MAC440 – Lista 4: critérios de convergência e séries alternadas

1. Calcule a razão da série geométrica de decida se é convergente. Em caso afirmativo, calcule sua soma.

   (a) $\displaystyle\sum_{n-1}^{+\infty} \frac{2^{n+2}}{(-3)^n}$ (b) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-3)^{n-2}}{4^n}$ (c) $\displaystyle\sum_{n=0}^{+\infty} \frac{4^{n+1}}{3^{2n}}$ (d) $\displaystyle\sum_{n=0}^{+\infty} \frac{2^{3n}}{5^{n+2}}$

   Resp. (a) $r = -\frac{2}{3}$, $S = -\frac{8}{5}$, (b) $r = -\frac{3}{4}$, $S = -\frac{1}{21}$, (c) $r = \frac{4}{9}$, $S = \frac{9}{20}$, (d) $r = \frac{8}{5}$ diverge.

2. Seja $a_k = \dfrac{2n^3}{5n^3 - n^2 + 1}$.

   (a) Calcule $\displaystyle\lim_{a_k \to +\infty} a_k$ e decida se a sequência (a_k) converge ou não.

   (b) Determine se a série $\sum a_k$ converge ou não.

   Resp. (a) $\displaystyle\lim_{a_k \to +\infty} a_k = \frac{2}{5}$, logo a sequência converge. (b) A série diverge, pelo critério do termo geral (ou $n-$ésimo termo).

(1) Verificar, usando o teste da razão, se a série é convergente ou divergente:

(a) $\displaystyle\sum_{1}^{+\infty} n(\frac{5}{7})^n$ (c) $\displaystyle\sum_{1}^{+\infty} \frac{n!}{5^n}$ (e) $\displaystyle\sum_{1}^{+\infty} \frac{3^{n-1}}{(n-1)!}$ (g) $\displaystyle\sum_{1}^{+\infty} \frac{n!}{n^n}$

(b) $\displaystyle\sum_{1}^{+\infty} \frac{5^{n-1}}{n^2+1}$ (d) $\displaystyle\sum_{1}^{+\infty} \frac{n^2}{n!}$ (f) $\displaystyle\sum_{0}^{+\infty} \frac{n}{e^{2n+3}}$ (h) $\displaystyle\sum_{0}^{+\infty} \frac{n(n+1)(n+2)}{3^n}$

Resp. (a) converge, (b) diverge, (c) diverge, (d) converge, (e) converge, (f) converge, (g) converge, (h) converge

(2) Verificar, usando o teste da raiz, se a série é convergente ou divergente:

(a) $\displaystyle\sum_{1}^{+\infty} \left(\frac{n+3}{2n+1}\right)^n$ (c) $\displaystyle\sum_{0}^{+\infty} n\left(\frac{3n-1}{4}\right)$ (e) $\displaystyle\sum_{0}^{+\infty} \left(\frac{n}{3n+5}\right)^{2n+1}$

(b) $\displaystyle\sum_{1}^{+\infty} \left(\frac{3n+4}{5n+1}\right)^{n/2}$ (d) $\displaystyle\sum_{1}^{+\infty} \left(\frac{n-1}{n+1}\right)^{n^2}$ (f) $\displaystyle\sum_{1}^{+\infty} \left(\frac{n+1}{3n+1}\right)^{n/3}$

Resp. (a) converge, (b) converge, (c) diverge, (d) converge, (e) converge, (f) converge

(3) Verificar, usando o teste da integral, se a série é convergente ou divergente:

(a) $\displaystyle\sum_{1}^{+\infty} \frac{1+\sqrt{n}}{n}$ (c) $\displaystyle\sum_{1}^{+\infty} \frac{1}{(3n+5)^3}$ (e) $\displaystyle\sum_{1}^{+\infty} \frac{\ln n}{n}$

(b) $\displaystyle\sum_{1}^{+\infty} \frac{\ln(n+2)}{n+2}$ (d) $\displaystyle\sum_{1}^{+\infty} \frac{n}{n^2+5}$ (f) $\displaystyle\sum_{1}^{+\infty} \frac{1}{(3n+2)^2}$

Resp. (a) diverge, (b) diverge, (c) converge, (d) diverge, (e) diverge, (f) converge

(4) Verificar, usando os testes de comparação (por desigualdade ou forma-limite), se a série é convergente ou divergente:

(a) $\displaystyle\sum_{1}^{+\infty} \frac{1}{n-0{,}3}$ (c) $\displaystyle\sum_{0}^{+\infty} \frac{1}{(2n+9)^2}$ (e) $\displaystyle\sum_{1}^{+\infty} \frac{1}{\sqrt{n}\sqrt{n+1}}$ (g) $\displaystyle\sum_{0}^{+\infty} \frac{1}{(n+2)(n+3)(n+4)}$

(b) $\displaystyle\sum_{1}^{+\infty} \frac{1}{ne^{n+1}}$ (d) $\displaystyle\sum_{0}^{+\infty} \frac{1}{\ln n^2}$ (f) $\displaystyle\sum_{0}^{+\infty} \frac{\operatorname{sen} n + 2^n}{n+4^n}$ (h) $\displaystyle\sum_{1}^{+\infty} \ln\left(1+\frac{1}{2^n}\right)$

Resp. (a) diverge, (b) converge, (c) converge, (d) diverge, (e) diverge, (f) converge, (g) converge, (h) converge

(5) Verificar se a série é condiconalmente convergente (CC), absolutamente convergente (AC) ou divergente (D):

(a) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^{n+1}5^n n!}{3^n n^n}$ (c) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^n}{\ln(n+1)}$ (e) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^{n+1}n}{2n+1}$ (g) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^n}{2n-1}$

(b) $\displaystyle\sum_{0}^{+\infty} \frac{(-1)^n(3n-1)}{10n-2}$ (d) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^{n+1}}{n^2+1}$ (f) $\displaystyle\sum_{1}^{+\infty} \frac{(-1)^{n+1}(1+\sqrt{n})}{n}$ (h) $\displaystyle\sum_{1}^{+\infty}(-1)^n(\sqrt{n+1}-\sqrt{n-1})$

Resp. (a) AC, (b) DV, (c) CC, (d) AC, (e) DV, (f) AC, (g) CC, (h) CC