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Steering Rate Controller based on Curvature of Trajectory for Autonomous Driving Vehicles

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Abstract— **We propose a steering control method for an autonomous vehicle by controlling the steering rate instead of the steering angle and describe a method for extracting the steering rate from the reference path using the relationship with curvature and slip angle. The proposed steering control method can be used not only for planning steering control input from the desired trajectories at the planning stage but also for the instantaneous planning of a driving path during vehicle motion. From the desired trajectory plotted in a two-dimensional XY coordinate system, the steering rate is extracted from the reference path, as a function of *x*. The extracted steering rate functions as the control input to the vehicle's controller for changing the front wheel's angle. MATLAB simulations of a double-lane-change maneuver were conducted for two scenarios, i.e., when the desired trajectory is given and when the driving path is planned instantaneously. For the double-lane-change maneuver, the curvature and the steering rate input to the controller are extracted from the virtually generated instantaneous reference path. The robustness of the proposed model and method are verified by re-constructing the trajectory traveled when the calculated steering rate is input into the controller.**

I. Introduction

This study is focused on the design of a lateral controller for the dynamic trajectory tracking of unmanned autonomous vehicles. The development of a steering control system is central to achieving lateral control of autonomous vehicles. Many articles on the lateral control of vehicles have been published [1]-[6]. Most of the reported lateral controllers determine the steering angle control based on trajectory planning and by sensing vehicle state parameters such as position error, yaw rate, and speed.

Ackermann [7] was one of the pioneers of lateral control. His approach involves merging active steering with yaw rate feedback to robustly decouple yaw and lateral motion. The yaw rate is measured using a gyro sensor, and the steering wheel angle measured using a potentiometer is used as the reference input. The difference between the reference input and the measured yaw rate acts as the input to the steering-angle actuator [8].

There exist many proposed and implemented steering mechanisms that generate optimal trajectories using cost function optimization techniques. Trajectory generation is followed by lateral control error tracking considering the front wheels' orientations [9].

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During the VisLab Intercontinental Autonomous Challenge, Broggi and group's real-time motion planning system that was based on the estimation of feasible trajectories on a cost map was capable of generating and following a trajectory in real-time [10]. For executing steering control, the best among all feasible trajectories is determined using the cost map. The use of a cost map enables fusion of the information generated by several sensors installed on the vehicle, thus allowing for assessments of the generated trajectory's reliability.

However, steering-angle-based autonomous vehicle control results in inaccurate tracking on the desired path because the steering wheel direction is changed in real-time as a vehicle moves with velocity $v(t)$.

When the steering angle is fixed, the vehicle follows a constant-curvature circular trajectory, but when the steering angle is varied to execute turn or lane-change maneuvers, the curvatures change. Therefore, when the steering angle is varied, the vehicle's motion is estimated using a clothoid. The curvature of a road is the inverse of its radius. The clothoid spiral is a curve that is used to transition from a straight road to a circular road.

To overcome the abovementioned drawbacks, a steering rate control method was studied instead of the steering-angle-based control [11]. Tan reported a driver steering model that captures driver key steering mechanisms by analyzing vehicle test data on individual driver steering behavior during a standard double-lane-change maneuver. To execute this maneuver, drivers modulate the steering rate in proportion to the perceived target angle error. Tan's driver steering model indicates that for steering vehicles, drivers apply steering rate control instead of the conventional steering angle control [12].

In this paper, we propose a steering rate controller based on the curvature of a vehicle's desired trajectory. The curvature of the desired curves is extracted at the planning stage; then, the steering rate and time derivative of the steering angle are generated such that they follow the curvature of the desired path. The vehicle's lateral kinematics is modeled using a linear "bicycle model" based on the steering angle of the front wheels [13].

This paper is organized as follows: Section II describes the bicycle kinematic model and the proposed process for extracting steering rate input from the desired trajectory. Section III presents sample paths for simulations. In section IV, we present simulation results for a few trajectories. Section V shows experimental result using a miniature unmanned ground vehicle (UGV), conclusions and future works are provided in Section VI.

