



The Hong Kong Olympiad in Informatics

香港電腦奧林匹克競賽

2021/22

初賽 Heat Event

初級組 Junior Group

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Time: 2:00 p.m. – 3:30 p.m.

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K

主辦

香港電腦教育學會

Organizer

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Format 類型	# Questions 題目數	Total Marks 佔分
Section A1 甲部 (一)	True or False 真假題	5
Section A2 甲部 (二)	Multiple Choice 多項選擇題	20
Section B 乙部	Fill-in-the-blanks 填充題	7 (A - K)
Total 總分		45

- (1) Assume that all variables without declaration shown in the following program segments have already been declared properly as 32-bit signed integers (Pascal: longint, C / C++: int).

下列程序段中所有未有列出宣告的變量，均假設已經適當地宣告為 32 位元有符號的整數 (Pascal: longint, C / C++: int)。

- (2) The following code is added to the beginning of all C and C++ programs.

在所有 C 和 C++ 程序的頂部加入以下程式碼:

C

```
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <stdlib.h>
#include <stdbool.h>
```

C++

```
#include <cstdio>
#include <cmath>
#include <cstdlib>
#include <string>
#include <iostream>
using namespace std;
```

For C, stdbool.h defines the boolean data type bool and values true (equivalent to 1) and false (equivalent to 0).

對於 C，stdbool.h 定義了布爾數據類型 bool 及值 true (等同 1) 及 false (等同 0)。

- (3) Other than questions that mention compilation, assume all programs are compiled properly in Ubuntu 20.04 using the compilers and commands below.

除了有提及編譯的題目之外，假設所有程序都在 Ubuntu 20.04 下使用以下編譯器及指令正確地編譯。

Pascal: Free Pascal (fp-compiler 3.0.4)	fpc program.pas
C: GNU GCC (gcc-4.9 4.9.4)	gcc -std=c99 program.c -o program
C++: GNU G++ (g++-4.9 4.9.4)	g++ -std=c++98 program.cpp -o program

Section A1 甲部 (一) (5 marks 分)

For each question, determine whether the statement is true or false, then mark the corresponding box (T: true or F: false) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請判斷下列每題的陳述句的真假，然後把答題紙對應的空格 (T: 真或 F: 假) 填滿。答對得一分，答錯不扣分。

1. The definition of "most" is more than 50% of the population. If most doctors are smart people, and most smart people are rich, then most doctors are rich.

「大部份」的定義為超過 50% 的人口。如果大部份的醫生都是聰明人，而且大部份的聰明人都是富有的，那麼大部份的醫生都是富有的。

2. Suppose a is an array of four 32-bit signed integers (Pascal: array a[0..3] of longint, C / C++: int a[4]). The following two program segments always output equivalently:

假設 a 是有四個 32 位元有符號整數的陣列 (Pascal: array a[0..3] of longint, C / C++: int a[4])。以下兩程序段的輸出總是相等：

Pascal

```
for w := 1 to a[0] do
  for x := 1 to a[1] do
    for y := 1 to a[2] do
      for z := 1 to a[3] do
        writeln('HKOI')
```

C

```
for (w = 1; w <= a[0]; w++)
  for (x = 1; x <= a[1]; x++)
    for (y = 1; y <= a[2]; y++)
      for (z = 1; z <= a[3]; z++)
        printf("HKOI\n");
```

C++

```
for (w = 1; w <= a[0]; w++)
  for (x = 1; x <= a[1]; x++)
    for (y = 1; y <= a[2]; y++)
      for (z = 1; z <= a[3]; z++)
        cout << "HKOI" << endl;
```

Pascal

```
for i := 0 to 3 do
  for j := 1 to a[i] do
    writeln('HKOI')
```

C

```
for (i = 0; i <= 3; i++)
  for (j = 1; j <= a[i]; j++)
    printf("HKOI\n");
```

C++

```
for (i = 0; i <= 3; i++)
  for (j = 1; j <= a[i]; j++)
    cout << "HKOI" << endl;
```

3. In a party, some participants have shaken hands with each other. If a total of n participants have shaken hands with other participants an odd number of times, n must be even.

在一個派對上，有些參加者互相握了手。如果一共有 n 個參加者握了奇數次其他參加者的手， n 必定為偶數。



4. When determining whether an element exists in a sorted array with at least 10 elements, binary search always requires fewer comparisons than linear search.

在判斷一個元素是否存在於至少有 10 個元素及經排序的陣列中時，二分檢索法所需要的比較次數總是比線性檢索法所需要的少。

5. Assume that function $r()$ returns a random integer between 0 and 65535 inclusive uniformly and independently. $f()$ always returns true.
假設函數 $r()$ 會均等和獨立地傳回一個 0 至 65535 之間 (含) 的隨機整數。函數 $f()$ 總是傳回 true。

Pascal

```
function A(): longint;
begin
  A := r();
end;

function B(): longint;
begin
  B := r();
end;

function f(): boolean;
begin
  f := (A() > B()) or (A() <= B());
end;
```

C / C++

```
int A() {
  return r();
}

int B() {
  return r();
}

bool f() {
  return (A() > B()) || (A() <= B());
}
```

Section A2 甲部 (二) (20 marks 分)

For each question, choose the **most appropriate** answer and mark the corresponding box (A, B, C, or D) on the answer sheet. One mark for each correct answer. No marks will be deducted for wrong answers.

請為下列每題各選一個**最適合**的答案，然後把答題紙對應的空格（A、B、C、或D）填滿。

答對得一分，答錯不扣分。

6. Heung Shing City has 15000 citizens, each citizen has a unique citizen ID. What is the minimum bytes required to represent the citizen ID?

