Topics: Related rates, approximations, linearization

Related Rates.

Example 1. *Compute*

$$\frac{d}{dx}x^2 =$$
$$\frac{d}{dx}y^2 =$$
$$\frac{d}{dt}x^2 =$$
$$\frac{d}{dt}V^2 =$$

Example 2. *If* $xyz = 9$ *and* $dx/dt = 5$ *and* $dy/dt = 4$ *then find* dz/dt *when* $(x, y, z) = (3, 3, 1)$.

Example 3. *I throw a pebble into a pond and watch the circular ripple which emanates from the place of impact. If I know that the radius is changing at a rate of* $1m/s$ *then how fast is the area of the circular region changing when the radius is* $5m$*?*

Example 4. *A boat is being pulled at a constant rate of* $1m/s$ *into a dock by a rope attached* 1 *meter above the plane of the bow. At what speed is the boat approaching the dock when it is* 2 *meters away?*

Linear approximations.

Example 5. *Approximate* $\sqrt{3.98}$.

Why do this when we can just use calculators?

1. Computer science: doing many calculations with linearization is faster than many calculations with original function and perhaps is practically the same.

2. Physics: Many physics equations are simplified by linearization. For example, a pendulum's motion is described by the equation

$$x'' = -\sin(x),$$

where x is the angle of the pendulum. If angles are small then $\sin x \approx x$ so the equation is often approximated by $x'' = -x$ which is much easier to study!

3. Biology: models of population dynamics in ecology use differential equations. Linearization allows us to make predictions about whether predators and prey will survive in the long term.

Example 6. *Justify the approximation that if x is small then*

$$\sin(x) \approx x,$$

by linearizing around $a = 0$.

Example 7. *Use the linearization of*

$$e^x \cos(x) \approx 1 + x$$

when x is near 0 to approximate $e^{.1}\cos(.1)$.