II. MODELING OF PROPOSED STEERING METHOD

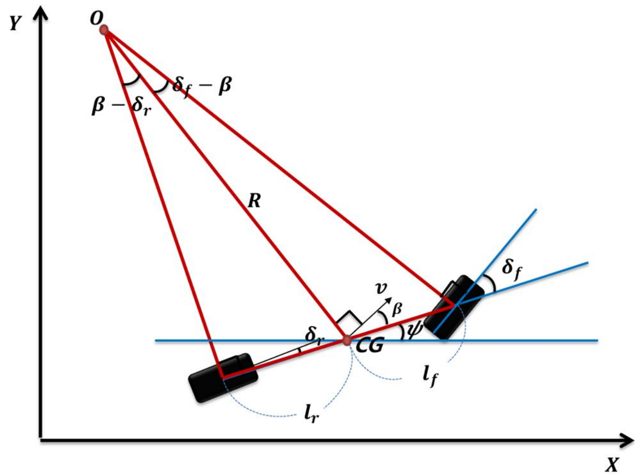

Fig. 1 Bicycle model of lateral vehicle kinematics.

A kinematic model of the lateral motion is developed using a bicycle model of the vehicle. This is achieved by lumping the right and left wheels together at the centers of the front and rear axles, as shown in Fig. 1 [13].

The vehicle parameters are as follows:

$\boldsymbol{l_f(l_r)}$ = Distance of front (rear) axle from *CG*
$\boldsymbol{CG}$ = Center of gravity of the vehicle

Variables are

$\boldsymbol{x, y}$ = Coordinates of *CG* location
$\boldsymbol{v_x(v_y)}$ = Velocity of vehicle to $\boldsymbol{x(y)}$ direction
$\boldsymbol{V}$ = Speed of motion
$\boldsymbol{\Psi}$ = Heading angle
$\boldsymbol{\dot{\Psi}}$ = Yaw rate
$\boldsymbol{\beta}$ = Slip angle at *CG* with respect to longitudinal axis
$\boldsymbol{\delta_f(\delta_r)}$ = Front (rear) steering angle
$\boldsymbol{\dot{\delta}_f}$ = Front steering rate ($= \frac{d\delta_f}{dt}$)
$\boldsymbol{R}$ = Turning radius at *CG*
$\boldsymbol{K}$ = Curvature of path

The vehicle motion is assumed as having planar geometry in the Cartesian coordinate system. Assuming front wheel only steering, we set the rear wheel angle δ_r to zero.

According to the bicycle kinematic model, the vehicle's position is expressed in terms of its velocity, slip angle, and heading angle.

The derivatives of the vehicle location are given by the following equation:

$$\frac{dx}{dt} = \boldsymbol{v_x} = V\cos(\beta + \Psi) \quad (1)$$

$$\frac{dy}{dt} = \boldsymbol{v_y} = V\sin(\beta + \Psi) \quad (2)$$

Assuming a negligible tire-slip, we express the relationship between the front steering angle δ_f and slip angle β as follows [13]:

$$\tan\beta = \frac{l_f \tan\delta_r + l_r \tan\delta_f}{l_f + l_r} \quad (3)$$

Given our assumption that the rear wheel cannot be steered, the relationship between the slip angle β and the front steering angle δ_f is simplified as follows:

$$\tan\beta = \frac{l_r}{l_f + l_r}\tan\delta_f \quad (4)$$

The vehicle's turning radius R is calculated from the kinematic model as follows [13]:

$$R = \frac{l_f + l_r}{\tan\delta_f \cos\beta} = \frac{l_r}{\sin\beta} \quad (5)$$

Given that the vehicle's motion is circular, the relationship between the heading angle and velocity is as follows:

$$V = R\frac{d\Psi}{dt} = R\,\dot{\Psi} \quad (6)$$

$$\Psi = \int \frac{V}{l_r}\sin\beta\, dt \quad (7)$$

For a curve with *y* as a function of *x* in the two-dimensional XY coordinate system, curvature $K(x)$ is calculated using the following equation [14]:

$$K(x) = \frac{y''}{(1 + y'^2)^{\frac{3}{2}}} = \frac{1}{R} \quad (8)$$

where $y' = \frac{dy}{dx}$, and $y'' = \frac{d^2y}{dx^2}$.

The ideal heading angle Ψ of a given path is given as follows:

$$\Psi = \tan^{-1}\left(\frac{dy}{dx}\right) \quad (9)$$

From Eq. (5) and Eq. (8), we obtain

$$\beta = \sin^{-1}(l_r \cdot K(x)) \quad (10)$$

We can extract the slip angle $\beta(x)$ because the curvature in Eq. (10) is a function of x.

Using Eq. (4), we get the steering angle δ_f as a function of the geometrical parameter x as follows:

$$\delta_f(x) = \tan^{-1}\left\{\frac{l_f + l_r}{l_r}\tan(\sin^{-1}(l_r \cdot K(x)))\right\} \quad (11)$$

We obtain steering rate $\dot{\delta}_f$ as a function of x.

$$\frac{d\delta_f}{dt}(x) = \frac{d\delta_f}{dx} \cdot \frac{dx}{dt} = V\cos(\beta + \Psi)\frac{d\delta_f}{dx} \quad (12)$$

Therefore, using Eq. (9)–(12), we can extract the steering rate input into the steering controller for lateral control of autonomous vehicles from the desired path on the map.

