MATERIAL CHARACTERIZATION: DETERMINING THE ELASTIC MODULUS OF BIRCH PLYWOOD

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ABSTRACT

With the material properties of wood being difficult to characterize due to the large number of variables that can affect its elastic modulus, the goal of this experiment is to analyze the elastic modulus of birch plywood, a material often used in prototyping. In order to find this material property we employ a three-point bend test with wood specimen modeled as a simply supported beam. By knowing the load applied to the wood through a load cell and the maximum wood deflection found with an optical linear encoder, we are able to manipulate the beam theory Eqn. (1) so the elastic modulus is the slope of our linearly fit data. A vice is used to model the three point bend test, with a load applied between two support points. As the vice is tightened the load cell measures the force on the wood and the linear encoder measures how far the vice lowers which directly correlates to the maximum deflection of the beam. Due to using multiple measurement tools to collect data, uncertainties arise from all of the tools used. For eight wood specimen the elastic modulus was found to range between 221 and 695 MPa. The varying results from previous models of plywood stiffness could be a result of hardware limitations and varying linear windows between deflection and load applied for each specimen.

NOMENCLATURE

E Elastic Modulus [MPa]
P Applied Load [N]
v_{max} Maximum Deflection [m]
L Distance between supporting points of a beam [m]
I Area Moment of Inertia [m^4]
b Width of beam (into page) [m]
h Thickness of beam [m]
F_{LC} Force measured by load cell [N]
V_{LC} Load cell voltage [V]
u_E Uncertainty in elastic modulus [MPa]
$u_{F_{LC}}$ Uncertainty in load cell force [N]
u_P Uncertainty in applied load [N]
u_L Uncertainty in length [m]
$u_{v_{max}}$ Uncertainty in maximum deflection [m]
u_I Uncertainty in moment of inertia [m^4]
u_b Uncertainty in width [m]
u_h Uncertainty in thickness [m]

1 INTRODUCTION

The material properties of wood are difficult to characterize because of its non isotropic properties. Factors that can affect these properties include direction of the grain, age of the tree, moisture content and much more. Our objective is to give an estimate of the elastic modulus of birch plywood, a common material used in laser cutting and rapid prototyping. Quantifying its properties can give engineers a better understanding of how prototypes will respond to stresses and strains. Plywood is manufactured from thin pieces of wood that are glued together with adjacent layers having their wood grain rotated by 90 degrees. This method of cross-graining makes the strength of plywood more consistent in all directions, so we are no longer subject to anisotropic properties depending on grain orientation [1]. To find the elastic modulus, we used a three-point-bend test modeled by the maximum deflection of a simply supported beam. This bend test gives a relationship between bending stress and deflection which provides the flexural modulus, or stiffness, of the wood. Rather than a single measurement system, we used multiple pieces of equipment including a load cell to measure the applied force and an optical linear encoder to measure the displacement in the center of the beam. Using these tools we can find the elastic modulus and quantify its uncertainty using both a linear fit and Gauss' method. Similar experiments found an elastic modulus of around 8000MPa which will be compared with our results [2].

2 MATERIALS AND METHODS

To demonstrate the full experimental procedure, we must cover the theory of the three-point bend test, our quantification of uncertainty, and the experimental equipment and setup.

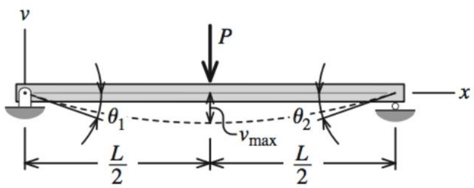

FIGURE 1. SIMPLY SUPPORTED BEAM [3]

2.1 Background and Theory

To relate the three-point bend test to the elastic modulus of a material, we model our specimen as a simply supported beam as seen in Fig. 1. We can relate our measured quantities of maximum deflection v_{max} and applied load P to our desired material property, the elastic modulus E, through Eqn. (1) [3].

$$v_{max} = \frac{PL^3}{48EI} \tag{1}$$

where the moment of inertia I is given by Eqn. (2).

$$I = \frac{bh^3}{12} \tag{2}$$

To find E, we can rearrange Eqn. (1) to solve for E or we can rearrange Eqn. (1) to make E the slope to find a slope fit for the many data points of P and v_{max} as seen in Eqn. (3).

$$\frac{PL^3}{48I} = Ev_{max} \tag{3}$$

2.2 Experimental Setup and Procedure

Our setup, as seen in Fig. 2, consists of a vice with an FC22 compression load cell fastened to the moving plate of the vice. The load cell is applying force to a thin metal block meant to distribute the load evenly across the beam cross section. The displacement of the vice's moving plate is measured by the EM1 transmissive optical encoder module. The specimen is placed on two semicircle supports acting as line contacts attached to the vice's lower plate. Additionally, the high level system diagram is shown in Fig. 3.

