Learning Objectives:

1. Define dimension of a vector space
2. Describe the relationship between dimensions of a vector space and its subspaces, including geometric intuition from $\mathbb{R}^n$
3. Calculate the dimension of subspaces

1 Dimension

Theorem: If a vector space V has a basis of n vectors, then every basis of V has n vectors.

Remark: This theorem tells us that any set which has more than n vectors must be linearly dependent and any set which has fewer than n vectors cannot span the vector space.

Definition: If V is spanned by a finite set, then V is **finite-dimensional** and the dimension of V, written $\dim V$ is the number of vectors in any basis for V. The dimension of $\{\mathbf{0}\}$ is 0. Any vector space not spanned by a finite set is infinite-dimensional.

Example 1. *Find the dimension of the subspace*

$$\left\{ \begin{pmatrix} s-2t \\ s+t \\ 3t \end{pmatrix} : s, t \in \mathbb{R} \right\}.$$

Example 2. *In general, the subspaces of* $\mathbb{R}^3$ *are classified by their dimension:*

- 1 *dimensional subspaces:*
- *2-dimensional subspaces:*
- *3-dimensional subspaces:*

> **Theorem:** If H is a subspace of a finite dimensional space V then ______________. Moreover any basis of H can be extended to a basis of V.

> **Theorem:** If $\dim V = p$ then any set of p linearly independent vectors is a basis of V, and any set of p vectors that spans V is a basis of V.

Example 3. *Given a matrix* A*, what are the dimensions of* $\operatorname{Nul} A$ *and* $\operatorname{Col} A$*?*

Example 4. T/F: *If* $\dim V = p$ *and* S *is a linearly dependent set in* V*, then* S *has more than* p *vectors.*