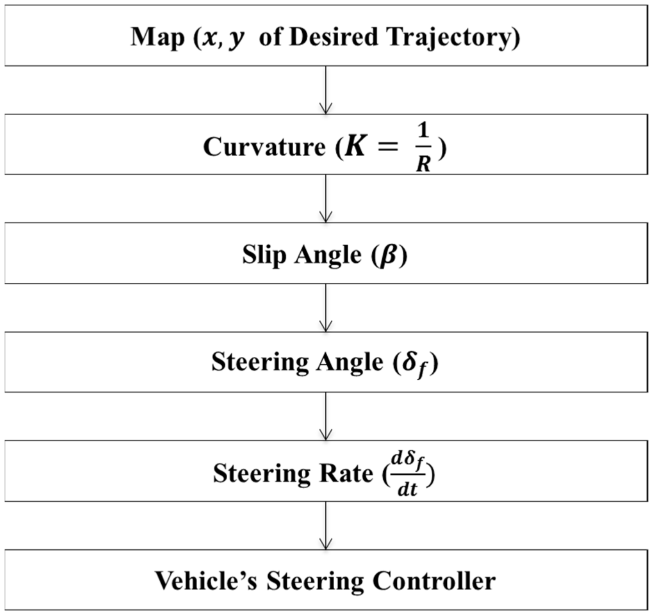

Fig. 2 Proposed process for extracting steering rate input from desired trajectory.

III. EXAMPLE PATHS AND SIMULATION

From the desired trajectory in the two-dimensional XY coordinate system, the steering rate is extracted from the reference path as a function of x. Fig. 3 shows a configuration of the proposed curvature-based steering rate controller. The proposed steering control method can be used not only for planning steering control input from the desired trajectories in the planning stage but also for instantaneous planning of driving path during motion. The extracted steering rate acts as the control input to the vehicle's controller for varying front wheel angle. To evaluate the proposed steering rate controller, we conducted MATLAB simulations for two lane-change maneuvers, i.e., when the desired trajectories are given and when the driving path is planned instantaneously. We assumed constant vehicle velocity and initial heading angle along the longitudinal direction of the desired path at the starting point.

A. Case 1 Left Turn

This is a general case of a cornering section or an intersection on the road (Fig 4). The vehicle is assumed to move at a constant velocity of 15 Km/h. We placed 43 waypoints for the reference trajectory, and the travel distance is 174 m.

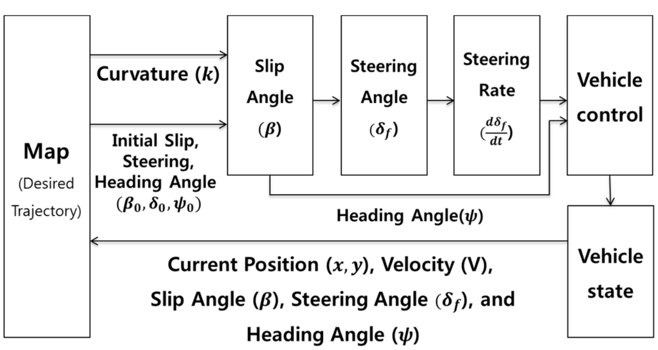

Fig. 3 Simulator structure using MATLAB.

B. Case 2 Acute Angle Course

This case is a left-turn maneuver but with a sharper curvature than that in Case 1 (Fig. 5). The vehicle is assumed to move at a constant velocity of 15 Km/h. There are 40 waypoints for reference trajectory, and the travel distance is 162 m.

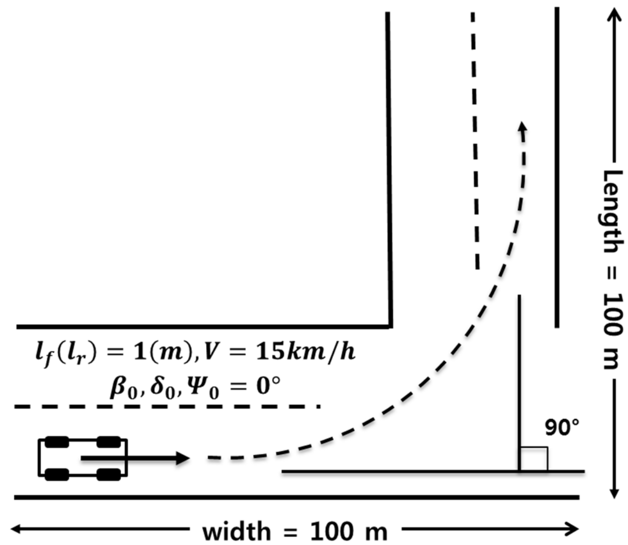

Fig. 4 Case 1: Path for left-turn maneuver.