香城有 15000 個市民，每個市民均有一個獨一無二的身分證編號。表示身分證編號最少需要多少位元組？

- A. 3
- B. 2
- C. 13
- D. 14

7. What is the output of the following program? 以下程序的輸出是什麼？

$$\begin{array}{r}
 13 \times 11 \times 10 \\
 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\
 \hline
 \begin{array}{l}
 \times 13 \\
 \hline
 130 \\
 26 \\
 \hline
 156
 \end{array}
 \quad
 \begin{array}{l}
 \times 11 \\
 \hline
 1560 \\
 156 \\
 \hline
 17160
 \end{array}
 \end{array}$$

Pascal

```

var x, y, z, i: longint;
begin
  x := 0;
  y := 0;
  z := 0;
  for i := 1 to 2022 do
  begin
    if (i mod 10 = 0) then
      inc(x)
    else if (i mod 5 = 0) then
      inc(y)
    else
      inc(z)
  end;
  write(x, ' ', y, ' ', z)
end.
  
```

C

```

int x, y, z, i;
int main() {
  x = 0;
  y = 0;
  z = 0;
  for (i = 1; i <= 2022; i++) {
    if (i % 10 == 0)
      x++;
    else if (i % 5 == 0)
      y++;
    else
      z++;
  }
  printf("%d %d %d", x, y, z);
  return 0;
}
  
```

C++

```

int x, y, z, i;
int main() {
  x = 0;
  y = 0;
  z = 0;
  for (i = 1; i <= 2022; i++) {
    if (i % 10 == 0)
      x++;
    else if (i % 5 == 0)
      y++;
    else
      z++;
  }
  cout << x << ' ' << y << ' ' << z;
  return 0;
}
  
```

- A. 404 808 810
- B. 202 202 1618
- C. 404 202 1618
- D. 202 404 1416

8. Consider the following program: 考慮以下程序：

Pascal

```
var
  TOP: longint = 150;
  up, down: longint;
  d, h: longint;
begin
  read(up, down);
  d := 0;
  h := 0;
  while (true) do
  begin
    inc(d);
    h := h + up;
    if (h >= TOP) then
      break;
    h := h - down;
  end;
  write(d)
end.
```

C

```
int TOP = 150;
int up, down;
int d, h;
int main () {
  scanf("%d %d", &up, &down);
  d = 0;
  h = 0;
  while (true) {
    d++;
    h = h + up;
    if (h >= TOP)
      break;
    h = h - down;
  }
  printf("%d\n", d);
  return 0;
}
```

C++

```
int TOP = 150;
int up, down;
int d, h;
int main () {
  cin >> up >> down;
  d = 0;
  h = 0;
  while (true) {
    d++;
    h = h + up;
    if (h >= TOP)
      break;
    h = h - down;
  }
  cout << d << endl;
  return 0;
}
```

Which of the following inputs gives the least output number?

以下哪項輸入使得輸出數字最小？

- | | |
|-----------|----|
| A. 102 99 | 3 |
| B. 33 24 | 9 |
| C. 10 0 | 10 |
| D. 50 43 | 13 |

9. John is 35 years old, a smart and outgoing person. In his secondary school days, he joined the Computer Team, participated the Hong Kong Olympiad in Informatics (HKOI) Junior once and HKOI Senior twice, won 1 Silver Medal and 2 Gold Medal in total. He even represented Hong Kong for the International Olympiad in Informatics (IOI), and since then his love towards programming never dropped.

約翰是一名三十五歲，聰明又外向的人。在他的中學時期，他加入了學校的電腦隊，參加了一次香港電腦奧林匹克競賽 (HKOI) 初級組和兩次 HKOI 高級組，共取得一個銀獎和兩個金獎。他更代表過香港參加 國際資訊奧林匹克競賽 (IOI)，自此之後他對編程的熱愛從未減退。

Given the above description about John, which of the following is the most likely to be true?

已知上述對約翰的描述，以下哪一項最有可能為真？

- A. John is a lawyer who plays programming competitions on Codeforces regularly in his free time.
約翰是一名在空餘時間定期參加 Codeforces 的編程競賽的律師。
- B. John is a lawyer who specializes in cases involving big tech firms.
約翰是一名專門處理涉及科技巨頭的案件的律師。
- C. Cannot determine which of the other three options is the most likely to be true.
無從判斷其他三個選項哪一項最有可能為真。
- D. John is a lawyer.
約翰是一名律師。

10. Which of the following Boolean expressions are logically equivalent to $P \text{ AND } (\text{NOT } (Q \text{ AND } R))$?
以下哪些布爾表達式與 $P \text{ AND } (\text{NOT } (Q \text{ AND } R))$ 在邏輯上等價？

- i. $P \text{ AND } ((\text{NOT } Q) \text{ OR } (\text{NOT } R))$
- ii. $(\text{NOT } P) \text{ OR } (Q \text{ AND } R)$

- A. ii only 只有 ii
B. None of them 無
C. i only 只有 i
D. i and ii i 和 ii

11. What is the output of the following program? 以下程序的輸出是什麼？ ~~X~~

Pascal

```
var
  x: longint;
begin
  x := 12;
  if ((x div 10 = 2) and (x > 15) or (x mod 3 = 0)) then
    write('YES')
  else
    write('NO');
  if ((x div 5 = 2) or (x < 13) and (x mod 5 = 1)) then
    write('YES')
  else
    write('NO')
end.
```

C

```
int x;
int main () {
  x = 12;
  if ((x / 10 == 2) && (x > 15)
      || (x % 3 == 0))
    printf("YES ");
  else
    printf("NO ");
  if ((x / 5 == 2) || (x < 13)
      && (x % 5 == 1))
    printf("YES");
  else
    printf("NO");
  return 0;
}
```

C++

```
int x;
int main() {
  x = 12;
  if ((x / 10 == 2) && (x > 15)
      || (x % 3 == 0))
    cout << "YES ";
  else
    cout << "NO ";
  if ((x / 5 == 2) || (x < 13)
      && (x % 5 == 1))
    cout << "YES";
  else
    cout << "NO";
  return 0;
}
```

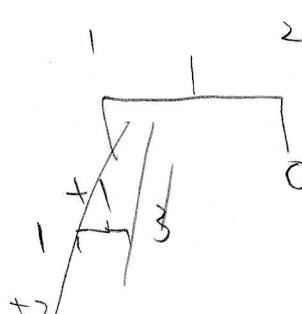
- A. NO YES
B. NO NO
C. YES YES
D. YES NO

12. Five teams participate in a basketball league. Every two teams play one match between them. The winner gets one point and the loser gets zero points. There are no draws.
五隊球隊參加一籃球聯賽。每兩隊對賽一場，勝方得一分，負方得零分，沒有和局。

Which of the following cannot be the total points the teams have at the end of the league?

