MAT127 **Supplementary Notes**

Chapter 4.9+5.4 Antiderivatives and Indefinite Integrals

Topics: Antiderivatives, Indefinite integrals

So far: We have taken **derivatives!** Given a position, we find the *velocity.* Given $f(x)$ compute $f'(x)$, etc. However, in a lot of situations we may need to go in the **other direction...**

Motivation. For many experimental setups, it is often easiest to measure a rate of change of a quantity, rather than the quantity itself. For example:

1. Biologists measure the population growth of a bacterial colony more easily than measuring the number of individual bacteria.
2. Physicists can calculate forces and acceleration of a falling object more easily than writing an equation for its position.
3. Chemists calculate reaction rates of chemical reactions rather than write formulas for exact number of molecules over time.
4. My bank tells me my interest rate, but does not tell me how much money I will have over time.

Definition: A function F is called an **antiderivative** of f on an interval I if

Example 1. *Let $f(x) = 2x^2$. What is an antiderivative of f?*

Definition: Suppose that $F(x)$ is an antiderivative of $f(x)$. The **indefinite integral** is

Take-away: The indefinite integral takes a function and outputs the most general antiderivative. In some way, taking an indefinite integral is something like the opposite of taking the derivative!

Example 2. *Determine*

$$\int \sin(x)dx =$$

$$\int e^x dx =$$

$$\int x^n dx =$$

(assuming $n \neq -1$). Check your answers by differentiating.

Example 3. *What is*

$$\int \frac{1}{x} dx?$$

Table of antiderivatives/indefinite integrals:

$$\int k dx =$$

$$\int x^n dx =$$

$$\int \frac{1}{x} dx =$$

$$\int e^x dx =$$

$$\int \cos(x) dx =$$

$$\int \sin(x) dx =$$

$$\int \sec^2(x) dx =$$

$$\int \sec(x)\tan(x) dx =$$

$$\int \frac{1}{\sqrt{1-x^2}} dx =$$

$$\int \frac{1}{1+x^2} dx =$$

$$\int a^x dx =$$

Example 4. *Suppose*

$$F'(x) = f(x), \text{ and } G'(x) = g(x).$$

Determine which of the following are true:

$$\int kf(x)dx = kF(x) + C$$

$$\int f(x) + g(x)dx = F(x) + G(x) + C$$

$$\int f(x)g(x)dx = F(x)G(x) + C$$

$$\int \frac{f(x)}{g(x)}dx = \frac{F(x)}{G(x)} + C$$

Example 5. *Find the most general antiderivative of*

$$f(x) = (x+5)(4x-1).$$

Take-away:

Example 6. *Not all of the following integrals can be computed (with what we know so far)! Compute the ones that are, and briefly explain why the others are not possible.*

1. $\displaystyle\int \frac{1}{\sqrt{t}} dt =$

2. $\displaystyle\int \frac{e^x}{e^x + 1} dx =$

3. $\displaystyle\int \frac{x^2 - 1}{x} dx =$

4. $\displaystyle\int \frac{4}{\sqrt{1 - x^2}} dx =$

5. $\displaystyle\int e^{x^2} dx =$

6. $\displaystyle\int \sin(\theta) + \cos(\theta) + 4 d\theta =$

7. $\displaystyle\int (1 + x^2)^{-1} + \sec^2(x) dx =$

8. $\displaystyle\int \tan(y) \sec(y) dy =$

9. $\displaystyle\int \frac{\cos(x)}{\sin^2(x)} dx =$

Example 7. *Find $f(x)$ when*

$$f''(x) = 32x^3 - 15x^2 + 8x.$$

Example 8. *Find $f(t)$ where $f''(t) = \sqrt{t} - 2\cos t$.*

Take-away:

Differential equations

A differential equation is an equation involving derivatives. The goal of the equation is to solve for the unknown function. This is the language of STEM. Many famous equations in physics, chemistry, and biology are differential equations: Newton's Second Law, Schrodinger's Equation, epidemiological models in biology, Maxwell's Equations, Navier-Stokes...

Example 9. *Find* $f(x)$ *if* $f'(x) = e^x + 20(1 + x^2)^{-1}$ *and* $f(0) = -2$.

Example 10. *Find* f *if* $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ *and* $f(1) = 1$. *What is* $f(-1)$*?*