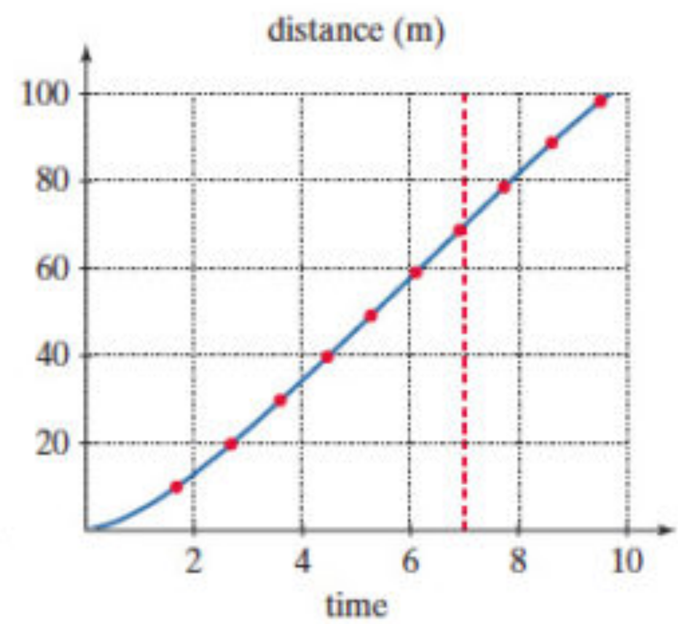
Topics: Derivative and rates of change; tangent lines; derivative as a function; differentiability and continuity; higher derivatives

1 Rates of change and instantaneous velocity

Example 1 (Motivation: calculating velocity).

Usain Bolt is said to be the fastest human being in the world. In the Beijing 2008 Olympics he reached 27.8 mph as a top speed around the 70m mark. For simplicity, convert $27.8mph = 12.43m/s$.



Calculus for the Life Sciences, Wiley

time (s)	*distance (m)*
1.7	*10*
2.8	*20*
3.8	*30*
4.2	*40*
5.5	*50*
6.2	*60*
7	*70*
7.8	*80*
8.4	*90*
9.6	*100*

Question: *How do we calculate the* **average velocity***?*

Question: *Why doesn't the* **average velocity** *give the correct answer?*

Question: *How can we better determine how fast Usain Bolt was going?*

Example 2 (Relation to tangent problem)
Suppose that I drop a ball from a high tower. Denote by $s(t)$ the distance (in meters) the ball falls after t seconds. Then

$$s(t) = 4.9t^2. \tag{1}$$

Question: *What is the average velocity over the interval $[5, 5.1]$?*

Question: *Sketch $s(t)$ and draw a line between the points $(5, s(5))$ and $(5.1, s(5.1))$. How does this line relate to the previous question?*

First take-away:

Question: *What happens if we take smaller intervals?*

Second take-away:

2 Derivative

Computing instantaneous rates of change arises so often that we give it a special name: derivative.

Definition: The *derivative of a function f at the point a* denoted $f'(a)$ is

$$f'(a) =$$

provided the limit exists.

Example 3 *Find the derivative of the function $f(x) = 2x^2 - x + 1$ at the number 1.*

Example 4 (Derivative as instantaneous rate of change).

The position of a particle in meters given by $f(t) = \frac{1}{1+t}$. What is its velocity and speed after 2 seconds?

The equation of the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ is given by

Example 5 (Derivative as the slope of the tangent line).

Find the equation of the tangent line to the curve

$$y = \sqrt{1 - 2x}$$

at the point $x = 0$*.*

Example 6 *A rock thrown upward on the planet Mars has height (in meters) after* t *seconds given by* $h(t) = 2t - t^2$*. What is the velocity of the rock the instant it hits the ground?*

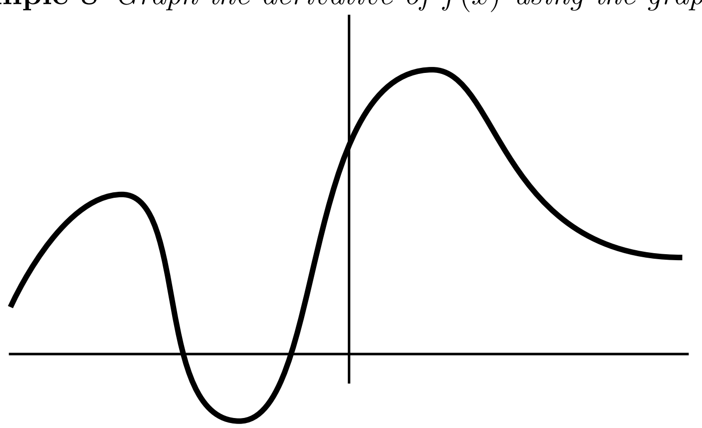
Example 7 *Draw a function with*

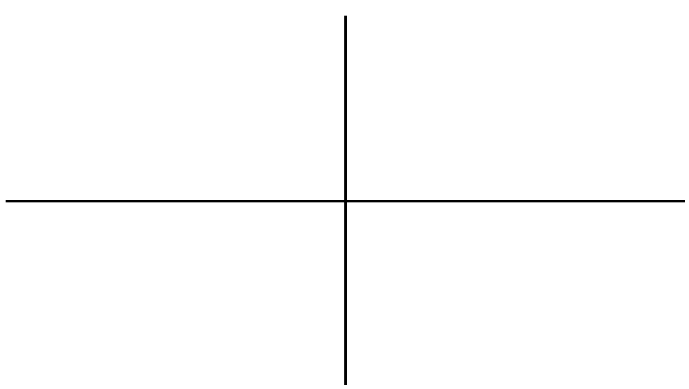
$$g(0) = g(2) = g(4) = 0, g'(1) = g'(3) = 0, g'(0) = g'(4) = 1, g'(2) = -1.$$

3 The derivative as a function

> **Definition:** A function f is differentiable at a if $f'(a)$ exists. It is differentiable on an open interval if it is differentiable at every number on the interval.

Example 8 *Graph the derivative of $f(x)$ using the graph of $f(x)$ alone:*





4 Non-differentiability

We next investigate what causes a function to fail to be differentiable.

Example 9 *What are some* **non-differentiable** *functions?*

Example 10 (Relationship between differentiability and continuity)

Can you construct a function that, at $x = 0$, is:

			Yes/No
Differentiable	&	*continuous ?*	
Not-differentiable	&	*continuous ?*	
Differentiable	&	*not continuous ?*	
Not-differentiable	&	*not continuous ?*	

Theorem.

5 Higher derivatives

Example 11 *The distance (m) a dropped ball travels in t seconds is $s(t) = 4.9t^2$. What is the acceleration of the ball at $t = 2$ seconds?*