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Proposition:There is a two-coloring-edges K_n with at most $\binom{n}{4} \cdot 2^{-5}$ monochromatic K_4.

proof1:Indeed, $\binom{n}{4} \cdot 2^{-5}$ is the expected number of monochromatic copies of K_4 in a random 2-edge-coloring of K_n, and hence a coloring as above exists.

proof2: Deterministical proof. Turn expectation of K_4 in all K_n into expectation of all K_4 in a sertain K_n.

First we have graph K_n:

In [28]:
```
import networkx as nx
import matplotlib.pyplot as plt
import numpy as np
from itertools import combinations

#K_n
n=50
G = nx.complete_graph(n)
```

Then define W,$w(K)$:K is K_4 in K_n,If at least one edge of K is colored red and at least one edge is colored blue then $w(K) = 0$. If no edge of K is colored, then $w(K) = 2^{-5}$, and if $r \geq 1$ edges of K are colored, all with the same color, then $w(K) = 2^{r-6}$. Also define the total weight $W = \sum w(K)$, as K ranges over all copies of K_4 in K_n.

we computer W from no edge is colored,noted as W_0.$W_0 = \binom{n}{4} \cdot 2^{-5}$.

In [29]:
```
def w(graph):
    no_color=0
    red=0
    blue=0

    #count colors of edges
    for u,v,attr in graph.edges(data=True):#Can't leave out "data=True"
        if attr.get('color') == 'black':
            no_color=no_color+1
        elif attr.get('color') == 'red':
            red = red+1
        elif attr.get('color') == 'blue':
            blue=blue+1

    if no_color == 6:
        return 2**-5
    if red >= 1 and blue >=1:
        return 0
    if red>=1:
        return 2**(red-6)
    if blue>=1:
        return 2**(blue-6)
```

Now we color edges$\{e_1, \ldots, e_{\binom{n}{2}}\}$ one by one and update W_{i-1} to W_i(by updating $w(K_4)$ where those K_4 contain present e_i).

By defination of W_{i-1},we get

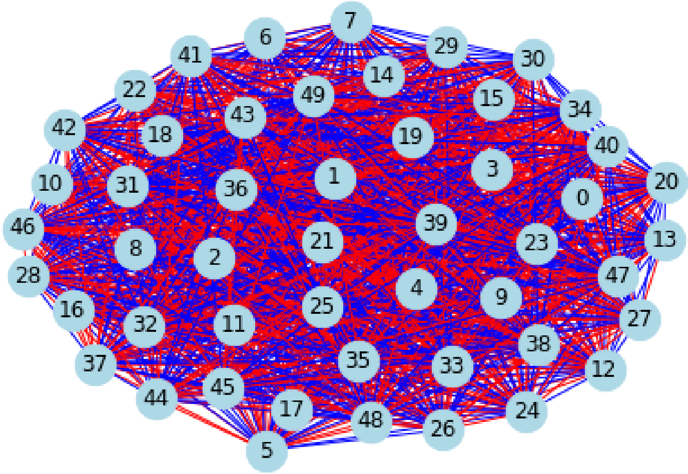
$$W_{i-1} = \frac{W_{\text{ired}} + W_{\text{iblue}}}{2}.$$

where W_{ired} is $\sum w(K)$ when we color e_i red,blue situation is similiar. So if $W_{\text{ired}} \leq W_{\text{iblue}}$ then we color e_i red,and $W_i = W_{\text{ired}}$, otherwise, we color it blue. Then we get $W_{\binom{n}{2}} \leqslant \cdots \leqslant W_0$.That is, monochromatic K_4 in this graph is less than $\binom{n}{4} \cdot 2^{-5}$

In [30]:

```
nx.set_edge_attributes(G,name='color',values='black')
points = range(0, n)

k4s = list(combinations(points, 4))
edges = list(combinations(points,2))
w_k = np.full(len(k4s),2**-5)


for i in range(0,len(edges)):#color edges one by one
    Gred=G.copy()
    nx.set_edge_attributes(Gred, {edges[i]: {'color': 'red'}})
    w_k_red =w_k.copy()

    Gblue=G.copy()
    nx.set_edge_attributes(Gblue, {edges[i]: {'color': 'blue'}})
    w_k_blue =w_k.copy()

    for j in range(0,len(k4s)):
        if set(edges[i]) <= set(k4s[j]):#if edge is in this K_4,change the related w(k)
            w_k_red[j] = w(Gred.subgraph(k4s[j]))
            w_k_blue[j] = w(Gblue.subgraph(k4s[j]))

    if np.sum(w_k_red)<=np.sum(w_k_blue):
        G=Gred.copy()
        w_k =w_k_red.copy()

    else:
        G=Gblue.copy()
        w_k = w_k_blue.copy()

mono_k4s=0

for s in k4s:
    if w(G.subgraph(s))==1:
        mono_k4s=mono_k4s+1
        example=s


if mono_k4s <= 2**-5*n*(n-1)*(n-2)*(n-3)/24:
    print("there is ",mono_k4s,"monochromatic K_4s.The result satisfis proposition")
if  mono_k4s!=0:
    print("example is",example)

edge_colors = [G[u][v]['color'] for u, v in G.edges()]
nx.draw(G, with_labels=True, edge_color=edge_colors, node_color='lightblue', node_size=500)
plt.show()
```

```
there is  5402 monochromatic K_4s.The result satisfis proposition
example is (39, 44, 46, 47)
```



Remark: 1.This alogthrim gives a certain graph for fixed n.Because every step is one and only.
2.To find K_4 in K_n, just confirm 4 vertices.But it's hard to update $w(K)$ if take 4 vertices as 4 indices.This program use natural order such as (0,1,2,3),(0,1,2,4)...(0,1,2,n),(0,1,3,4)......and check if certain edge in some of them one by one.
3.Ramsay theory says there must be at least one monochromatic K_4 in 2-coloring K_{18}.R(4,4)=18.
4.In Python,same object can have different name.Simple exchanging works in C/C++,but '.copy' is necessary in Python like below.

In [4]:
```
x=np.array([1,2])
y=np.array([2,3])
z=x
y=z
x=y
print(x,y,z)
```

```
[1 2] [1 2] [1 2]
```