

MAC440 - LISTA 5: SÉRIES DE POTÊNCIAS - SÉRIES DE TAYLOR/MACLAURIN

1. Determine o raio e o intervalo de convergência de cada série abaixo:

(a) $\sum_{n=1}^{+\infty} \frac{(x+1)^n}{n^3 + 2}$ Resp: $r = 1, I = [-2, 0]$

(b) $\sum_{n=0}^{+\infty} \frac{(-1)^n(x+2)^n}{2^n}$ Resp: $r = 2, I =]-4, 0[$

(c) $\sum_{n=1}^{+\infty} \frac{(-1)^n(x-2)^n}{2n+1}$ Resp: $r = 1, I =]1, 3]$

(d) $\sum_{n=0}^{+\infty} \frac{(-7)^n(x-1)^n}{3^n}$ Resp: $r = 3/7, I =]4/7, 10/7[$

(e) $\sum_{n=1}^{+\infty} \frac{3^{n-1}(x+1)^n}{\sqrt{n^3}}$ Resp: $r = 1/3, I = [-4/3, -2/3]$

(f) $\sum_{n=1}^{+\infty} \frac{(x-3)^n}{3n+5}$ Resp: $r = 1, I = [2, 4[$

(g) $\sum_{n=1}^{+\infty} (-1)^{n-1} \frac{x^n}{3^n \sqrt[3]{n}}$ (esp: $r = 3, I = [-2, 0]$)

2. Determine a série de Taylor da função dada em torno de x_0 em cada caso, bem como seu raio e intervalo de convergência:

(a) $f(x) = e^x, x_0 = 0$

(c) $f(x) = \cos x, x_0 = 0$

(e) $f(x) = \operatorname{arctg} x, x_0 = 0$

(b) $f(x) = \sin x, x_0 = 0$

(d) $f(x) = \ln x, x_0 = 1$

(f) $f(x) = \ln(1+x), x_0 = 0$

Resp.

(a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{+\infty} \frac{x^n}{n!}, r = \infty, I = \mathbb{R}$

(b) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, r = \infty, I = \mathbb{R}$

(c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{(2n)!}, r = \infty, I = \mathbb{R}$

(d) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots = \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{(x-1)^n}{n}, r = 1, I =]0, 2[$

(e) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, r = 1, I = [-1, 1]$

(f) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{+\infty} (-1)^n \frac{x^n}{n}, r = 1, I =]-1, 1]$