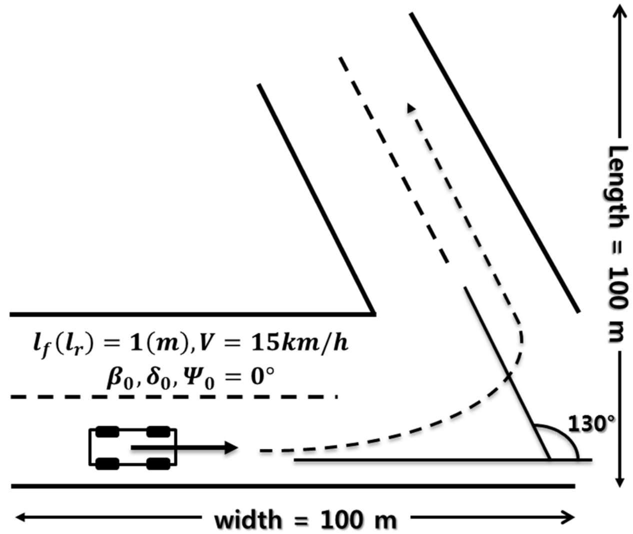

Fig. 5 Case 2: Path for left-turn manuver with a course of acute angle.

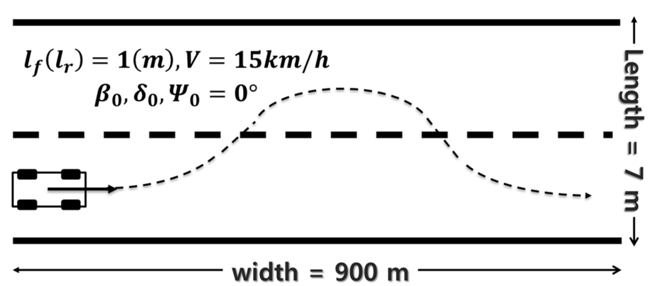

Fig. 6 Case 3: Double-lane-change maneuver.

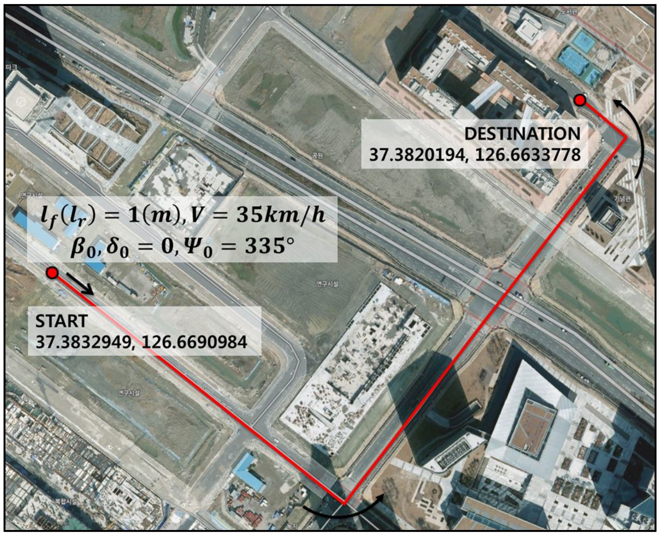

Fig. 7 Case 4: Course of Yonsei University's International Campus entrance.

C. Case 3 Double-Lane-Change Maneuver

This maneuver is executed when an autonomous vehicle needs to overtake other vehicles or avoid obstacles in the lane (Fig 6). In this case, there is no reference path on the map because the desired trajectory is not specified at the journey start. Thus, the steering rate input is generated in real-time during driving for changing lanes twice and returning to the original lane. The lane width is 3.5 m according to the highway standard in Korea [15].

D. Case 4 Trajectory of Real Map

This trajectory is a road around Yonsei University's International Campus (Fig. 7). The waypoints were collection of positioning data using real-time kinematic differential GPS (RTK-DGPS). During the test, the vehicle moved with a constant velocity of 35 Km/h. This is a bended course consisting of three straight and two curved sections. The total travel distance is 729.1 m. The collected position data were re-aligned by curve fitting using the smooth filter in MATLAB. The number of waypoints is 76, and they are located at 9.72-m intervals.

