

HW5

1. The initial-value problem

$y' = 1 + (y/t) + (y/t)^2$, $1 \leq t \leq 2$, $y(1) = 0$ has the exact

solution $y(t) = t \tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and

compare it with the actual values of y .

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the

solution, and compare it with the actual values of y .

comparing the results of Euler and Taylor :

t	Euler	Exact	Error	Taylor	Error
1.0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
1.1	0.10000000	0.10515982	0.00515982	0.10500000	0.00015982
1.2	0.20991736	0.22124277	0.01132542	0.22091916	0.00032362
1.3	0.33047056	0.34912113	0.01865057	0.34861239	0.00050875
1.4	0.46235355	0.48968166	0.02732812	0.48895375	0.00072791
1.5	0.60628547	0.64387533	0.03758986	0.64288278	0.00099255
1.6	0.76304149	0.81275274	0.04971125	0.81143817	0.00131457
1.7	0.93347503	0.99749413	0.06401910	0.99578673	0.00170740
1.8	1.11853673	1.19943864	0.08090191	1.19725172	0.00218692
1.9	1.31929261	1.42011584	0.10082322	1.41734353	0.00277230
2.0	1.53694328	1.66128176	0.12433848	1.65779466	0.00348709

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5 \cos t - \frac{1}{3} \sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9 \cos t + \frac{1}{3} \sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3} \cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3} \cos t.$$

Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

comparing the results (h=0.05):

t	u1(RK)	u1(Exact)	Error	u2(RK)	u2(Exact)	Error
0.00	1.33333333	1.33333333	0.00000000	0.66666667	0.66666667	0.00000000
0.05	1.72188026	1.91205863	0.19017838	-0.49959934	-0.90907659	0.40947724
0.10	1.72691505	1.79306259	0.06614754	-0.83259771	-1.03200245	0.19940475
0.15	1.61716063	1.60196676	0.01519387	-0.89037299	-0.96145871	0.07108572
0.20	1.48168729	1.42390240	0.05778489	-0.86104209	-0.87468103	0.01363894
0.25	1.34894503	1.26764562	0.08129941	-0.80750453	-0.79522077	0.01228376
0.30	1.22706330	1.13157652	0.09548678	-0.75034063	-0.72499857	0.02534206
0.35	1.11747812	1.01299856	0.10447957	-0.69588591	-0.66305963	0.03282629
0.40	1.01952546	0.90940859	0.11011687	-0.64573176	-0.60821421	0.03751755
0.45	0.93197667	0.81862953	0.11334713	-0.59993424	-0.55938925	0.04054499
0.50	0.85354051	0.73878784	0.11475267	-0.55809249	-0.51565767	0.04243482
0.55	0.78301727	0.66827466	0.11474261	-0.51970627	-0.47622475	0.04348152
0.60	0.71933702	0.60570965	0.11362737	-0.48429030	-0.44041076	0.04387955
0.65	0.66156029	0.54990941	0.11165088	-0.45140706	-0.40763534	0.04377172
0.70	0.60886766	0.49986025	0.10900741	-0.42067262	-0.37740382	0.04326880
0.75	0.56054684	0.45469474	0.10585211	-0.39175408	-0.34929551	0.04245857
0.80	0.51598005	0.41367148	0.10230857	-0.36436468	-0.32295352	0.04141116
0.85	0.47463257	0.37615771	0.09847486	-0.33825859	-0.29807605	0.04018254
0.90	0.43604262	0.34161435	0.09442827	-0.31322610	-0.27440884	0.03881727
0.95	0.39981231	0.30958300	0.09022930	-0.28908926	-0.25173868	0.03735057
1.00	0.36559983	0.27967491	0.08592492	-0.26569799	-0.22988784	0.03581015

comparing the results (h=0.1):

t	u1(RK)	u1(Exact)	Error	u2(RK)	u2(Exact)	Error
0.00	1.33333333	1.33333333	0.00000000	0.66666667	0.66666667	0.00000000
0.10	-3.05243707	1.79306259	4.84549965	8.98930534	-1.03200245	10.02130780
0.20	-23.84779486	1.42390240	25.27169726	51.19270400	-0.87468103	52.06738503
0.30	-130.16520171	1.13157652	131.29677824	269.26919317	-0.72499857	269.99419174
0.40	-680.23148509	0.90940859	681.14089368	1399.36858350	-0.60821421	1399.97679771
0.50	-3531.29958537	0.73878784	3532.03837320	7258.24183884	-0.51565767	7258.75749651
0.60	-18312.79505221	0.60570965	18313.40076186	37634.95548298	-0.44041076	37635.39589373
0.70	-94951.33190728	0.49986025	94951.83176753	195131.87173536	-0.37740382	195132.24913918
0.80	-492306.46563948	0.41367148	492306.87931096	1011721.87207795	-0.32295352	1011722.19503147
0.90	-2552513.62386741	0.34161435	2552513.96548176	5245578.82658988	-0.27440884	5245579.10099871
1.00	-13234278.78916787	0.27967491	13234279.06884278	27197287.20658695	-0.22988784	27197287.43647479