Topics: Limits at infinity, horizontal asymptotes

Definition: Let f be defined on (a, ∞). Then

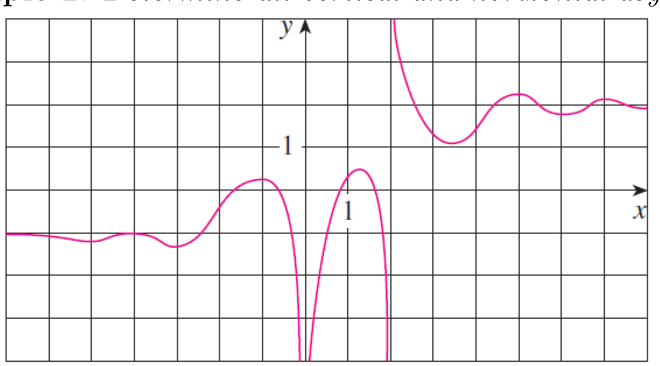
means that $f(x)$ becomes arbitrarily close to L for sufficiently large x.

Definition: Let f be defined on $(-\infty, a)$. Then

means that $f(x)$ becomes arbitrarily close to L for sufficiently large negative x.

Definition: The line $y = L$ is a *horizontal asymptote* of the curve $y = f(x)$ if

Example 1. *Determine all vertical and horizontal asymptotes of the following function*



Computing infinite limits

Some infinite limits can be reasoned through by thinking about the behavior of the function.

Example 2. *Compute*

$$\lim_{x \to 2^-} \tan^{-1}\left(\frac{1}{x-2}\right).$$

Example 3. *Compute*

$$\lim_{x \to \infty} x^2 - x^3.$$

Theorem. (i). For all $r > 0$ we have

$$\lim_{x \to \infty} \frac{1}{x^r} =$$

and if $\frac{1}{x^r}$ is defined for negative x then similarly

$$\lim_{x \to -\infty} \frac{1}{x^r} =$$

(ii).

$$\lim_{x \to -\infty} e^x =$$

and

$$\lim_{x \to \infty} 1/e^x =$$

Example 4. *Evaluate*

$$\lim_{x\to\infty} \frac{2x^2 + x - 100}{4x^2 - 300x + 4}.$$

Rule of thumb:

Example 5. (Rational functions) *Compute*

$$\lim_{x\to\infty} \frac{x^2 + x}{3 - x}$$

and

$$\lim_{x\to-\infty} \frac{x^2 - 3x + 3}{x^3 + \pi x}.$$

Discuss a general pattern.

Example 6. (Conjugate). *Compute* $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$.

Example 7. (Exponentials).

Determine the horizontal asymptotes of

$$f(x) = \frac{8e^x}{e^x - 3}.$$

Example 8. (Squareroots). *(Hint: $1/x = 1/\sqrt{x^2}$ since $x > 0$)*

Compute

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5}.$$