The specimens act as beams to which Eqn. (1) will apply. We measured data for eight specimens of Russian birch plywood whose dimensions are given in Fig. 4. The specimens thicknesses are indicated in Table 1. The data for specimen B was thrown out due to recording errors.

This displacement measured by the encoder is equivalent to the maximum deflection experienced by the beam because the load is applied in the center. The applied load P experienced by the beam is displayed in Eqn. (4)

$$P = F_{LC} + W_{block} \tag{4}$$

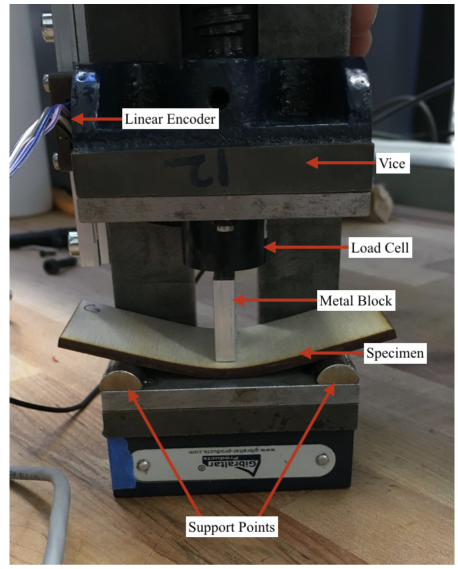

FIGURE 2. EXPERIMENTAL SETUP

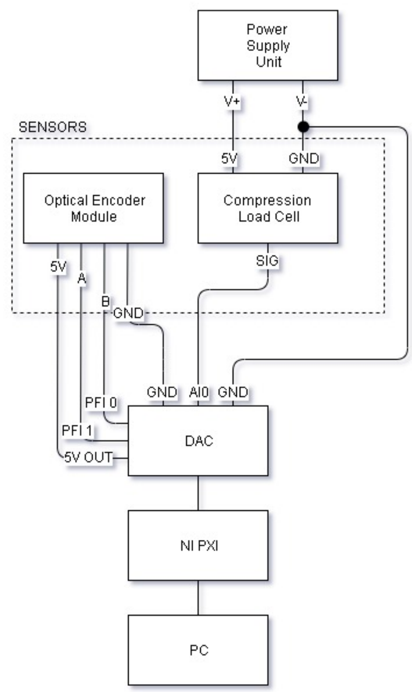

FIGURE 3. HIGH LEVEL SYSTEM DIAGRAM

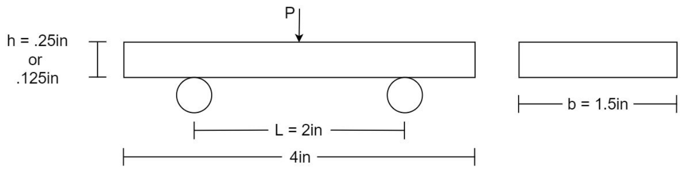

FIGURE 4. SPECIMEN DIMENSIONS

where F_{LC} is the force measured by the load cell and $W_{block} = 0.1001N$ is the weight of the block.

Because the load cell only outputs a voltage, we must correlate that voltage to a force by finding a calibration function. We can find this function by measuring the voltage of the load cell when known weights are applied to it. From the data sheet, we can assume this will be a linear fit given by Eqn. (11).

$$F_{LC} = AV_{LC} + B \quad \pm u_{F_{LC}} \tag{5}$$

where V_{LC} is load cell voltage and A, B are linear fit coefficients.