IV. Simulation Results

We used MATLAB to simulate the proposed model and method. First, we extracted the curvature given the desired trajectory for Cases 1, 2, and 4. The slip angle and steering rate input were calculated from the extracted curvature. To verify the robustness of the proposed model and method, we re-constructed the traveled trajectory when the calculated steering rate was used as the input to the controller. For Case 3, i.e., the double-lane-change maneuver, first, a virtual trajectory was generated. This virtual trajectory served as an instantaneous reference path for extracting the curvature and the steering rate input to the controller.

A. Cases 1 and 2

The simulated results for the left-turn maneuvers of Cases 1 and 2 are shown in Figs. 8, 9 and 10, respectively. The calculated steering rate is an almost linear function of time in Case 1. The curvature and corresponding steering rate input change very sharply when the vehicle traverses through an acute-angled section following a straight section. This shows that the simulated trajectories track the desired path very accurately.

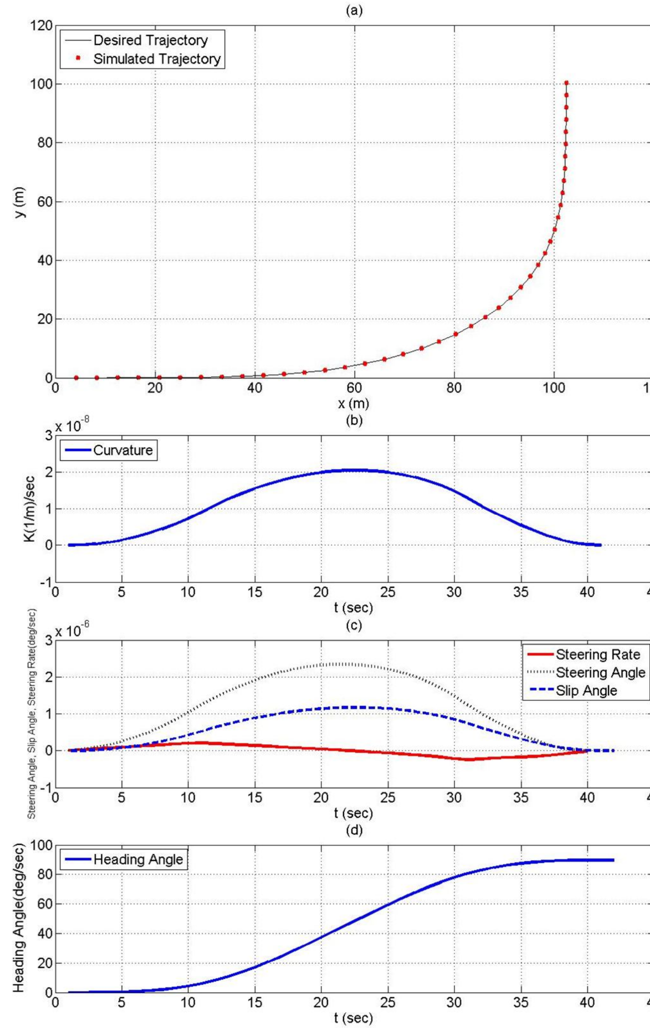

Fig. 8 Simulated results of Case 1. The given and simulated trajectories (a), extracted curvature (b). The steering rate input, slip angle and steering angle (c), and heading angle (d).

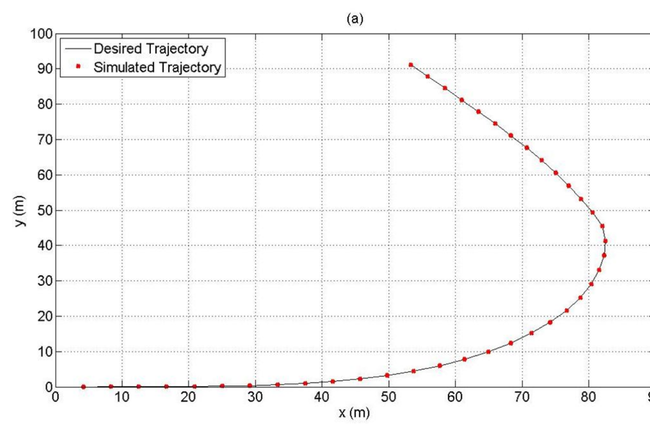

Fig. 9 Simulated results for Case 2. The given and simulated trajectories (a).