下列哪項不可能是聯賽完結後各隊的得分？

- A. 4, 2, 2, 1, 1 10
B. 4, 3, 2, 1, 0 10
C. 2, 2, 2, 2, 2 10
D. 4, 4, 1, 1, 0 10



13. Suppose you now have an empty stack and a queue of 7 numbers. You can perform the following actions arbitrarily:

假設你有一個空的堆疊（棧）和一個裝有 7 個數的隊列。你可以任意進行以下操作：

1. If the queue is not empty: pop the front element from the queue and push it into the stack.
如果隊列非空：把隊首元素推入堆疊（棧）中，並將其從隊列中刪除。
2. If the stack is not empty: pop an element from the stack and output it.
如果堆疊（棧）非空：從堆疊（棧）中彈出一個元素，並將其輸出。

Given the queue has 4, 7, 3, 2, 5, 1, 6 (in this order) with 4 being the front element initially. Which of the following can be an output sequence?

給定隊列中有 4, 7, 3, 2, 5, 1, 6 (以此順序) 4 為初始的隊首元素。以下哪一個序列可能是輸出序列？

- A. 2, 3, 1, 5, 7, 6, 4
- B. 3, 4, 7, 5, 2, 1, 6
- C. 6, 1, 5, 2, 7, 3, 4
- D. 4, 7, 3, 1, 2, 5, 6

11 2 3 4 5 6 X

14. What is the output of the following program? 以下程序的輸出是什麼？

Pascal

```
var
  a: array[0..7] of longint =
    (3, 1, 4, 1, 5, 9, 2, 6);
  i, j: longint;
begin
  for i := 1 to 7 do
    for j := 0 to 7 - i do
      if (a[j] > a[j + 1]) then
        begin
          a[j] := a[j + 1];
          a[j + 1] := a[j];
        end;
  write(a[1], ' ', a[6])
end.
```

C

```
int a[8] = {3, 1, 4, 1, 5, 9,
            2, 6};
int i, j;
int main() {
  for (i = 1; i <= 7; i++)
    for (j = 0; j <= 7 - i; j++)
      if (a[j] > a[j + 1]) {
        a[j] = a[j + 1];
        a[j + 1] = a[j];
      }
  printf("%d %d", a[1], a[6]);
  return 0;
}
```

C++

```
int a[8] = {3, 1, 4, 1, 5, 9,
            2, 6};
int i, j;
int main() {
  for (i = 1; i <= 7; i++)
    for (j = 0; j <= 7 - i; j++)
      if (a[j] > a[j + 1]) {
        a[j] = a[j + 1];
        a[j + 1] = a[j];
      }
  cout << a[1] << " " << a[6];
  return 0;
}
```

- A. 3 9
- B. 1 6
- C. 1 2
- D. 6 1

15. Given the sequence: 2, 4, 5, 1, 3. What is the minimum number of adjacent swaps required to sort the sequence in increasing order?

給予序列：2, 4, 5, 1, 3。如果要把序列由小至大排列，最少需要進行多少次相鄰的數字交換？

- A. 4
- B. 6
- C. 7
- D. 5

2, 4, 5, 1, 3

4, 2, 5, 1, 3

4, 5, 2, 1, 3

4, 5, 2, 3, 1

16. What is the output of the following program? 以下程序的輸出是什麼?

Pascal

```
var
  x: longint = 101;
  y: longint = 100;
  i: longint;
  dx: array [0..3] of longint =
    (1, 2, 3, 4);
  dy: array [0..3] of longint =
    (4, 3, 2, 1);
begin
  for i := 1 to 100 do
  begin
    x := x + dx[x mod 4];
    y := y + dy[y mod 4]
  end;
  write(x, ' ', y)
end.
```

C

```
int dx[4] = {1, 2, 3, 4};
int dy[4] = {4, 3, 2, 1};
int x = 101, y = 100;
int i;
int main() {
  for(i = 1; i <= 100; i++) {
    x = x + dx[x % 4];
    y = y + dy[y % 4];
  }
  printf("%d %d", x, y);
  return 0;
}
```

C++

```
int dx[4] = {1, 2, 3, 4};
int dy[4] = {4, 3, 2, 1};
int x = 101, y = 100;
int i;
int main() {
  for(i = 1; i <= 100; i++) {
    x = x + dx[x % 4];
    y = y + dy[y % 4];
  }
  cout << x << ' ' << y;
  return 0;
}
```

- A. 1 4 ~~X~~
 B. 351 350 ~~X~~
 C. 499 500
 D. 495 500

$$\begin{array}{r} 4 \sqrt{101} \\ \underline{\quad} \\ 2 \end{array}$$

$$\begin{array}{r} 5 \sqrt{500} \\ \underline{\quad} \\ 5 \end{array}$$

17. What is the output of the following program? 以下程序的輸出是什麼?

Pascal

```
var
  a: array[0..7] of longint =
    (2, 0, 2, 1, 1, 1, 1, 3);
  x: longint;
begin
  x := 0;
  if (a[3] = 2) then
    x := x + 1
  else if (a[6] mod 2 = 1) then
    x := x + 2;
    if (a[4] = a[7]) then
      x := x + 3
  else
    x := x + 4;
  write(x)
end.
```

C

```
int a[8] =
  {2, 0, 2, 1, 1, 1, 1, 3};
int x;
int main() {
  x = 0;
  if (a[3] == 2)
    x = x + 1;
  else if (a[6] % 2 == 1)
    x = x + 2;
    if (a[4] == a[7])
      x = x + 3;
  else
    x = x + 4;
  printf("%d", x);
  return 0;
}
```

