Problem: Find $\left(\frac{f(x)}{g(x)}\right)'$.

Solution: If $\left(\frac{f(x)}{g(x)}\right) = f(x)\cdot\left(g(x)\right)^{-1}$

Therefore, using the power rule $f'(x)\cdot\left(g(x)\right)^{-1} + \left(g'(x)\right)^{-1}\cdot f(x)$,

and $\left(g'(x)\right)^{-1} = \left(\frac{1}{g(x)}\right)'$, which we proved to equal $-\frac{g'(x)}{g(x)^2}$, so

$$f'(x)\cdot\frac{1}{g(x)} + f(x)\cdot -\frac{g'(x)}{g(x)^2},$$

which simplifies to $\frac{f'(x)}{g(x)} - \frac{g'(x)F(x)}{g(x)^2}$

Next, multiply the first term by $\frac{g(x)}{g(x)}$ to get a common denominator of $g(x)^2$, which then gives you the final solution,

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{g(x)^2}$$