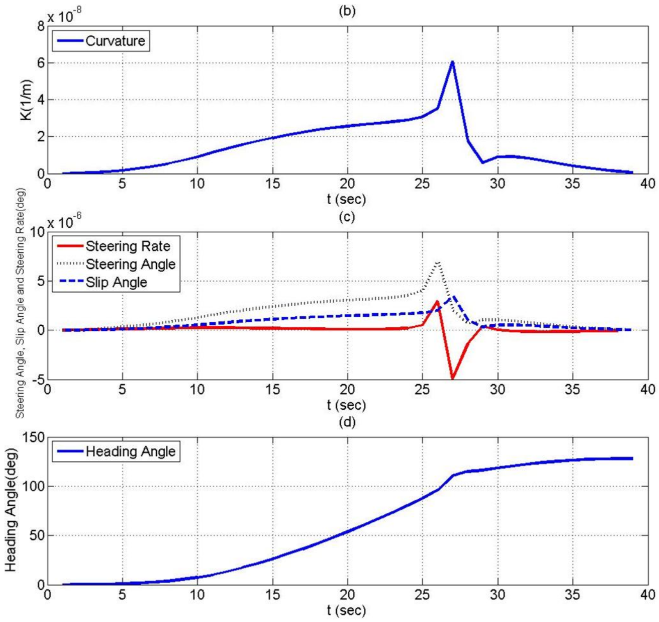

Fig. 10 Simulated results for Case 2. extracted curvature (b). The steering rate input, slip angle and steering angle (c), and heading angle (d).

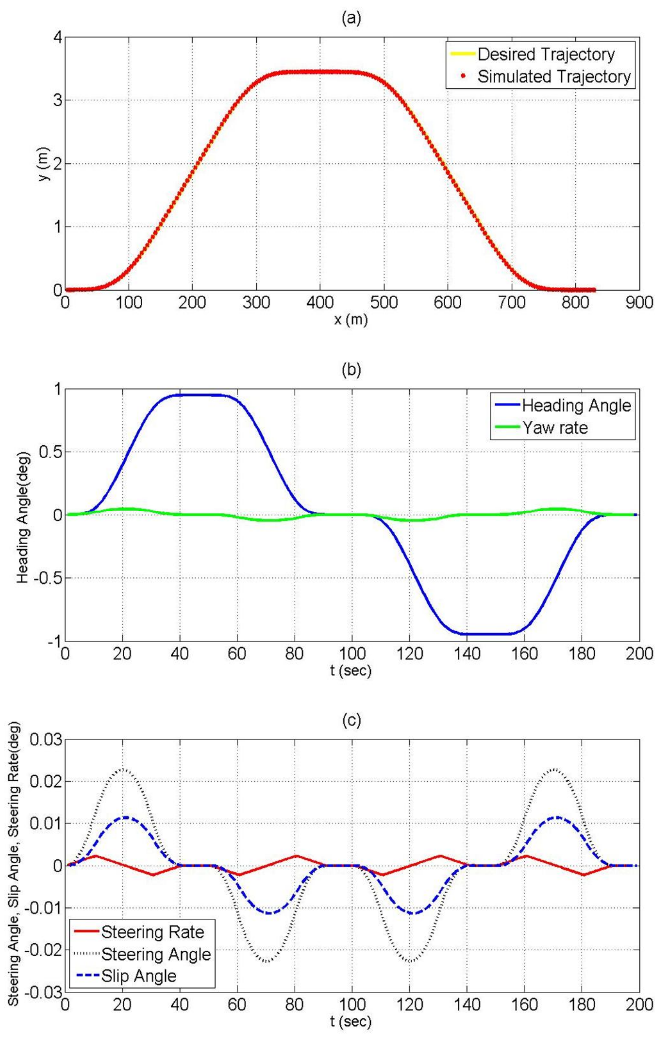

Fig. 11 Simulated results of Case 3 for double-lane-change maneuver. simulated trajectory (a), heading angle and yaw rate (b), steering rate input, slip angle and steering angle (c).

