1.integrate

$$\int \frac{d\Theta}{\sin^6\Theta + cos^6\Theta}$$

Solution:

$$\int \frac{d\Theta}{\sin^6\Theta + cos^6\Theta}$$

$$= \int \frac{d\Theta}{(sin^2\Theta)^3 + (cos^2\Theta)^3}$$

$$= \int \frac{d\Theta}{(sin^2\Theta + cos^2\Theta)^3 - 3\,sin^2\Theta cos^2\Theta(sin^2\Theta + cos^2\Theta)}$$

[you know $a^3+b^3 = (a+b)^3 - 3ab(a+b)$]

$$= \int \frac{d\Theta}{1 - 3\,sin^2\Theta cos^2\Theta}$$

[as $sin^2\Theta + cos^2\Theta = 1$]

$$= \int \frac{d\Theta}{1 - \frac{3}{4}(2sin^2\Theta)(2cos^2\Theta)}$$

$$= \int \frac{d\Theta}{1 - \frac{3}{4}(1 - cos2\Theta)(1 + cos2\Theta)}$$

[using $2sin^2A = 1 - cos2A$

and $2cos^2A = 1 + cos2A$]

$$= \int \frac{d\Theta}{1 - \frac{3}{4}(1 - cos^2 2\Theta)}$$

[$(a+b)(a-b) = a^2 - b^2$]

$$= \int \frac{4d\Theta}{4 - 3(1 - cos^2 2\Theta)}$$

$$= \int \frac{4d\Theta}{1 + 3cos^2 2\Theta}$$

$$= 4\int \frac{sec^2 2\Theta d\Theta}{sec^2 2\Theta + 3}$$

[multiplying by $sec^2 2\Theta$ to numerator and denominator]

$$=4\int \frac{sec^2 2\theta d\,\theta}{1+tan^2 2\theta+3}$$

$$=2\int \frac{2sec^2 2\theta d\,\theta}{4+tan^2 2\theta}$$

$$=2\int \frac{d\,z}{2^2+z^2}$$ [let $tan\,2\theta = z \rightarrow 2sec^2 2\theta d\theta = dz$]

$$=2.\frac{1}{2}\tan^{-1}\frac{z}{2} + c$$

$$= \tan^{-1}\frac{tan2\theta}{2} + c$$ **[Answer]**