Learning Objectives:

1. Calculate eigenvalues, eigenvectors, and eigenspaces of a square matrix
2. Describe the geometric interpretation of eigenvectors under transformation

1 Eigenvectors

Definition: An **eigenvector** of an $n \times n$ matrix A is a nonzero vector $\mathbf{x}$ such that ______________________ for some scalar λ. The scalar λ is called the **eigenvalue** corresponding to $\mathbf{x}$.

Example 1. *Why is it important that we require* $\mathbf{x} \neq 0$*?*

Example 2. *Let* $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$ *and* $\mathbf{u} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. *Are* $\mathbf{u}$ *and* $\mathbf{v}$ *eigenvectors of* A*?*

Example 3. *What is the geometric interpretation of eigenvectors?*

Example 4. *Show that* 7 *is an eigenvalue of* $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$.

Remark: In the next section we will see how to find the right λ without being told what it is.

2 Eigenspace

Example 5. *What connections are there between eigenvectors and subspaces?*

Definition: The **eigenspace** of A corresponding to the eigenvalue λ is the subspace corresponding to the set of all solutions to ____________________

Example 6. *Let* $A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$. *An eigenvalue is* 2. *Find the dimension of the corresponding eigenspace.*

Example 7. *If* $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$*, then what are the eigenvalues of* A*?* **Hint:** *When does* $A - \lambda I$ *become singular?*

Theorem: The eigenvalues of a triangular matrix are

Theorem: If $\mathbf{v}_1, \ldots, \mathbf{v}_r$ are eigenvalues corresponding to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ is linearly independent.

Example 8. T/F: *An* $n \times n$ *matrix can have at most* n *eigenvalues.*