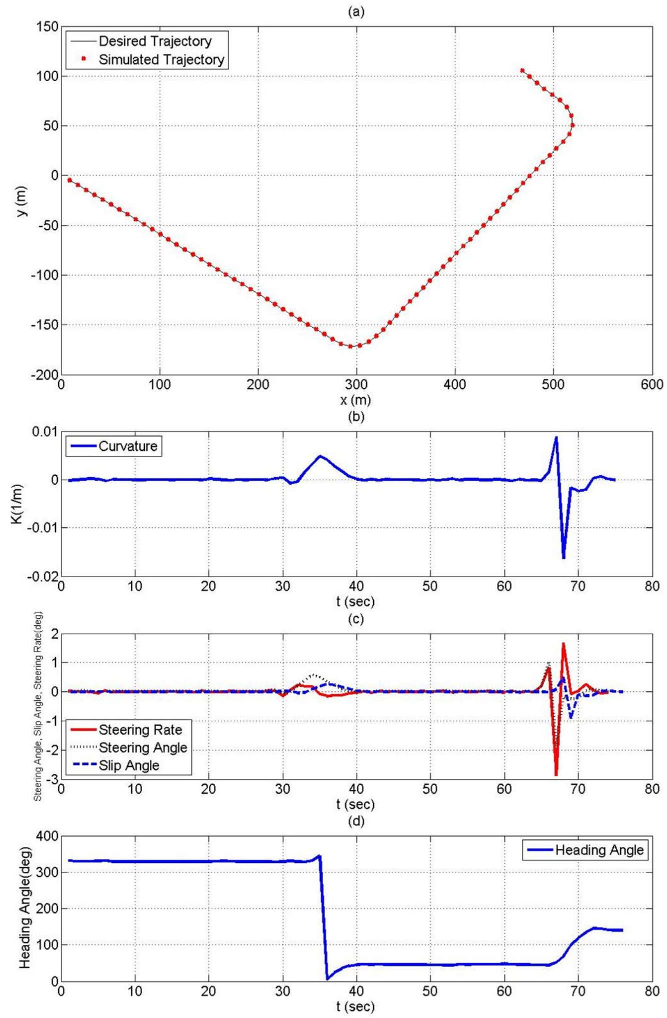

Fig. 12 Case 4: Results at main entrance of Yonsei University's International Campus (non-real scale). The given and simulated trajectories (a), extracted curvature (b). The steering rate input, slip angle and steering angle (c), and heading angle (d).

B. Case 3 Double-Lane-Change Maneuver

Simulated results for Case 3, which involves a double-lane-change maneuver, are shown in Fig. 11. From the virtual trajectory, we calculated heading angle and yaw rate (b), and steering rate input, slip angle, and heading angle as functions of time (c). The heading angle and yaw rate change very smoothly in the simulation results. The steering rate is almost a linear function of time, which is consistent with the results of the experimental driver steering model [12].

C. Case 4 Trajectory of Real Map

The simulation results of the real road path with three straight sections and two curved sections show that the simulated trajectory was well fitted to the desired trajectory (Fig. 12). The simulated curvatures were almost zero in the straight sections and changed in the curved sections.

V. Experimental Result

In order to verify the feasibility of proposed steering rate control method, we performed an experiment using a miniature sized unmanned ground vehicle that has Real-

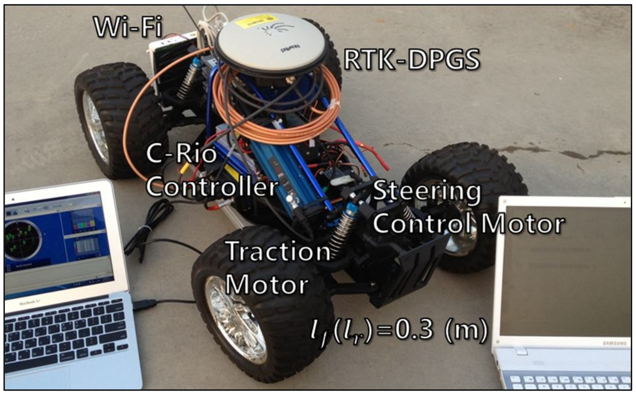

Fig. 13 The miniature sized UGV for the experiment.

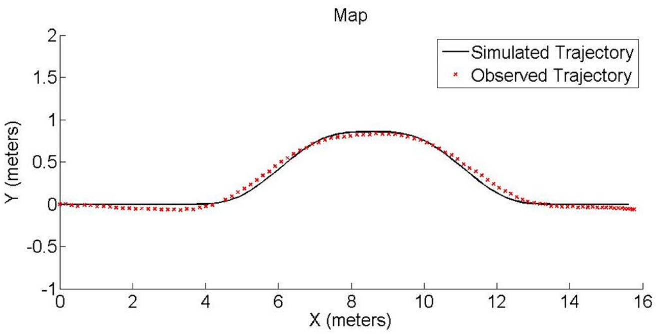

Fig. 14 Experimental and simulated trajectories of UGV for double-lane-change maneuver.

Time Kinematic Differential Global Positioning System (RTK-DGPS), Wi-Fi wireless communication interface and in-vehicle controller for autonomous driving. The 2-wheel-driven UGV that is scaled by 1/5 size of the real car manufactured by Askaracing is used. UGV is equipped with NI Compact-Rio as a in-vehicle real-time controller. The measured trajectory of vehicle using DGPS was saved as data log files during autonomous driving. Fig. 14 shows measured and simulated trajectories for double-lane-change maneuver for low speed test run at 1.57 m/sec. The width of lanes was 85 cm in the experiment. The simulation result shows good match with the experimental data.

