Related rates and linearization

Chapter 3.9, 3.10

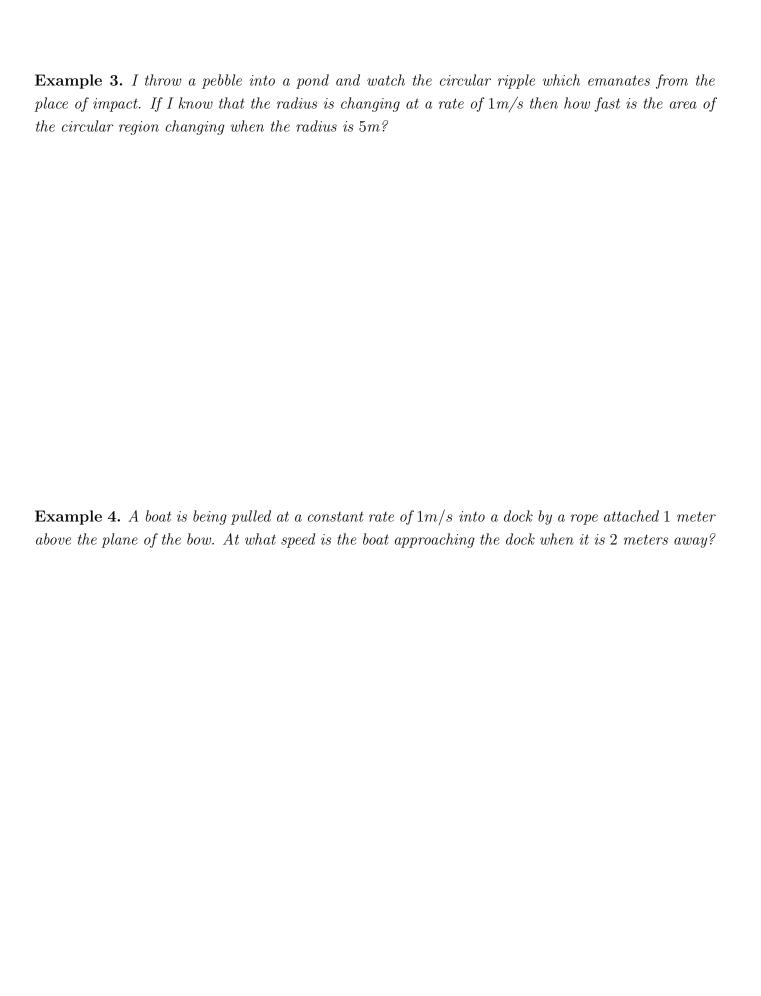
Topics: Related rates, approximations, linearization

Related Rates.

Example 1. Compute

$$\frac{d}{dx}x^2 = \frac{d}{dx}y^2 = \frac{d}{dt}x^2 = \frac{d}{dt}V^2 =$$

Example 2. If xyz = 9 and dx/dt = 5 and dy/dt = 4 then find dz/dt when (x, y, z) = (3, 3, 1).



Linear approximations.

Example 5. Approximate $\sqrt{3.98}$.

Why do this when we can just use calculators?

- 1. Computer science: doing many calculations with linearization is faster than many calculations with original function and perhaps is practically the same.
- 2. Physics: Many physics equations are simplified by linearization. For example, a pendulum's motion is described by the equation

$$x'' = -\sin(x),$$

where x is the angle of the pendulum. If angles are small then $\sin x \approx x$ so the equation is often approximated by x'' = -x which is much easier to study!

3. Biology: models of population dynamics in ecology use differential equations. Linearization allows us to make predictions about whether predators and prey will survive in the long term.

Example 6. Justify the approximation that if x is small then

$$\sin(x) \approx x$$

by linearizing around a = 0.

Example 7. Use the linearization of

$$e^x \cos(x) \approx 1 + x$$

when x is near 0 to approximate $e^{.1}\cos(.1)$.