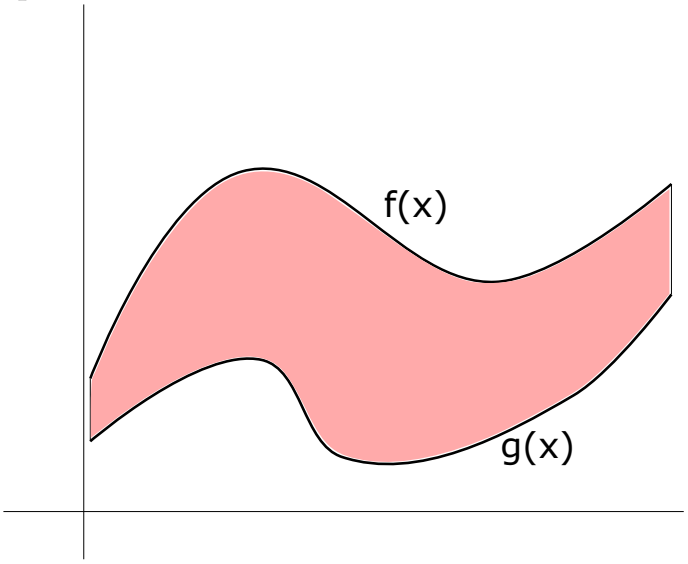
Area Between curves

Goal: Determine how to calculate the area between two curves: $f(x)$ and $g(x)$. *For this section we will only calculate area, not signed area.*

Example 1. *Determine a method to calculate the area between $f(x)$ and $g(x)$:*



The area between the curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ is

Example 2. *Why did we write* $|f(x) - g(x)|$ *rather than* $f(x) - g(x)$*? How do we evaluate this integral in practice?*

Example 3. *Find the area of the region bounded above by* $y = e^x$*, bounded below by* $y = x$*, and bounded on the sides by* $x = 0$ *and* $x = 1$*.*

Example 4. *Find the area of the region enclosed by the parabolas* $y = x^2$ *and* $y = 2x - x^2$.

Example 5. *Find the area of the region between the curves* $y = \sin x$, $y = \cos x$, $x = 0$ *and* $x = \pi/2$.

Area between curves in y-direction

Suppose that $x = f(y)$ and $x = g(y)$ are two curves which are functions of x. Then, $f(y) \geq g(y)$ means that $f(y)$ is _____ $g(y)$. Then, the area between the curves $f(y)$, $g(y)$ and $y = c$ and $y = d$ is

Example 6. *Find the area enclosed by the line* $y = x - 1$ *and the parabola* $y^2 = 2x + 6$.

Example 7. Application: *The* Gini index *is a measure of wealth inequality (usually within a nation).*

Take x to be the poorest $100x\%$ of the population in a country (so $x \in [0, 1]$).

Let $L(x)$ be the percentage of the total country's income earned by the the poorest $100x\%$ people (again scaled so that $L(x) \in [0, 1]$).

Example 8. *What does $L(x) = x$ mean?*

Example 9. *The* Gini index *is the area between the curves $y = x$ and $y = L(x)$, overall divided by the area under $y = x$. What does the Gini index tell us?*

The Gini index in the US has grown from .43 in 1990 to .49 in 2018. According to the CIA, the US falls around rank 39 in the world, with Lesotho (South Africa) rank 1.