1　準備:幾何学

定義 1. Let X be a set. A topology on X is a collection of subsets of X, such that:
(1) the empty set ϕ and the set X are open;
(2) the union of an arbitrary collection of open sets is open;
(3) the intersection of a finite number of open sets is open.

問題 2. X の部分集合族 $\mathcal{O}$ を用いて、上記の位相空間の定義を書き直せ.

定義 3. 位相空間 M が次の条件 $(1),(2)$ を満足する時, M を m dimensional topological manifold という.
(1) M is Hausdorff space.
(2) For ${}^{\forall}p \in M$ m 次元座標近傍 (U, φ) が存在する.

定義 4. Given any C^k manifold M, of dimension n, with $k \geq 1$, for any $p \in M$, a tangent vector to M at p is any equivalence class of C^1 curves through p on M, modulo the equivalence relation defined in below.

$$\gamma_1 : (-\epsilon_1, \epsilon_1) \to M \gamma_2 : (-\epsilon_2, \epsilon_2) \to M \text{ are equivalent } \Leftrightarrow^{\exists} (U, \varphi) \text{ s.t } \frac{d(\varphi \circ \gamma_1)}{dt}(0) = \frac{d(\varphi \circ \gamma_2)}{dt}(0) \tag{1}$$

次に逆写像定理が重要となる.

定理 5. M, N を C^r 級多様体, $f : M \to N$ を C^r 級写像とする. $(df)_p : T_p(M) \xrightarrow{\sim} T_{f(p)}N$ なら, $p \in^{\exists} U, f(p) \in^{\exists} V$ s.t $f|U : U \xrightarrow{\sim} V$.