Mean Shift clustering algorithm

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Outline

- Mean Shift is a non-parametric clustering algorithm
- It is based on *Kernel Density estimation*
- The only parameter is the *bandwidth*
- $O(n^2)$ computational cost
- Common application in computer vision: image segmentation
- It is embarassingly parallel

- At each step a kernel function is applied to each point to make it shift towards the local maxima
- Most used kernel: Gaussian kernel

$$K(x) = e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

- New position x' where x has to be shifted is computed as:

$$x' = \frac{\sum_{x_i \in N(x)} K(dist(x, x_i))x_i}{\sum_{x_i \in N(x)} K(dist(x, x_i))} \tag{2}$$

$N(x)$ is the neighborhood of x, a set of points for which K(x_i) $\neq 0$

- Algorithm stops when all points have stopped shifting, that is they have reached the local maxima

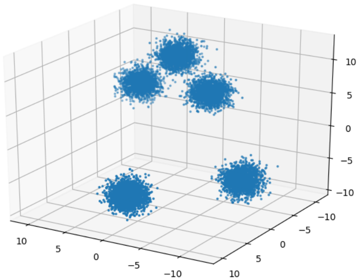
Sequential implementation

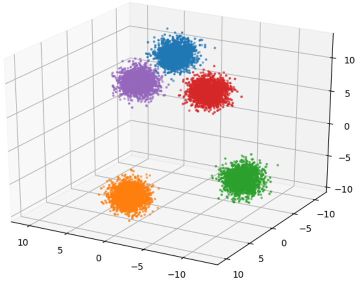
Algorithm 1 Mean shift core

function MEANSHIFT(*originalPoints*)
    shiftedPoints ← *originalPoints*
    while iterationIndex $<$ *MAX_ITERATIONS* **do**
        for each point *p* in *shiftedPoints* **do**
            $p \leftarrow$ SHIFTPOINT(*p*, *originalPoints*)

Algorithm 2 Shift a single point

function SHIFTPOINT(*p*, *originalPoints*)
    shiftedP ← 0
    weight ← 0
    for each point *x* in *originalPoints* **do**
        $dist \leftarrow dist(p, x)$
        $w \leftarrow GKernel(dist, BW)$
        $shiftedP \leftarrow shiftedP + w * x$
        $weight \leftarrow weight + w$
    return *shiftedP* / *weight*





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Algorithm 3 OpenMP Mean shift core

```
function OpenMPMeanShift(originalPoints)
    shiftedPoints ← originalPoints
    while iterationIndex < MAX_ITERATIONS do
        #pragma parallel for schedule(static)
        for each point p in shiftedPoints do
            p ← ShiftPoint(p, originalPoints)
```

Algorithm 4 Shift a single point

```
function ShiftPoint(p, originalPoints)
    shiftedP ← 0
    weight ← 0
    for each point x in originalPoints do
        dist ← dist(p, x)
        w ← GKernel(dist, BW)
        shiftedP ← shiftedP + w * x
        weight ← weight + w
    return shiftedP / weight
```

- just a pragma directive
- static scheduling: loop divided statically in chunks of equal size

Taking advantage of GPUs:

- One thread for each point to shift
- Coalescing of accesses to memory:
  - Points stored as a *Structure of Arrays*

$$[x, \ldots, x_n, y_1, \ldots, y_n, z_1, \ldots, z_n] \tag{3}$$

  - Access to the array in the form of:

$$blockDim.x * blockIdx.x + threadIdx.x \tag{4}$$

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Algorithm 5 CUDA Naive version Mean Shift core

```
function NaiveCUDAMS(originalPoints)
    shiftedPoints ← originalPoints
    while iterationIndex < MAX_ITERATIONS do
        NaiveKernel(shiftedPoints, originalPoints)
```

Algorithm 6 CUDA Naive version Kernel

```
function NaiveKernel(shiftedPts, originalPts)
    tx ← threadIdx.x
    bx ← blockIdx.x
    idx ← bx * blockDim.x + tx
    if idx < |originalPts| then
        shiftedP ← 0
        weight ← 0
        p ← shiftedPts[idx]
        for each point x in originalPts do
            dist ← dist(p, x)
            w ← GKernel(dist, BW)
            shiftedP ← shiftedP + w * x
            weight ← weight + w
        shiftedPts[idx] ← shiftedP / weight
```

A further optimization is possible:

- Each thread reads *O(n)* points from global memory to compute the shift
- Shared Memory can be exploited with the Tiling pattern
- *TILE_WIDTH* = *BLOCK_DIM*
- *O(n)* accesses reduced to *O(n/TILE_WIDTH)*

Algorithm 7 CUDA Tiling version Mean Shift core

```
function TilingCUDAMS(originalPoints)
    shiftedPoints ← originalPoints
    while iterationIndex < MAX_ITERATIONS do
        TilingKernel(shiftedPoints, originalPoints)
```