C++

```
int a[8] =
  {2, 0, 2, 1, 1, 1, 1, 3};
int x;
int main () {
  x = 0;
  if (a[3] == 2)           x = 1
    x = x + 1;
  else if (a[6] % 2 == 1)  x = 2
    x = x + 2;
    if (a[4] == a[7])      x = 3
      x = x + 3;
  else
    x = x + 4;
  cout << x;
  return 0;
}
```

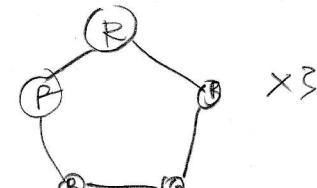
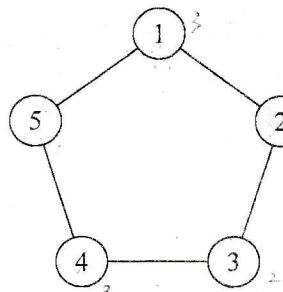
- A. 4
 B. 6
 C. 2
 D. 1

18. You are given a pentagon with 5 vertices labeled from 1 to 5 (see the figure below). Find the number of ways to color each vertex in one of red, green or blue so that no two neighbouring vertices have the same color.

Note: Two ways of coloring are considered different if there exists a vertex which is colored differently.

你有一個五邊形，當中 5 個頂點分別被標記為 1 至 5（見下圖）。你可以為每一個頂點填上紅、綠、藍其中一種顏色。求相鄰之頂點顏色不重複的填色方法數。

註：對於任何兩種填色方法，如存在一頂點被填上不同顏色，此兩種填色方法將被視為相異的。



P	B	R G
B	R	B G
G	R X	
G	B X	

- A. 30
- B. 5
- C. 15
- D. 20

19. Suppose a and b are 32-bit signed integer variables (Pascal: `longint`, C / C++: `int`) each storing a positive integer, which of the following expressions are equivalent to i ?

假設 a 和 b 是 32 位元有符號的整數變量 (Pascal: `longint`, C / C++: `int`)，各儲存一個正整數，以下哪些表達式與 i 相等？

Pascal

i. $a = b$

C / C++

$a == b$

ii. $(a \text{ xor } b) = 0$

$(a \wedge b) == 0$

iii. $(a \text{ or } b) = (a \text{ and } b)$

$(a | b) == (a & b)$

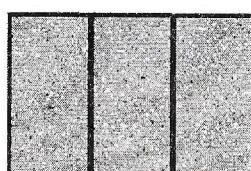
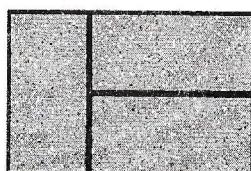
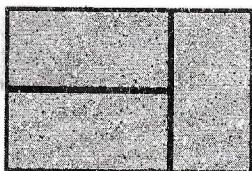
iv. $(a + b) \text{ div } 2 = a$

$(a + b) / 2 == a$

- A. iii only 只有 iii
- B. ii only 只有 ii
- C. ii, iii and iv ii、iii 和 iv
- D. ii and iii only 只有 ii 和 iii

20. Below are the all 3 arrangements of tiling a 2×3 grid with only 1×2 tiles :

以下是以 1×2 方塊填滿 2×3 網格的全部 3 種方法：



What is the number of arrangements tiling a 2×10 grid with only 1×2 tiles?

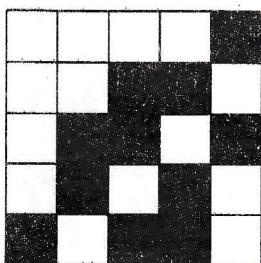
有多少種不同方法以 1×2 方塊填滿 2×10 網格？

2×10

- A. 81
- B. 89
- C. 55
- D. 144

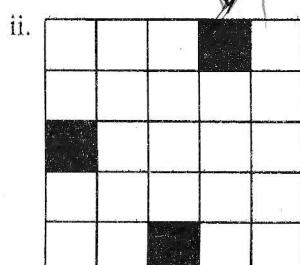
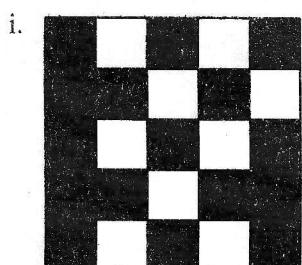
21. Consider the following 5×5 grid with black and white cells.

現有以下 5×5 的網格，其中有着黑色和白色的格子。



You can perform the following action an arbitrary number of times: Choose a 2×2 sub-grid completely within the grid, change each cell in the sub-grid into its opposite colours, i.e. black to white, white to black. Which of the following grids can be produced from the grid above?

你可以進行以下操作任意多次：在網格內選取一個 2×2 的子網格，將子網格中每個格子變成其相反的顏色，即由黑變白、由白變黑。請問以下哪些網格能從以上的網格轉變而成？



- A. None of them 無
- B. ii only 只有 ii
- C. i and ii i 和 ii
- D. i only 只有 i

22. What is the output of the following program? 以下程序的輸出是什麼?

Pascal

```
var
  n, cnt: longint;
  i, j, k: longint;
begin
  n := 10;
  cnt := 0;
  for i := 0 to n - 1 do
    for j := i + 2 to n - 1 do
      for k := j - 1 to n - 2 do
        cnt := cnt + 1;
  write(cnt);
end.
```

C

```
int n, cnt;
int i, j, k;
int main() {
  n = 10;
  cnt = 0;
  for (i = 0; i <= n - 1; i++)
    for (j = i + 2; j <= n - 1;
         j++)
      for (k = j - 1; k <= n -
           2; k++)
        cnt++;
  printf("%d", cnt);
  return 0;
}
```