TABLE 1. MATERIAL THICKNESS

Specimen	Thickness (in.)
A-D	0.25
E-H	0.125

2.3 Uncertainty

Several uncertainties must be measured including that for calibrated load cell and for the elastic modulus. The load cell is calibrated using Eqn. (11) where $u_{F_{LC}}$ demonstrates the uncertainty of the force measured from this linear fit. Assuming that this data is normally distributed, the uncertainty is calculated for a 95% confidence interval by taking the standard deviation σ of the residuals. This standard deviation is transformed into uncertainty by

$$u_{F_{LC}} = 2z\sigma = 1.96\sigma \tag{6}$$

where z is determined from 95% of the area under a normally distributed curve [4].

The uncertainty of the elastic modulus u_E will be calculated in two forms: the slope of a linear fit and through Gauss' method. The linear fit is calculated from Eqn. (3) in a similar method to that of the load cell. However, the method of finding uncertainty described by load cell fit would give the uncertainty of the quantity $\frac{PL^3}{48I}$, but we want the uncertainty of E. To find this uncertainty, we used the MATLAB function `fit` for a linear relationship which outputs the 95% confidence intervals of the polynomial coefficients.

While this uncertainty demonstrates how well the data fits that linear relationship, it does not take into account uncertainties from all of the other variables in the equation, for instance, the uncertainty from the applied load $u_P = u_{F_{LC}}$. All of the quantities from Eqn. (1) have uncertainties. We can use Gauss' method to calculate the uncertainty for every data point and take the maximum value to determine a more comprehensive uncertainty for the elastic modulus [4]. The uncertainty of the elastic modulus from Gauss' method is given by Eqns. (7) and (8).

$$u_E = \sqrt{(\frac{\partial E}{\partial P}u_P)^2 + (\frac{\partial E}{\partial L}u_L)^2 + (\frac{\partial E}{\partial v_{max}}u_{v_{max}})^2 + (\frac{\partial E}{\partial I}u_I)^2} \tag{7}$$

$$= \left(\left(\frac{L^3}{48 v_{max} I} u_P \right)^2 + \left(\frac{3PL^2}{48 v_{max} I} u_L \right)^2 + \left(\frac{-PL^3}{48 v_{max}^2 I} u_{v_{max}} \right)^2 + \left(\frac{-PL^3}{48 v_{max} I^2} u_I \right)^2 \right)^{\frac{1}{2}} \tag{8}$$

Similarly, the uncertainty of the moment of inertia in Eqn. (2) can be found with Eqns. (9) and (10).

$$u_I = \sqrt{\left(\frac{\partial I}{\partial b}u_b\right)^2 + \left(\frac{\partial I}{\partial h}u_h\right)^2} \tag{9}$$

$$= \sqrt{\left(\frac{h^3}{12}u_b\right)^2 + \left(\frac{bh^2}{4}u_h\right)^2} \tag{10}$$

3 RESULTS AND DISCUSSION

For the material characterization experiment of birch plywood, a total of eight specimens were analyzed. The results of each experiment are summarized in Table 2. Figures 12 and 13 show representative results from three-point bend test performed with 0.25" and 0.125" ordinary birch plywood specimens. Moreover, a compression load cell was calibrated against five known weights for determining a relationship between output voltage and compression force. Results from both three-point bend tests, load cell calibration experiments, and reasons for discrepancies between expected and experimental results are discussed.

3.1 Three-point Bend Tests

Linear fit plots for each specimen are shown in Appendix A. The output voltage and linear encoder displacement for each

TABLE 2. ELASTIC MODULUS EXPERIMENTAL VALUES

Spec.	Elastic Modulus (MPa)	Slope Uncertainty (MPa)	Gaussian Uncertainty (MPa)
A	2687.95	24.19	4281.80
C	695.95	1.89	150.10
D	691.72	1.31	138.77
E	288.83	0.30	217.21
F	262.79	0.12	238.36
G	278.12	0.17	218.07
H	287.70	0.19	271.85

test can be seen before truncation in Figures 6 and 7 for 0.25" and 0.125" thick plywood respectively. The windowing truncation was decided based on the load cell data for 0.25" specimen and the linear encoder data for the 0.125" specimen. This is because the load cell can only handle up to 5V but the 0.25" wood requires a higher load than supplied by 5V fracture, so the window chosen needs to be prior to the load cell saturation. Without truncation, the deflection will appear to increase with no increase of load. Similarly, the 0.125" wood bottoms out against the vice before fracture occurs, so the windowing section for these specimen must be prior to this which is depicted as a plateau in the linear encoder graph. If not truncated, the graph will show an increase in load for no deflection.

