1 Vectors

Vectors are mathematical objects which arise quite frequently in all sciences.

Definition: A **vector** is an ordered list of numbers. We normally name vectors using lowercase, boldface letters, or letters with arrows over them:

$$\mathbf{v} = \overrightarrow{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

If the vector is arranged vertically, we call it a **column vector**. If it is arranged horizontally, we call it a **row vector**, e.g.,

$$\mathbf{v} = (2 \;\; 4).$$

A vector we will use often this semester is the zero vector. For example, $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ or $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

The number of components in any particular zero vector will always be clear from context.

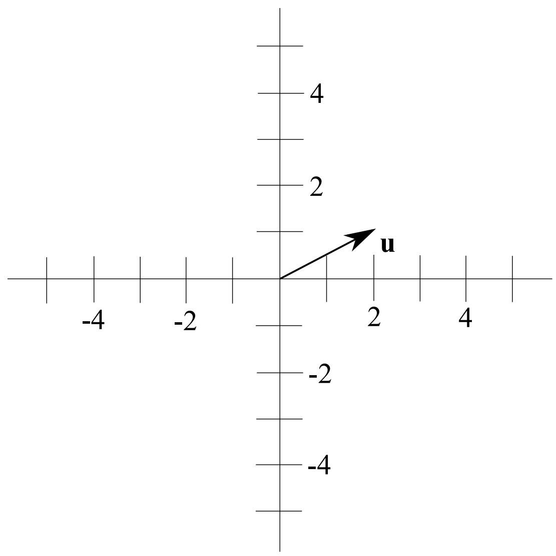
Applications: Vectors appear in any setting where a single object must be described using several independent quantities. For example:

1. In *physics* a vector can encode (x, y, z) coordinates of an object, or the direction and size of a force applied to an object.
2. In *computer science* a vector can encode colors by their red, green, and blue components.
3. In *finance* a vector can represent a portfolio of many stocks.
4. In *biology* a vector can represent different levels of protein expression in an organism.

What are other places where vectors may be useful to manage data?

2 Geometric interpretation

A vector with n components is from the space $\mathbb{R}^n$. For example, $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \in \mathbb{R}^2$. We can visualize this vector as the arrow on the plane pointing from the origin to the point $(2, 1)$.



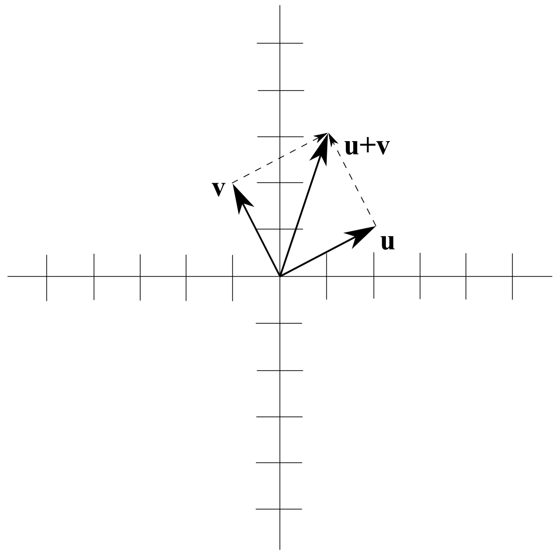
A vector from $\mathbb{R}^3$ would look like an arrow in 3D space, and vectors from $\mathbb{R}^4$ are tough to visualize...

2.1 Vector addition

Two vectors from the same $\mathbb{R}^n$ can be added component-wise. If $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ then

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 1+2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

Geometric interpretation: The vector $\mathbf{u} + \mathbf{v}$ points to the 4th corner of a parallelogram with other corners at the origin, $\mathbf{u}$, and $\mathbf{v}$. This is sometimes referred to as "head-to-tail" addition, since you can think of $\mathbf{u} + \mathbf{v}$ by shifting $\mathbf{v}$ so that its tail matches with the head of $\mathbf{u}$. Then $\mathbf{u} + \mathbf{v}$ is where the head of $\mathbf{v}$ is now pointing. See the picture below for these visualizations.



3 Scalar multiplication

A vector can also be scaled by a real number by multiplying each component. If $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $c = -2$ then

$$c\mathbf{u} = -2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}.$$

Geometric interpretation: The number c is often called a *scalar* because it scales the original vector by a factor of c (where negative values additionally flip the vector to point in the opposite direction).