Topics: Limit laws, evaluating limits, Squeeze Theorem

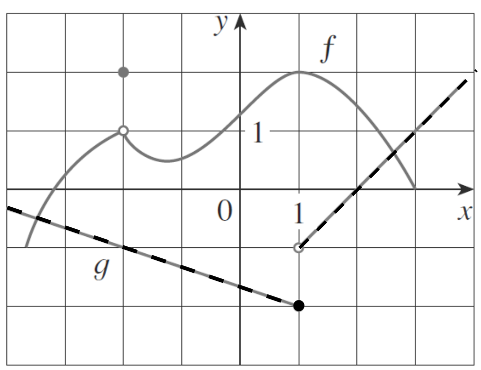
Limit laws. Let c be a constant and suppose that the limits

$$\lim_{x\to a} f(x) \text{ and } \lim_{x\to a} g(x)$$

exist. Then

1. $\lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} f(x) - g(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$ where c is a constant number.
4. $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$
5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$

Example 1 *Consider the functions f and g with the following graphs:*



Compute

$$\lim_{x\to -2} f(x) - 2g(x)$$

and

$$\lim_{x\to 2} f(x)/g(x).$$

Direct substitution

Theorem. (Direct substitution rules) (i). Let $f(x)$ be a polynomial or rational function. Then

provided that a is in the domain of $f(x)$.

(ii). Assume that $\lim_{x \to a} f(x)$ exists. Then

where, if n is even, we assume that

Squeeze Theorem

Theorem. If $f(x) \leqslant g(x)$ for all x near $x = a$ (except possibly at $x = a$) then

provided the limits exist.

Squeeze Theorem. If $f(x) \leqslant g(x) \leqslant h(x)$ for all x near $x = a$ (except possibly at $x = a$) and

then

Example 2 *Compute*

$$\lim_{x \to 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x}.$$

Solving limits: Jigsawed

Instructions: We will split into groups, called "expert" groups. Each expert group will work on one exercise. Beyond solving the problem, groups should

1. Write down the most important steps to (a) identify and (b) solve the problem
2. Create a new problem which uses the same principle.

We will then form "jigsawed" groups with one representative from each expert group. Each expert will have 4 minutes to explain their problem to the group and answer questions.

Example 3 (Factor). *Compute*

$$\lim_{t \to -3^+} \frac{t^2 - 9}{(t+3)^2}.$$

Example 4 (Expand). *Compute*

$$\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}.$$

Example 5 (Conjugate). *Compute*

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}.$$

Example 6 (Check one-sided limits) *Compute*

$$\lim_{x \to 4} f(x),$$

where

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4. \end{cases}$$

Example 7 (Check one-sided limits) *Compute*

$$\lim_{x \to -6} \frac{2x + 12}{|x + 6|}.$$

Example 8 (Common denominators) *Compute*

$$\lim_{h \to 0} \frac{\dfrac{1}{(2+h)^2} - \dfrac{1}{4}}{h}.$$

Example 9 (Extra) *Evaluate the limit, if it exists:*

$$\lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$$