C++

```
int n, cnt;
int i, j, k;
int main() {
  n = 10;
  cnt = 0;
  for (i = 0; i <= n - 1; i++)
    for (j = i + 2; j <= n - 1;
         j++)
      for (k = j - 1; k <= n -
           2; k++)
        cnt++;
  cout << cnt;
  return 0;
}
```

- A. 1000
- B. 900
- C. 120
- D. 240

$\alpha = 50$

23. Consider the following program: 考慮以下程序：

Pascal

```
var a: longint;
begin
  read(a);
  if (a mod 2 = 0) then
    a := a div 2;
  if (a mod 3 = 0) then
    a := a div 3;
  if (a mod 4 = 0) then
    a := a div 4;
  if (a mod 5 = 0) then
    a := a div 5;
  write(a);
end.
```

C

```
int a;
int main() {
  scanf("%d", &a);
  if (a % 2 == 0)
    a = a / 2;
  if (a % 3 == 0)
    a = a / 3;
  if (a % 4 == 0)
    a = a / 4;
  if (a % 5 == 0)
    a = a / 5;
  printf("%d", a);
  return 0;
}
```

C++

```
int a;
int main() {
  cin >> a;
  if (a % 2 == 0)
    a = a / 2;
  if (a % 3 == 0)
    a = a / 3;
  if (a % 4 == 0)
    a = a / 4;
  if (a % 5 == 0)
    a = a / 5;
  cout << a;
  return 0;
}
```

Given that the input is an integer between 1 and 100 inclusive, for how many inputs will the program output 1?
若輸入為一個 1 至 100 (含) 之整數，有多少個輸入會令程序輸出 1?

- A. 11
- B. 15
- C. 12
- D. 9

4+

20

(60)

4

23, 45

2 3 ④ 5 6 7 ⑧ 9 10
 11 ② 13 14 ⑤ 16 17 18 19
 ⑥ 21 22 23 ⑦ 25 26 27 28
 29

K - 11

24. There are 6 numbers 0, 1, 2, 3, 5 and 8. You can take any of them (from none to all). How many ways you can take such that the sum of the numbers you take is between 10 and 20 inclusive?

現有 6 個數字 0, 1, 2, 3, 5 和 8。你可以拿走任意數量的數字（從不拿到拿走所有均可）。有多少種你拿走數字的方法使得你拿走的數字之和介乎 10 和 20 之間（含）？

- A. $31 \times 8 + 32 + 53 = 1$ ②
 B. $29 \times 8 - \sqrt{5} \times 8$ ⑧
 C. $30 \times 5 - \sqrt{3} - 4$ ④
 D. $32 \checkmark - \times 3 - 2, 21$ ②

$$\checkmark 8+0+1+\cancel{x}+3+5 = 18$$

$$\checkmark 8+1+2+3+5$$

$$\cancel{4}+2 \quad 4$$

$$4 \times 3 \times 2$$

25. Alice and Bob are in a survival game, in which they must cross an elevated bridge in order. There are 3 steps to the end of the bridge, with each step having 3 normal glasses and 1 tempered glass a player can step on. For each step, a player steps on 1 of the 4 glass panels ahead uniformly at random. If the glass is tempered, they proceed to the next step. Otherwise, the glass breaks and the player is eliminated.

When the first player is eliminated, the next player begins to cross the bridge. Assumed that they always follow tempered glass steps that the previous player has stepped on.

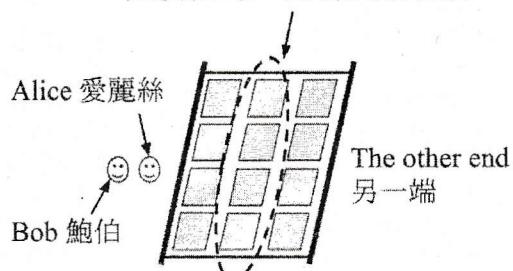
A player wins if and only if they arrive at the other end of the bridge first. If Alice goes first, what is the probability of Bob winning the game?

愛麗絲和鮑伯正在玩一個生存遊戲，當中他們必須依序橫越一道架高橋。通往橋的另一邊需要走 3 步，每步分別有 3 塊普通玻璃和 1 塊強化玻璃可以踏。對於每步，玩家會踏上面前 4 塊中的隨機 1 塊玻璃。如果玻璃是強化玻璃，玩家可繼續下一步。否則玻璃會破裂而玩家會被淘汰。

當第一位玩家被淘汰，下一位玩家便可以開始過橋。假設他總是會跟隨前一位玩家踏過的強化玻璃。

一位玩家勝出當且僅當他先抵達橋的另一邊。如果愛麗絲先走，鮑伯勝出的機率是多少？

Glass panels a player can step on for their second step
玩家第二步可以踏的玻璃板



Game layout 遊戲佈局

- A. $\frac{1}{16}$
 B. $\frac{3}{64}$
 C. $\frac{9}{256}$
 D. $\frac{5}{128}$

END OF SECTION A 甲部完

Section B 乙部 (20 marks 分)

The blanks are labeled from A to K. Please fill in the blanks on the answer sheet.
下列各空格分別命名為 A 至 K，請在答題紙上對應的地方填上答案。

Note 注意：

- (1) Select exactly one programming language on the Answer Sheet. Answers must be in that language.
您必須在答題紙上選擇剛好一種編程語言，並只使用該語言作答。
- (2) For C and C++, you must not use the ?: operator.
對於 C 及 C++，答案不可以包括 ?: 運算元。
- (3) You must not use any library function unless the appropriate library has been included. (See Page 1)
除非適當的函數庫已被引用 (見頁一)，否則答案不可以包括任何函數庫內的函數。
- (4) You can write only one character in each box on the answer sheet.
答題紙上每個小格只可填上一個字符。
- (5) Answers must not exceed the designated number of boxes.
答案長度不得多於該題提供的小格數目。
- (6) Write legibly. Unrecognizable answers will be regarded as incorrect.
字體須端正清楚，無法辨別之答案當錯誤論。
- (7) If blank X is divided into N parts X1, X2, ..., XN, it means that marks will only be given when X1, X2, ..., XN are all correct.
如果空格 X 分為 N 部份 X1、X2、...、XN，那麼 X1、X2、...、XN 皆為正確才會給分。

