Topics: Implicit Differentiation, derivatives of inverse trig functions/logarithmic functions

Implicit Differentiation.

Example 1. *How do I calculate*

$$\frac{d}{dx}(y(x))^2?$$

Example 2. *Find* $\frac{dy}{dx}$ *given the relation* $x^2 + y^2 = 25$. *Use this to find the slope of the tangent line at the point* $(3, 4)$.

Example 3. *Find* y' *if* $\tan(x-y) = \frac{y}{1+x^2}$.

Take-away:

Example 4. *Find* y'' *given* $x^3 + y^3 = 1$.

Derivatives of inverse trigonometric functions.

Example 5. *Compute*

$$\frac{d}{dx}(\sin^{-1}(x)).$$

Derivatives of inverse trigonometric functions.

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}x = \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cos^{-1}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec^{-1}x = \qquad \frac{\mathrm{d}}{\mathrm{d}x}\csc^{-1}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}x = \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cot^{-1}x =$$

Example 6. *Differentiate* $y(t) = \cos^{-1}(a\sin^{-1}t)$ *with respect to* t*.*

Logarithms

Definition: The **logarithm** is the *inverse function* to the exponential function. That is:

$$\log_b(x) = y \iff b^y = x.$$

Written another way, the exponential and logarithm functions "undo" each other:

$$\log_a(a^x) = x, \quad a^{\log_a(x)} = x.$$

The *domain* of $\log_b(x)$ is $(0, \infty)$ and the *range* is $(-\infty, \infty)$.

The **natural logarithm** is $\ln(x)$ which is the same as $\log_e(x)$.

Logarithm Rules:

1. $\log_b(a^x) = x \log_b(a)$
2. $\log_b(xy) = \log_b(x) + \log_b(y)$
3. $\log_b(x/y) = \log_b(x) - \log_b(y)$
4. $\log_b(x) = \frac{\ln(x)}{\ln(b)}$.

Example 7. *Solve*

$$1 + e^{4x+1} = 20.$$

Derivatives of logarithmic functions.

Derivative of logarithmic functions.

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x =$$

Example 8. *Differentiate* $y = \ln(x^3 + 1)$.

Example 9. *Differentiate* $y = \ln(\ln(x))$.

Boot Camp

Example 10. *Find y' where*

$$\tan^{-1}(y) = xy.$$

Example 11. *Find the equation of the tangent line to the curve at the given point*

$$x^2 + 2xy - y^2 + x = 2, \quad (1, 2).$$

Example 12. *Find y'' where*

$$x^4 + \ln y = a^4.$$

Example 13. *Differentiate $y(x) = \frac{\ln(\sin(x))}{\sec(x)}$.*