Figures 12 and 13 show that specimens D and E under a variable compression force, in the range of 17N-100N and 4N-28N respectively, resulted in a linear relationship between the quantities in Eqn. (3). From these figures, we can see that the 0.25" thick plywood specimen was able to sustain higher loads and deflections compared to the 0.125" about the center of the beam. It was observed that the 0.25" thick plywood had much more resistance against plastic deformation than the 0.125" thick plywood. This was due to the increased number of wood veneer layers making up the specimen. Furthermore, the 0.125" thick plywood yielded before the 0.25" thick plywood. The load cell output voltage did not experience signal saturation for the 0.125" plywood whereas the 0.25" plywood did. On the other hand, 0.5" plywood did not experience signal saturation for encoder displacement measurements whereas the 0.125" plywood did.

3.2 Load Cell Calibration Experiments

Figure 8 shows a plot of applied force and load cell output voltage. The applied force was determined by applying five known weights against the load cell. The following weights were used during the calibration experiment: 0.1555, 0.2072, 0.3110, 0.3626 kg. For each trial, the output voltage of the load cell was recorded until a steady state value was met. This value was determined to be the voltage associated with the known weight. These values were determined for each known weight and plotted against known weights. Figure 9 shows the results of these tests. The following calibration curve was used throughout the rest of the experiment:

$$F_{LC} = 25.56V_{LC} - 11.34 \pm 0.090 \tag{11}$$

where the uncertainty is given by Eqn. (6).

This calibration curve resulted in a range of 0 - 25 lbf (0 - 111N). Because the the voltages measured were only between 0.5-0.6V while we used the full range from 0.5-4.8V, we extrapolated this calibration function through the full voltage range.

3.3 Reasons for Discrepancies

A similar study on plywood using a three-point bend test to determine the elastic modulus of wood measured a value around 8000MPa [2]. However, our data suggest values in the range of 250-700MPa causing us to deviate by an order of 10. This can be seen by comparing the results in Table 2, and the results determined by the literature in Figure 5.

With the exception of the data for specimen A, the uncertainties calculated in Table 2 do not account for this large of a deviation. It is also clear how much larger the Gaussian uncertainty is in comparison to the slope fit uncertainty. From Table 3, we can see the many uncertainties that arise from the measurement tools used and how they propagate. We can see that using only a slope fit uncertainty would not sufficiently describe the 95% confidence interval.

The reason for possible discrepancies include: hardware calibration, various wood properties and treatments, and appropriate windowing for each specimen. Additionally, the calibration curve determined was only valid for loads between the range of 0 - 25 lbf (0 - 111N). According to the manufacturer's data sheet, a range of 0 - 100 lbf (0 - 444N) was expected to be a standard range. A calibration of the load cell for the full range of values could have produced better results. A possible solution to this problem includes the use of a load cell with a range appropriate for the application. Another discrepancy that contributed to our results was the assumption made to assume that every specimen possessed constant properties and similar treatments. Plywood is a composite structure that consists of veneers across each other connected by a bonded joint. Each manufacturing process involved contribute to the material properties, including tree species, age of tree, surface appearance, grain orientation, glue joint, condition it is in, and much more. All of these properties and treatments involved during the manufacturing process affect

TABLE 3. MEASUREMENT UNCERTAINTIES

Uncertainties	Measurement Tool	Value
u_p	Load cell	$\pm 0.091 N$
u_L	Calipers	$\pm 0.5 mm$
$u_{v_{max}}$	Linear optical encoder	$\pm 0.005 in.$
u_b	Laser cutter	$\pm 0.5 mm$
u_h	Calipers	$\pm 0.001 in$
u_I (0.25”)	Laser cutter/Calipers	$\pm$1.9e-10m^4
u_I (0.125”)	Laser cutter/Calipers	$\pm$4.8e-11m^4

the material’s properties such as the elastic modulus. Possible solutions include the use of plywood’s made by different manufacturers, the use requirements in accordance with US product standard structural plywood standards (i.e. wood species, veneer grade, adhesive bond requirements, etc.) [1]. Lastly, selecting different windows of data for each specimen contributed to the results determined. Depending on the time interval selected for each trial a completely different value for elastic modulus was found. Each value varied significantly due to the curve veering away from linearity as the specimen began to experience plastic deformation. So to ensure the region of interest was linear, appropriate time intervals were selected. A possible solution to this problem would have been to use an instron machine with precise controls and accuracy.

