HW 11

Given the equation $y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}$, $0 \le x \le 1$, $y(0) = 1$,

$y(1) = 2$

use $h = 0.1$

Questions:

a. Use the shooting method to approximate the solution of the problem

b. Use the finite-difference method to approximate the solution

c. Use the variation approach to approximate the solution.

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METHOD 1: SHOOTING METHOD
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Boundary conditions: y(0) = 1, y(1) = 2
Optimal initial slope: y'(0) = 0.024000
Achieved final value: y(1) = 2.000001
Error in boundary condition: 0.00000095

Solution points:
x = 0.0, y = 1.000000, y' = 0.024000
x = 0.1, y = 1.016634, y' = 0.302355
x = 0.2, y = 1.059211, y' = 0.544550
x = 0.3, y = 1.124314, y' = 0.754099
x = 0.4, y = 1.209025, y' = 0.934403
x = 0.5, y = 1.310524, y' = 1.089549
x = 0.6, y = 1.426233, y' = 1.223938
x = 0.7, y = 1.554456, y' = 1.340453
x = 0.8, y = 1.693765, y' = 1.441832
x = 0.9, y = 1.842716, y' = 1.531383
x = 1.0, y = 2.000001, y' = 1.612523
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METHOD 2: FINITE DIFFERENCE METHOD
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Grid points: 11
Step size: h = 0.1
Boundary conditions: y(0) = 1, y(1) = 2

Solution points:
x = 0.0, y = 1.000000
x = 0.1, y = 0.992158
x = 0.2, y = 0.994922
x = 0.3, y = 1.010846
x = 0.4, y = 1.043396
x = 0.5, y = 1.097230
x = 0.6, y = 1.178600
x = 0.7, y = 1.295925
x = 0.8, y = 1.460607
x = 0.9, y = 1.688187
x = 1.0, y = 2.000000
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METHOD 3: VARIATIONAL APPROACH (GALERKIN METHOD)
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Number of basis functions: 5
Coefficients for higher-order terms: [-0.97616757  0.51691436 -0.1677539   0.01369669]
Boundary conditions satisfied: y(0) = 1.000000, y(1) = 2.000000

Solution points:
x = 0.0, y = 1.000000
x = 0.1, y = 1.016647
x = 0.2, y = 1.059298
x = 0.3, y = 1.124478
x = 0.4, y = 1.209112
x = 0.5, y = 1.310516
x = 0.6, y = 1.426372
x = 0.7, y = 1.554716
x = 0.8, y = 1.693922
x = 0.9, y = 1.842684
x = 1.0, y = 2.000000
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