those in HW #2 for $2E/N_0 = 9$ (i.e. plot your previous SKE and SKEP results). All ROC curves should be plotted on non-linear axes (probability paper) with the axes covering the ranges $P_d = [0.01, 0.99]$ and $P_f = [0.01, 0.99]$. Please include grid lines on your ROC plots in order to more easily determine the values you are obtaining for P_d and P_f . Show histograms of the test statistics obtained from the Monte Carlo simulations conditioned on H_0 and H_1 .

Note that the $N(\text{SNR}) \ll 1$ assumption used in the derivation of the SKEPII test statistic (the average of L periodograms $I_N(k)$) is not valid for the cases investigated. However, this test statistic should be used anyway for the analytical results and corresponding Monte Carlo simulations.

Include also Monte Carlo simulation results of the performance of the exact SKEPII receiver (no low SNR assumption) and separately compare these results with the approximate results for $L = 2, 4$, and 8 .

Reference

[1] S. Kay. Fundamentals of Statistical Signal Processing. Vol. II: Detection Theory. Prentice-Hall: 1998.

III. Correlated Noise

With reference to HW #3 (“Correlated Noise”), consider the following six signal types: (1) $s(n)$ as defined in HW #3 with $f_c = 1/16$, (2) $s(n)$ as in (1) except $f_c = 1/4$, (3) $s(n)$ as in (1) except $f_c = 3/8$, (4) $s(n)$ a FM chirp with the form:

$$s_{FM}(n) = A \sin \left[2\pi \left(f_c n + \frac{f'}{2} n^2 \right) \right]; \quad f_c = \frac{1}{16}, \quad f' = \frac{3}{8 \cdot 16} \quad (\text{chirp rate}), \quad n = 0, \dots, 15$$
$$= 0; \text{ otherwise,}$$

(5) $s(n)$ as in (4) except $f_c = 1/8$ and $f' = 1/(4 \cdot 16)$, and (6) $s(n)$ the optimum waveform for the correlated noise covariance matrix C (i.e. the $s(n)$ that maximizes d^2 for the optimum correlated noise processor) where the energy is kept the same as in signal types (1)–(3).

Plot the time series and corresponding spectrum $|S(f)|^2$ (dB) (just the positive frequencies) of each waveform $s(n)$. Also, for each waveform $s(n)$, overplot $|S(f)|^2$ (dB) and $|H(f)|^2$ (dB) (just the positive frequencies) as in Part II in HW #3.

Carry out Part III in HW #3 for the six signal types. In addition to your ROC curves, please include a table with the SNR and d^2 values for each of your cases. Compare the performance of the three processors for the different signal types and comment on the reasons for the differences seen. Note that the ROC curve for the optimum waveform is off the Pd/Pf scale. Please include grid lines on your ROC plots in order to more easily determine the values you are obtaining for Pd and Pf.

Include the theoretical expressions for d^2 and summarize their derivations along with how the noise covariance matrix C is obtained. Comment on how the optimum waveform is obtained.

Show analytically why the optimum colored noise processor and the mismatched processor have identical performance for the optimum signal waveform. Recall the expressions for d^2 and the solution for the optimum waveform in terms of an eigenvalue/eigenvector decomposition of the colored noise covariance matrix.

You should use the same amplitude for the FM chirps that you are using for the sinusoidal signals. Note that the correct expression for FM chirp energy is $s^T s$ not $N(A^2/2)$ as it is for a sinusoid with an integer number of cycles over N points. You should use the actual chirp energy $s^T s$ in your calculations for SNR and d^2 . Don't be concerned that the SNR's for the two chirps and the sinusoids are different (in fact, they are quite close but not exactly the same). The part that is of interest here is the comparison across the three processors for a given signal and not specifically the absolute performance comparison between the six signal types.