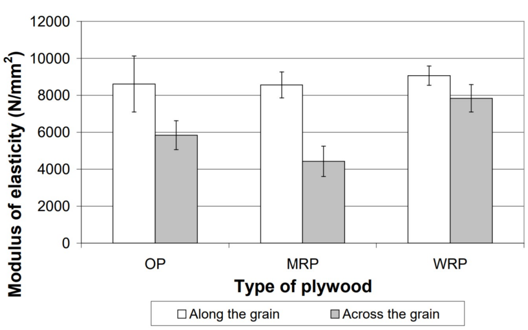

FIGURE 5. COMPARISON RESULTS FOR ELASTIC MODULUS [2]

4 CONCLUSION

Considering only the good data sets of specimen C-H, our results show that the elastic modulus of plywood varies depending on the thickness of the wood. The quarter inch specimen have an elastic modulus of around 693 MPa where the eigth-inch wood is around 280 MPa. Both of these values are an order of magnitude of 10 lower than the expected strength of the wood as found in a previous study. The reason for the difference in values could be due to hardware limitations, experiment repeatability, and the assumption of all other wood variables to be constant. For future work we would need to analyze the other variables that can affect wood strength and how they play into the results we are getting. Additionally, using a load cell that can handle the full range of loads applied and taller support points will allow us to analyze the specimen all the way up to fracture.

ACKNOWLEDGMENT

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[2] Siim K., Kask R., Lille H., Takker E., 2012. *“STUDY OF PHYSICAL AND MECHANICAL PROPERTIES OF BIRCH PLYWOOD DEPENDING ON MOISTURE CONTENT.”*

[3] R. C. Hibbeler., 2018. *Mechanics of Materials.* Harlow, England: Pearson.

[4] Richard S. Figliola, Donald E. Beasley, 2011. *Theory and Design for Mechanical Measurements Fifth Edition.* John Wiley Sons, Inc.

