One of the goals of this section is to determine an alternative way to calculate determinants. While cofactor expansions will always work, they can be somewhat tedious or calculation intensive. The motivating idea is the following:

1. If a matrix is triangular then calculating the determinant is easy.
2. We can row reduce matrices to make them triangular.

So, the only thing to figure out is: *how do the row operations affect the determinants of matrices?*

# 1 Calculating determinants using row operations

As a starting example, consider the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Its determinant is $\det A = ad - bc$. Suppose we performed a replacement row operation, to add 4 times the first row to the second:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \sim \begin{pmatrix} a & b \\ 4a+c & 4b+d \end{pmatrix}.$$

What do you think the determinant of the new matrix will be? (Before looking ahead, try to think about this!)

**Example 1.** *Calculate*

$$\begin{vmatrix} a & b \\ 4a+c & 4b+d \end{vmatrix}.$$

**Solution.** Since this is a $2 \times 2$ matrix, we still can use the standard formula:

$$\begin{vmatrix} a & b \\ 4a+c & 4b+d \end{vmatrix} = (a)(4b+d) - (b)(4a+c) = 4ab + ad - 4ab - bc = ad - bc.$$

Surprise! The value we get is the determinant of the original matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The determinant didn't change.

It turns out that no matter the size of the matrix, row replacement never changes the determinant! However, one needs to check what happens if we interchange or scale rows of matrices to see how determinants are affected. In those cases something *does* happen.

**Theorem:** Let $A$ be a square matrix.

a. If $B$ is produced by adding a multiple of one row to another, then $\det A = \det B$.

b. If $B$ is produced by interchanging two rows, then $\det B = -\det A$.

c. If $B$ is produced by scaling one row by $k$ then $\det B = k \cdot \det A$.

**Example 2.** *I encourage you to find examples of* $2 \times 2$ *matrices confirming the 2nd and 3rd parts of this Theorem!*

**Example 3.** *Suppose* $A$ *is a* $3 \times 3$ *matrix with* $\det A = 5$. *When tasked with computing* $\det(2A)$ *many students will say*

$$\det(2A) = 10.$$

*What do you think about this?*