1. Complete the following program such that its output is ARENA.
完成以下程序使得其輸出是 ARENA。

Pascal

```
var
  river: string = 'RIVER';
  a: array[1..5] of longint;
  i: longint;
begin
  for i := 1 to 5 do
    a[i] := ord(river[i])
    - ord('A');
  for i := 1 to 5 do
  begin
    a[i] := _____ A _____;
    a[i] := a[i] + ord('A');
    write(chr(a[i]));
  end
end.
```

C

```
char river[] = "RIVER";
int a[5];
int i;
int main() {
  for (i = 0; i <= 4; i++)
    a[i] = (int)river[i] - (int)
      'A';
  for (i = 0; i <= 4; i++) {
    a[i] = _____ A _____;
    a[i] = a[i] + (int)'A';
    printf("%c", (char)a[i]);
  }
  return 0;
}
```

C++

```
string river = "RIVER";
int a[5];
int i;
int main () {
  for (i = 0; i <= 4; i++)
    a[i] = (int)river[i] - (int)
      'A';
  for (i = 0; i <= 4; i++) {
    a[i] = _____ A _____;
    a[i] = a[i] + (int)'A';
    cout << (char)a[i];
  }
  return 0;
}
```

Answer 答案: _____ A _____ (2 marks 分)

2. In Heung Shing, the coin system has n different denominations. We use an integer array $c[0..n-1]$ (C/C++: $c[n]$) to represent the coin system, so that the n types of coins are worth, respectively, $c[0]$ cents, $c[1]$ cents, ..., and $c[n-1]$ cents, where $1 = c[0] < c[1] < \dots < c[n-1]$.

香城的硬幣系統有 n 種面額。以整數數組 $c[0..n-1]$ (C/C++: $c[n]$) 表示之，則硬幣的價值分別為 $c[0]$ 毫子、 $c[1]$ 毫子、...、 $c[n-1]$ 毫子，使得 $1 = c[0] < c[1] < \dots < c[n-1]$ 。

(Note that the English and Chinese versions use different monetary units.)

(注意，中文及英文版本使用不同的金額單位。)

Alice wants to solve the coin changing problem: what is the minimum number of coins needed to make a change of exactly m cents? She proposes the following greedy algorithm: first use as many as possible the most valuable coin, then use as many as possible the second most valuable coin, and so on.

愛麗絲想解決以下問題：作剛好 m 毫子的找續，至少需要幾多枚硬幣？她提議以下貪心演算法：先儘量多用價值最高的硬幣，然後儘量多用價值第二高的硬幣，如此類推。

(a) Complete the following program so that it implements the greedy algorithm described above. At the end of the program, $b[i]$ should equal the number of $c[i]$ -cent coins used. The input and output sections of the program are omitted. You may assume that $1 \leq n \leq 100$.

完成以下程序，以實現上述貪心演算法。在程序最後， $b[i]$ 應等於使用了的 $c[i]$ 毫子硬幣數量。程序的輸入輸出部分省略。假設 $1 \leq n \leq 100$ 。

Pascal

```
var
  b, c: array[0..99] of longint;
  n, m: longint;
  i: longint;
begin
  // input 輸入 n, m, c[0], ..., c[n-1]
  for _____ B1 _____ do
    begin
      b[i] := 0;
      while _____ B2 _____ do
        begin
          inc(b[i]);
          m := _____ B3 _____
        end
    end;
  // output 輸出 b[0], ..., b[n-1]
end.
```

C / C++

```
int b[100], c[100];
int n, m;
int i;
int main() {
  // input 輸入 n, m, c[0], ..., c[n-1]
  for ( _____ B1 _____ ) {
    b[i] = 0;
    while ( _____ B2 _____ ) {
      b[i]++;
      m = _____ B3 _____;
    }
  }
  // output 輸出 b[0], ..., b[n-1]
}
```

Answer 答案: _____ B1 _____ B2 _____ B3 _____ (2 marks 分)

(b) Unfortunately, Alice's greedy algorithm does not always work. Find a counter-example with $n = 3$, $1 = c[0] < c[1] < c[2] \leq 10$, and $m = 10$. Write down the values of $c[1]$ and $c[2]$.

不幸地，愛麗絲的貪心演算法並不總是正確。請找一個反例，符合 $n = 3$ 、 $1 = c[0] < c[1] < c[2] \leq 10$ 、及 $m = 10$ 。寫下 $c[1]$ 和 $c[2]$ 的值。

10

Answer 答案: _____ C1 _____ C2 _____ (1.5 marks 分)

(c) A coin system $c[0..n-1]$ is said to be *simple*, if Alice's greedy algorithm can produce an optimal answer for every positive integer m not greater than 10000. Which of the following coin systems is/are simple? Write down the corresponding indices of all simple coin systems and separate them with commas. (Example: 2, 3, 4). Only answers that are all correct will get credit.

如果一硬幣系統 $c[0..n-1]$ ，使得愛麗絲的貪心演算法對於所有不大於 10000 的正整數 m 都能輸出最佳答案，則稱該硬幣系統為簡單。以下哪些硬幣系統為簡單？請寫下所有簡單硬幣系統對應的序號，並用逗號分隔(例: 2, 3, 4)。全對才給分。

1. $c[0..2] = \{1, 8, 20\}$
2. $c[0..3] = \{1, 2, 5, 10\}$
3. $c[0..4] = \{1, 2, 4, 8, 10\}$
4. $c[0..6] = \{1, 4, 7, 10, 13, 16, 19\}$
5. $c[0..6] = \{1, 20, 30, 40, 50, 60, 70\}$

Answer 答案: D (2 marks 分)

3. Consider a $n \times m$ grid ($1 \leq n, m \leq 1000$), where every cell is placed with a light bulb that is initially turned off. Each of the n rows and m columns of the grid is connected to an individual button, which toggles (on→off, off→on) all the light bulbs in that corresponding row/column when pressed. Complete the following program so that it reads the status (pressed: 1, not pressed: 0) of every button and outputs the number of light bulbs that are ultimately on. $r[i]$ and $c[j]$ stores the status of the button of the i -th row and the j -th column respectively.

