Topics: Algebra review, functions, domain/range, difference quotients, lines

Algebra review

Example 1. (Exponent rules)

	Your Guess	***Correct Answer***
$x^{-n} =$		
$x^m x^n =$		
$\dfrac{x^m}{x^n} =$		
$x^0 =$		
$(x^m)^n =$		
$(xy)^n =$		

Example 2. *Simplify*

$$\left(\frac{3xy}{x^2y^{-1/2}}\right)\left(\frac{2x}{y^3}\right)^{-2}.$$

Factoring

Example 3. *Factor* $(x^3)^2 - 5x^3x^2 + 6x^4$

Example 4. *Simplify*

$$\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}.$$

Take-aways: Common denominators allows us to combine fractions; compound fractions can be simplified by reciprocals; factoring by difference of squares

Lines

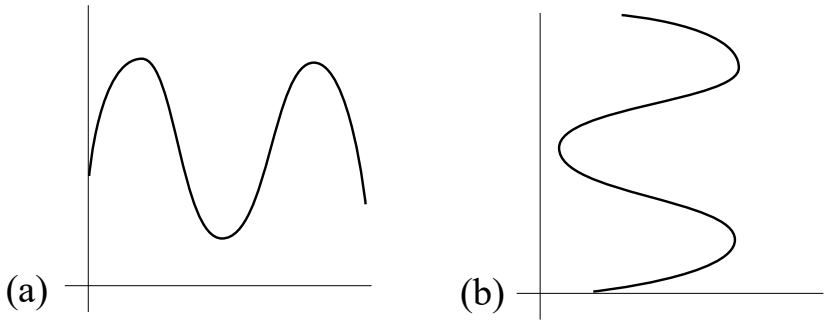
Example 5. *Find the equation of the line passing through*

$$(1, 2) \text{ to } (-1, 4).$$

Take-aways: Equation of a line requires two data: slope of the line, and a point on the line.

Functions

Example 6. *Which of the following graphs represents a function?*



(c) Both (a) and (b)

(d) Neither (a) nor (b)

Definition: A **function** f is a rule that assigns

- The **domain** of f is the set of values _____.
- The **range** of f is the set of values _____
- Two functions f and g are **equal** if
  1. The domains of f and g _____
  2. For every x in the domain,_____

Types of Functions

- **Definition**: **Power functions** are of the form $f(x) =$

  Example 7. *Sketch the graph of the following power functions. What other functions look similar to the one you've sketched?*

  1. $y = x^2$ *2.* $y = x^3$ *3.* $y = \sqrt{x}$ *4.* $y = \dfrac{1}{x}$

- **Definition**: A function P is called a **polynomial** if it can be written in the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

with $n \geq 0$ and each a_i constant.

Example 8. *Determine whether each of the following functions are polynomials.*

1. $f(x) = -0.3x^{13} + \pi x^2 + 4$

2. $f(x) = x^{-3} + 2x^2 + 3$

3. $f(x) = 11$

- **Definition**: A **rational function** f is the ratio of two polynomials P and Q: $f(x) = \dfrac{P(x)}{Q(x)}$.

Example 9. *Determine whether each of the following functions are rational functions.*

1. $f(x) = \dfrac{2x - 1}{x^4 + 7}$

2. $f(x) = x^7 + 3x - 21$

3. $f(x) = x^{-1}$

- **Definition**: Beyond polynomials and rational functions:
  - An **algebraic function** is a function constructed by performing algebraic operations (addition, subtraction, multiplication, division, powers) on polynomials.

    Write down an algebraic function that is NOT a polynomial or rational function:

  - A **transcendental function** is a function that is not algebraic.

    What functions have you previously learned about that are not algebraic?

- **Definition**: A **piecewise function** is defined by different formulas on different parts of its domain.

  Example 10. *Consider the piecewise function given by:*

$$f(x) = \begin{cases} 2, & x < -1 \\ 5, & x = -1 \\ x + 4, & -1 < x \leqslant 2 \\ x^2, & x > 2. \end{cases}$$

  a) Compute $f(6)$, $f(-1)$, $f(-23)$, and $f(2)$.

  b) Graph $f(x)$.

Domain/Range

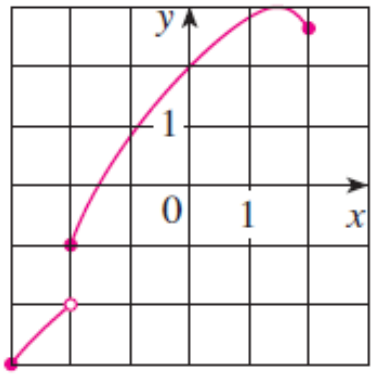
Typical problems for domain: Dividing by zero, negatives in even roots (i.e. $\sqrt{\cdot}, \sqrt[4]{\cdot}, \dots$), asymptotes (logaritms)

Example 11. *Determine the domain of*

$$f(x) = \frac{x-1}{x^2+2x-3}.$$

Example 12. *What is the domain of* $f(x) = \frac{1}{\sqrt{x-4}}$*?*

Example 13. *Determine the domain and range of the following function:*



Difference Quotients

Given a function f, the **difference quotient** of f at the point a is

$$\frac{f(a+h)-f(a)}{h}.$$

Here, h is some non-zero number.

Example 14. *Let $f(x) = x^2 + 1$. Compute the difference quotient at $a = 1$:*