Algorithm 8 CUDA Tiling version Kernel

```
function TilingKernel(shiftedPts, originalPts)
    tx ← threadIdx.x
    bx ← blockIdx.x
    idx ← bx * blockDim.x + tx
    tile ← SharedMemArray[TILE_WIDTH]
    shiftedP ← 0
    weight ← 0
    for tileIter < numTiles do
        tileIdx ← tileIter * TILE_WIDTH + tx
        if idx < |originalPts| then
            tile[tx] ← originalPts[tileIdx]
        else
            tile[tx] ← nullPoint
        __synchthreads()                                ▷ End of loading
        if idx < |originalPts| then
            p ← shiftedPts[idx]
            for i with i < TILE_WIDTH do
                x ← tile[i]
                if x! = nullPoint then
                    dist ← dist(p, x)
                    w ← GKernel(dist, BW)
                    shiftedP ← shiftedP + w * x
                    weight ← weight + w
        __synchthreads()                                ▷ End of computing
    if idx < |originalPts| then
        shiftedPts[idx] ← shiftedP / weight
```

Experimental results

- Performances compared with the speedup metric, computed as:

$$S = \frac{t_S}{t_P} \tag{5}$$

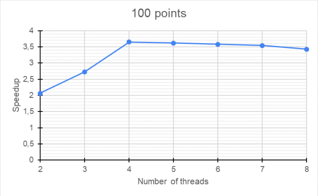
- Tests executed on a machine with:
  - OS: Ubuntu 18.04 LTS
  - CPU: Intel Core i7-8565U 1.8GHz up to 4.6GHz with Turbo Boost, 4 cores/8 threads
  - RAM: 16 GB DDR4
  - GPU: NVidia GeForce MX250 2GB with CUDA 10.1

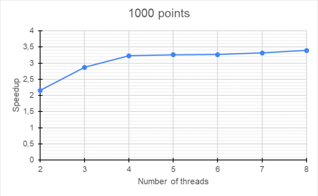
- Each time is the average of 5 experiments for the sequential and OpenMP versions and of 15 experiments for both CUDA implementations

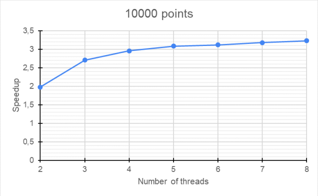
- The implementations have been evaluated on gaussian distributions composed by respectively 100, 1000, 10000, 100000 and 250000 3D points
- bandwidth set to 2
- *MAX_ITERATIONS* constant set to 10, which has been empirically estimated to be enough to make all points converge to the local maxima

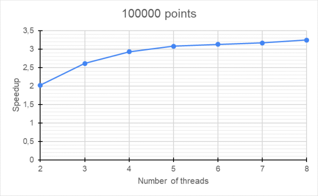
OpenMP: Increasing threads

Number of threads gradually increased for each dataset



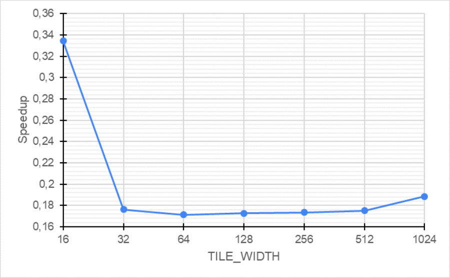






Not tested on 250k points dataset: too long execution time

Execution time for 10000 points for the CUDA tiling implementation varying *TILE_WIDTH*



TILE_WIDTH	CUDA Tiling
16	0.334405 s
32	0.176171 s
64	**0.171354 s**
128	0.172666 s
256	0.173306 s
512	0.175091 s
1024	0.18845 s

Naive vs Tiling CUDA

Dim	CUDA Naive	CUDA Tiling	Speedup
100	0.000401 s	0.000437 s	1.09
1000	0.003044 s	0.003359 s	1.11
10000	0.171354 s	0.191216 s	1.12
100000	15.79 s	18.29 s	1.16
250000	100.38 s	121.10 s	1.20

Dim	Sequential	CUDA Tiling	Speedup
100	0.005384 s	0.000401 s	13.43
1000	0.475485 s	0.003044 s	156.21
10000	49.29 s	0.171354 s	287.64
100000	4560.37 s	15.79 s	288.80
250000	† 27768 s	100.38 s	† 276.61

- † time has been estimated with a quadratic regression (due to the $O(n^2)$ computational cost)

Global comparison

Global comparison between sequential, OpenMP and Tiling CUDA best results:

- Greatest speedups with CUDA, at the expense of a more complicated implementation
- OpenMP lets to reach noticeable speedups with a simple implementation (just a directive)

Dim	Sequential	OpenMP	OpenMP Speedup	CUDA Tiling	CUDA Speedup
100	0.005384 s	0.001473 s	3.66	0.000401 s	13.43
1000	0.475485 s	0.140161 s	3.39	0.003044 s	156.21
10000	49.29 s	15.25 s	3.24	0.171354 s	287.64
100000	4560.37 s	1403.78 s	3.25	15.79 s	288.80
250000	† 27768 s	† 8720 s	† 3.18	100.38 s	† 276.61

Table: † times have been estimated with a quadratic regression

Conclusions

- The embarassingly parallel structure of Mean Shift makes it suitable for parallel implementations
- OpenMP has an excellent speedup and development cost ratio
- CUDA makes Mean Shift applicable to datasets intractable with a CPU