考慮一個 $n \times m$ 的格網 ($1 \leq n, m \leq 1000$)，在每個格子中放了一個原本關掉的燈泡。格網的 n 個列和 m 個行都分別連接着一個獨立的按鈕，按下去時會把對應的行/列的所有燈泡狀態切換掉（開 → 關，關 → 開）。完成以下程序使得它輸入每個按鈕的狀態（已按：1，未按：0）並輸出最終亮着的燈泡的數量。 $r[i]$ 和 $c[j]$ 分別儲存第 i 列和第 j 行的按鈕的狀態。

Pascal

```
var
  r: array[0..1000] of longint;
  c: array[0..1000] of longint;
  n, m, x, y, i, j: longint;
begin
  read(n, m);
  x := 0;
  for i := 1 to n do
  begin
    read(r[i]);
    x := x + r[i];
  end;
  y := 0;
  for j := 1 to m do
  begin
    read(c[j]);
    y := y + c[j];
  end;
  write(E)
end.
```

C

```
int r[1001];
int c[1001];
int n, m, x, y, i, j;
int main() {
  scanf("%d %d", &n, &m);
  x = 0;
  for (i = 1; i <= n; i++) {
    scanf("%d", &r[i]);
    x = x + r[i];
  }
  y = 0;
  for (j = 1; j <= m; j++) {
    scanf("%d", &c[j]);
    y = y + c[j];
  }
  printf("%d", E);
  return 0;
}
```

C++

```
int r[1001];
int c[1001];
int n, m, x, y, i, j;
int main() {
  cin >> n >> m;
  x = 0;
  for (i = 1; i <= n; i++) {
    cin >> r[i];
    x = x + r[i];
  }
  y = 0;
  for (j = 1; j <= m; j++) {
    cin >> c[j];
    y = y + c[j];
  }
  cout << E;
  return 0;
}
```

Answer 答案: E (1.5 marks 分)

4. The input to the following program is 401 integers between 1 and 1000 inclusive, given in non-descending order. Complete the following program so that it outputs an integer x , such that the sum of absolute difference of x and each input number is minimized. If there is multiple x that can achieve the same minimum sum, the program should output the least x .

以下程序的輸入是 401 個以非遞減次序給定的整數，每個數均在 1 及 1000 之間(含)。完成以下程序使得它輸出 x ，使得 x 和各輸入數字的絕對差的總和最小。假如有多於一個 x 能達到相同的最小總和，程序應輸出最小的 x 。

Pascal

```
var
  a: array[0..400] of longint;
  x, minSum: longint;
  temp: longint;
  i, j: longint;
begin
  for i := 0 to 400 do
    read(a[i]);
  x := -1;
  minSum := 1000000000;
  for i := 1 to 1000 do
  begin
    temp := 0;
    for j := 0 to 400 do
      temp := temp +
        _____ F1 _____;
    if (temp < minSum) then
    begin
      minSum :=
        _____ F2 _____;
      x := _____ F3 _____;
    end
  end;
  write(x)
end.
```

C

```
int a[401];
int x, minSum;
int temp;
int i, j;
int main() {
  for (i = 0; i <= 400; i++)
    scanf("%d", &a[i]);
  x = -1;
  minSum = 1000000000;
  for (i = 1; i <= 1000; i++) {
    temp = 0;
    for (j = 0; j <= 400; j++)
      temp = temp +
        _____ F1 _____;
    if (temp < minSum) {
      minSum =
        _____ F2 _____;
      x = _____ F3 _____;
    }
  }
  printf("%d", x);
  return 0;
}
```

C++

```
int a[401];
int x, minSum;
int temp;
int i, j;
int main() {
  for (i = 0; i <= 400; i++)
    cin >> a[i];
  x = -1;
  minSum = 1000000000;
  for (i = 1; i <= 1000; i++) {
    temp = 0;
    for (j = 0; j <= 400; j++)
      temp = temp +
        _____ F1 _____;
    if (temp < minSum) {
      minSum =
        _____ F2 _____;
      x = _____ F3 _____;
    }
  }
  cout << x;
  return 0;
}
```

Answer 答案: _____ F1 _____ F2 _____ F3 _____ (2 marks 分)

Complete the following program so that it produces the same output as the above program.
完成以下程序使得它的輸出與以上程序相同。

Pascal

```
var
  a: array[0..400] of longint;
  i: longint;
begin
  for i := 0 to 400 do
    read(a[i]);
  write(____ G ____)
end.
```

C

```
int a[401];
int i;
int main() {
  for (i = 0; i <= 400; i++)
    scanf("%d", &a[i]);
  printf("%d", _____ G _____);
  return 0;
}
```

C++

```
int a[401];
int i;
int main() {
  for (i = 0; i <= 400; i++)
    cin >> a[i];
  cout << _____ G _____;
  return 0;
}
```

Answer 答案: _____ G _____ (1.5 marks 分)

5. Complete the following program such that its output is HKOI.

完成以下程序使得其輸出 HKOI。

Pascal

```
var
  a: array[0..9] of longint =
    (1, 2, 12, 0, 58,
     74, 64, 92, 58, 0);
  i, sum: longint;
begin
  a[9] := _____ H _____;
  sum := 0;
  for i := 0 to 9 do
    sum := sum xor a[i];
  if sum = 118 then
    write('HKOI')
end.
```

C

```
int a[10] = {1, 2, 12, 0, 58,
             74, 64, 92, 58, 0};
int i, sum;
int main() {
  a[9] = _____ H _____;
  sum = 0;
  for (i = 0; i <= 9; i++)
    sum = sum ^ a[i];
  if (sum == 118)
    printf("HKOI");
  return 0;
}
```