VI. CONCLUSIONS & FUTURE WORKS

We proposed a steering rate controller for autonomous vehicles as an alternative to the control method based on the steering angle determined from the curvature of a trajectory. This paper describes a method for extracting the steering rate from the reference path using the relationship with curvature and slip angle of lateral kinematic model. A kinematic model for describing the lateral motion was developed using a bicycle model of the vehicle. The proposed steering control method can be used not only for planning the steering control input from the desired trajectories at the planning stage but also for instantaneously planning the driving path during vehicle motion. From the desired trajectory in the two-dimensional XY coordinate system, the steering rate is extracted from the reference path as a function of x. The extracted steering rate acts as the control input to the vehicle's controller to change the angle of the front wheels. MATLAB simulations were conducted for the following two cases of double-lane-change maneuver: when the desired trajectories were given and when instantaneous planning of driving path was done. For the double-lane-change maneuver, the curvature and steering rate input to the controller are extracted by generating the virtual trajectory of the instantaneous reference path. The robustness of the proposed model and method is verified by reconstructing the traveled trajectory when the calculated steering rate is input to the controller. The proposed method could be applied to trajectory planning and steering controller algorithms for the development of autonomous vehicles.

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REFERENCES

[1] M. A. Sotelo. "Lateral Control Strategy for Autonomous Steering of Ackerman-like Vehicles", Robotics and Autonomous Systems, Vol. 45, Issues 3-4, p223-233, 2003

[2] J. M. Snider, "Automatic Steering Methods for Autonomous Automobile Path Tracking", tech. report CMU-RI-TR-09-08, Robotics Institute, Carnegie Mellon University, Pittsburgh, 2009

[3] V. Milanés, J. Pérez, E. Onieva, C. González, and T. de Pedro, "Lateral power controller for unmanned vehicles", Elect. Rev., Vol. 86, no. 1, p207–211, 2010

[4] J. Pérez, V. Milanés, E. Onieva, "Cascade Architecture for Lateral Control in Autonomous Vehicles", IEEE Tr. on ITS, Vol. 12, No. 1, p73-82, 2011

[5] J. W. Lee, B. Litkouhi, "A Unified Framework of the Automated Lane Centering/Changing Control for Motion Smoothness Adaptation", International IEEE Conference on Intelligent Transportation Systems, Anchorage, Alaska, 2012

[6] M. H. Lee, K. S. Lee, H. G. Park, Y. C. Cha, D. J. Kim, B. Kim, S. Hong, H. H. Chun ,"Lateral Controller Design for an Unmanned Vehicle via Kalman Filtering", International Journal of Automotive Technology, Vol. 13, Issue 5, p801-807, 2012

[7] J. Ackermann, D. Odenthal, T. Bunte, "Advantages of active steering for vehicle dynamics control", In Proc. 32nd International Symposium on Automotive Technology and Automation, p263-270, Vienna, 1999

[8] J. Ackermann, "Robust Car Steering By Yaw Rate Control", Proceedings of the 29th Conference on Decision and Control Honolulu, Hawaii, 1990

[9] M. Montemerlo, S. Thrun, H. Dahlkamp, D. Stavens, "Winning the DARPA Grand Challenge with an AI robot", In Proceedings of the AAAI National Conference on Artificial Intelligence, 2006

[10] A. Broggi, P. Medici, E. Cardarelli, P. Cerri, A. Giacomazzo, "Development of the control system for the VisLab Intercontinental Autonomous Challenge", International IEEE Annual Conference on Intelligent Transportation Systems, Madeira Island, 2010

[11] J. Kim, I. Bae, S. Kim, "Modeling lateral vehicle trajectory based on steering rate control", KASE Annual Conference and Exhibition, 2012

[12] H. Tan, J. Huang, "Experimental development of a new target and control driver steering model based on DLC test data", IEEE Tr. on ITS, Vol. 13, No. 1, p375-384, 2012

[13] R. Rajamani, "Lateral Vehicle Dynamics, Chaper 2 of Vehicle Dynamics and control", p15-49, Springer, 2006

[14] J. Stewart, "Calculus: Concepts and Contexts, Section 10.3 of Arc Length and Curvature", p707-713, Cengage Learning, 2009

[15] Asian Highway Classfication and Design Standards, United Nations Economic and Social Commission for Asian and Pacific, http://www.unescap.org/