Appendix A: Plots

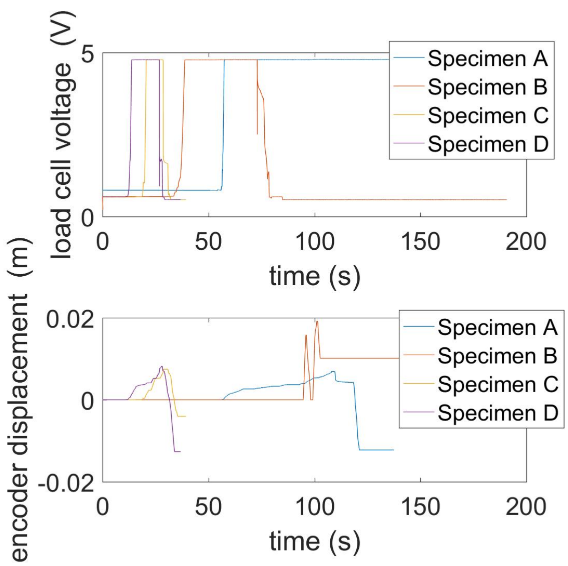

FIGURE 6. .25” SPECIMENS RAW DATA

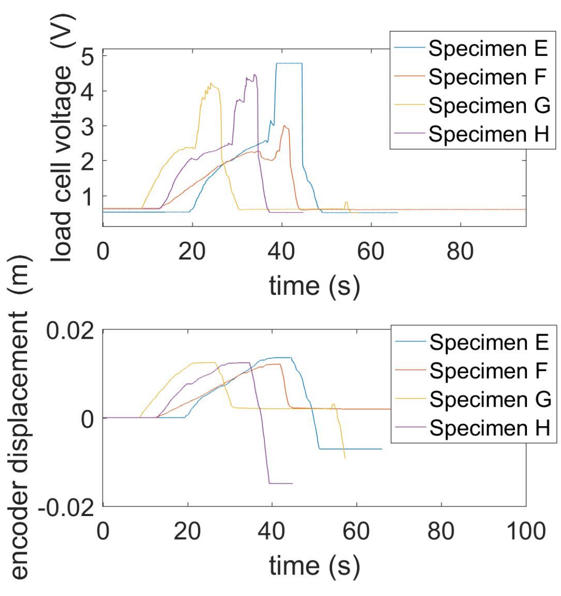

FIGURE 7. .125” SPECIMENS RAW DATA

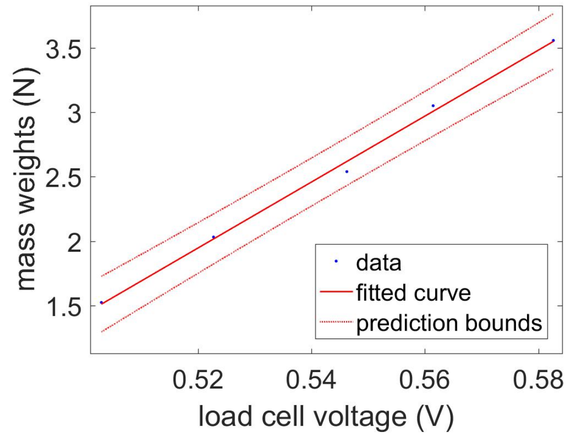

FIGURE 8. LOAD CELL CALIBRATION

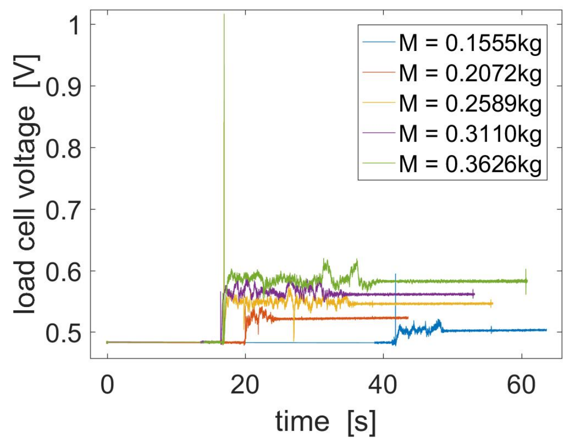

FIGURE 9. LOAD CELL CALIBRATION VOLTAGES

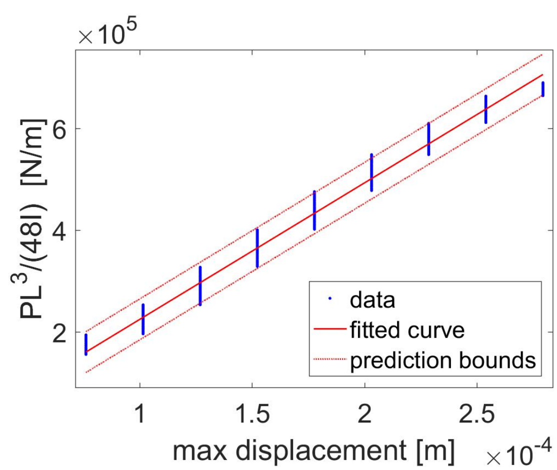

FIGURE 10. SPECIMEN A ELASTIC MODULUS

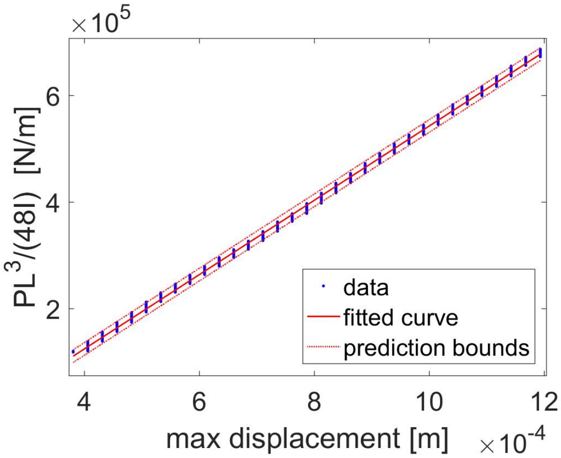

