

Topics: Basic differentiation rules, derivatives of polynomials and exponential functions, product/quotient rules

1 Power rule

Example 1. *Calculate*

$$\frac{d}{dx}x$$

and

$$\frac{d}{dx}x^2$$

Power rule. For any real number r :

Example 2. *Compute the following derivatives:*

1. $\frac{d}{dx}x^{2.4} =$

2. $\frac{d}{dx}\sqrt{x} =$

3. $\frac{d}{dx}\frac{1}{x^2} =$

2 Derivative rules

Theorem. If f and g are differentiable at x and c is a real number then:

$$\begin{aligned}\frac{d}{dx}cf(x) &= c\frac{d}{dx}f(x) \\ \frac{d}{dx}(f(x) + g(x)) &= \frac{d}{dx}f(x) + \frac{d}{dx}g(x), \\ \frac{d}{dx}(f(x) - g(x)) &= \frac{d}{dx}f(x) - \frac{d}{dx}g(x).\end{aligned}$$

Example 3. Compute the derivative of $f(a) = (1 + 3a^2)^2$.

Example 4. Compute the derivative of $f(x) = \frac{\sqrt{x}+2x^2-x^2\sqrt{x}}{x}$.

3 Derivative of the exponential function.

Example 5. Let $f(x) = e^x$. Then

x	$f(x)$	$f'(x)$
0		
$.5$		
1		
$-.5$		

$$\frac{d}{dx}e^x =$$

Example 6. For what value of x does $f(x) = e^x - 2x$ have a horizontal tangent?

4 Product rule.

Theorem. (Product Rule) If f and g are differentiable at x then

$$\frac{d}{dx}(f(x)g(x)) =$$

or in prime notation,

$$(fg)' =$$

Example 7. *Compute the derivative of x^2e^x .*

Example 8. *Compute the derivative of $(1 + x^2)xe^x$.*

5 Quotient Rule.

Theorem. (Quotient Rule) If f and g are differentiable at x and $g(x) \neq 0$ then

$$\frac{d}{dx} \frac{f(x)}{g(x)} =$$

or, in prime notation,

$$\left(\frac{f(x)}{g(x)} \right)' =$$

Example 9. *Find the slope of the tangent line to $y = e^x/(1 + x^2)$ at $x = 1$.*

6 Gallery Walk

Example 10. Differentiate $f(r) = \frac{r^2 e^r}{r + k e^r}$, where k is constant.

Example 11. Differentiate $f(x) = \frac{x}{1 + \frac{3}{x}}$.

Example 12. Differentiate $f(x) = \frac{qx^2 - e^x}{xe^x}$, where q is constant.

Example 13. Differentiate $f(t) = \frac{3 - t^{1/3}}{1 + te^t}$

Example 14. Differentiate $f(s) = \frac{s^2 e^s}{s + e^s}$.

Example 15. Differentiate $f(x) = \frac{x e^x}{2x^2 + 1}$.