problem 1

```
x: [4.  4.2 4.5 4.7 5.1 5.5 5.9 6.3]
y: [102.6 113.2 130.1 142.1 167.5 195.1 224.9 256.8]

1a. 二次多項式 (y = ax² + bx + c)
--------------------------------------------------
係數：a = 6.691184, b = -1.883746, c = 3.086393
二次逼近：y = 6.691184x² + -1.883746x + 3.086393
誤差：0.072427

1b. 指數形式 (y = be^(ax))
--------------------------------------------------
係數：a = 0.398495, b = 21.444544
指數逼近：y = 21.444544 * e^(0.398495x)
誤差：9.745923

1c. 冪函數形式最小平方逼近 (y = bx^a)
--------------------------------------------------
係數：a = 2.019634, b = 6.238952
冪函數逼近：y = 6.238952 * x^2.019634
誤差：0.108262
```

problem 2

```
f(x) = (1/2)cos(x) + (1/4)sin(2x)
find : P₂(x) = a₀ + a₁x + a₂x²

最小平方二次多項式逼近：
P₂(x) = 0.498279 + 0.326548x + -0.232631x²

L2 誤差 = √(∫₋₁¹ [f(x) - P₂(x)]² dx) = 0.056925
```

problem 3

```
函數：f(x) = x² sin(x)
區間：[0, 1]
m = 16

3a. 計算離散最小平方三角多項式 S₁(x)
--------------------------------------------------
a₀ = 0.41373785
a₁ = -0.32947399
b₁ = -0.29836171

S₁(x) = 0.41373785 + -0.32947399cos(πx) + -0.29836171sin(πx)
離散 L2 誤差：0.18715513

3b. 計算 ∫₀¹ S₁(x) dx
------------------------------
∫₀¹ S₁(x) dx = 0.22379488

3c. 與 ∫₀¹ x² sin(x) dx 比較
----------------------------------
∫₀¹ x² sin(x) dx = 0.22324428
∫₀¹ S₁(x) dx = 0.22379488
積分誤差：0.00055061

3d. 計算連續 L2 誤差 E(S4)
----------------------------------
E(S₁) = √(∫₀¹ [f(x) - S₁(x)]² dx) = 0.03915781
```