FIGURE 11. SPECIMEN C ELASTIC MODULUS

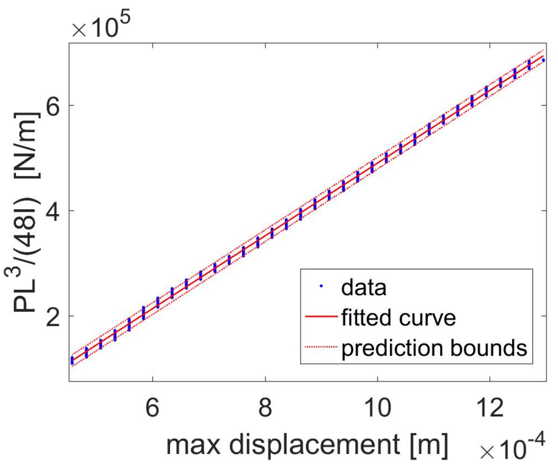

FIGURE 12. SPECIMEN D ELASTIC MODULUS

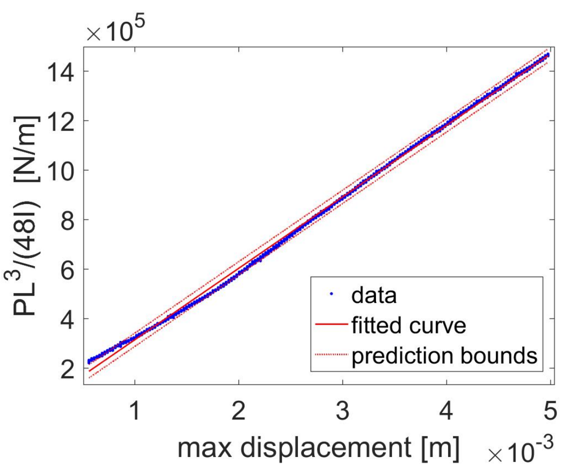

FIGURE 13. SPECIMEN E ELASTIC MODULUS

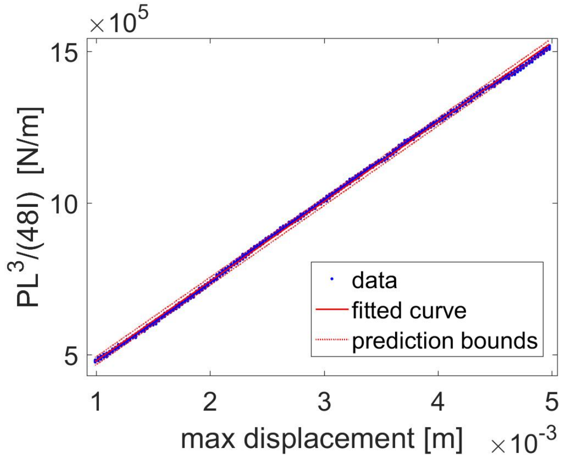

FIGURE 14. SPECIMEN F ELASTIC MODULUS

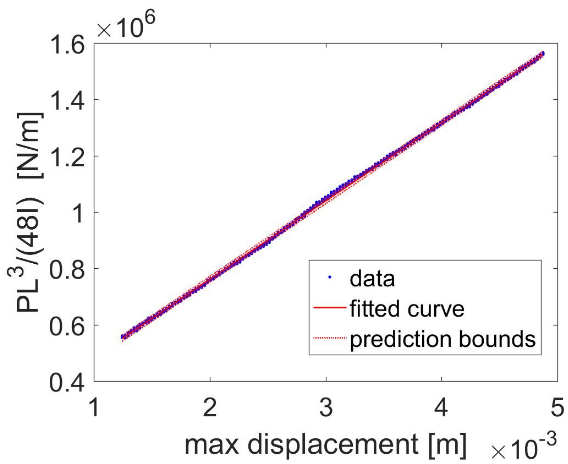

FIGURE 15. SPECIMEN G ELASTIC MODULUS

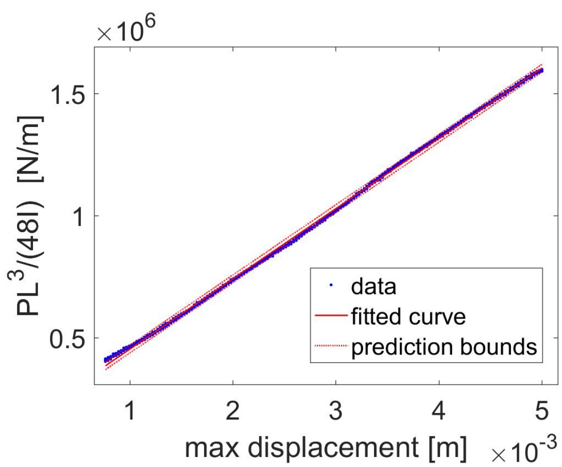

FIGURE 16. SPECIMEN H ELASTIC MODULUS