C++

```
int a[10] = {1, 2, 12, 0, 58,
             74, 64, 92, 58, 0};
int i, sum;
int main() {
  a[9] = _____ H _____;
  sum = 0;
  for (i = 0; i <= 9; i++)
    sum = sum ^ a[i];
  if (sum == 118)
    cout << "HKOI";
  return 0;
}
```

Answer 答案: _____ H _____ (1.5 marks 分)

Handwritten notes:
a/18
2/18
3/18
59
2118
18

6. The following program tries to delete all negative numbers in the array. However, the program contains an error.
以下程序嘗試刪除數組中的所有負數。可是，程式中有一個錯誤。

Pascal

```

1 var
2   n, i, j: longint;
3   a: array[0..99] of longint;
4 begin
5   read(n);
6   i := 0;
7   while (i < n) do
8   begin
9     read(a[i]);
10    inc(i);
11  end;
12  i := 0;
13  while (i < n) do
14  begin
15    if (a[i] < 0) then
16    begin
17      j := i + 1;
18      while (j < n) do
19      begin
20        a[j - 1] := a[j];
21        inc(j);
22      end;
23      dec(n);
24    end;
25    inc(i);
26  end;
27  i := 0;
28  while (i < n) do
29  begin
30    write(a[i], ' ');
31    inc(i);
32  end
33 end.

```

C

```

34 int n, i, j, a[100];
35 int main() {
36   scanf("%d", &n);
37   i = 0;
38   while (i < n)
39   {
40     scanf("%d", &a[i]);
41     i++;
42   }
43   i = 0;
44   while (i < n)
45   {
46     if (a[i] < 0)
47     {
48       j = i + 1;
49       while (j < n)
50       {
51         a[j - 1] = a[j];
52         j++;
53       }
54       n--;
55     }
56     i++;
57   }
58   i = 0;
59   while (i < n)
60   {
61     printf("%d ", a[i]);
62     i++;
63   }
64   return 0;
65 }

```

C++

```

67 int n, i, j, a[100];
68 int main() {
69   cin >> n;
70   i = 0;
71   while (i < n)
72   {
73     cin >> a[i];
74     i++;
75   }
76   i = 0;
77   while (i < n)
78   {
79     if (a[i] < 0)
80     {
81       j = i + 1;
82       while (j < n)
83       {
84         a[j - 1] = a[j];
85         j++;
86       }
87       n--;
88     }
89     i++;
90   }
91   i = 0;
92   while (i < n)
93   {
94     cout << a[i] << " ";
95     i++;
96   }
97   return 0;
98 }
99

```

The bug can be fixed by changing exactly one line. Find the line and correct it so that the program outputs the correct answers for all of the following cases:

只需修改一行便能修正程式的錯誤。找出該行並修正它，使程式對於下列的各個輸入均能輸出正確的答案：

Input 輸入	Output 輸出
5 1 2 -3 -4 5	1 2 5
2 -5 3	3
4 -3 -4 8 -6	8

Line number 行數: I1

Correction 改正: I2 (2 marks 分)

7. Given a function f that takes the coordinates of two horizontal line segments as parameters, where:
給予函數 f ，其參數為兩條橫線線段，當中：

- line segment a connects two points (ax_1, ay) and (ax_2, ay) and it is guaranteed that $ax_1 \leq ax_2$.
線段 a 連接兩點 (ax_1, ay) 和 (ax_2, ay) ，保證 $ax_1 \leq ax_2$ 。
- line segment b connects two points (bx_1, by) and (bx_2, by) and it is guaranteed that $bx_1 \leq bx_2$.
線段 b 連接兩點 (bx_1, by) 和 (bx_2, by) ，保證 $bx_1 \leq bx_2$ 。

Complete f **WITHOUT** using any function calls, so that it returns `true` if and only if a and b share at least one point.

在**不調用**任何函數的情況下完成函數 f ，使其返回 `true` 當且僅當 a 和 b 共享至少一點。

Pascal

```
function f(ay, ax1, ax2, by, bx1, bx2: longint):  
  boolean;  
begin  
  f := _____ J _____;  
end;
```

C / C++

```
bool f(int ay, int ax1, int ax2,  
       int by, int bx1, int bx2) {  
  return _____ J _____;  
}
```

Answer 答案: _____ J _____ (2 marks 分)

Given another function g that takes the coordinates of a rectangle and a line segment as parameters, where:
給予另一函數 g ，其參數為一個長方形和一條線段，當中：

- rectangle a has (ax_1, ay_1) as bottom-left vertex and (ax_2, ay_2) as top-right vertex. It is guaranteed that $ax_1 \leq ax_2$ and $ay_1 \leq ay_2$.
長方形 a 的左下角為 (ax_1, ay_1) ，右上角為 (ax_2, ay_2) ，保證 $ax_1 \leq ax_2$ 和 $ay_1 \leq ay_2$ 。
- line segment b connects two points (bx_1, by) and (bx_2, by) and it is guaranteed that $bx_1 \leq bx_2$.
線段 b 連接兩點 (bx_1, by) 和 (bx_2, by) ，保證 $bx_1 \leq bx_2$ 。

Complete g , so that it returns `true` if and only if a and b share at least one point. You may use function f in function g .

完成函數 g ，使其返回 `true` 當且僅當 a 和 b 共享至少一點。你可以在函數 g 中使用函數 f 。

Pascal

```
function g/ay1, ay2, ax1, ax2,  
        by, bx1, bx2: longint): boolean;  
begin  
  if (ay1 > by) or (ay2 < by) then  
    g := false  
  else  
    g := _____ K _____;  
end;
```

C / C++

```
bool g(int ay1, int ay2, int ax1, int ax2,  
       int by, int bx1, int bx2) {  
  if (ay1 > by || ay2 < by)  
    return false;  
  else  
    return _____ K _____;  
}
```

Answer 答案: _____ K _____ (2 marks 分)

END OF PAPER 全卷完