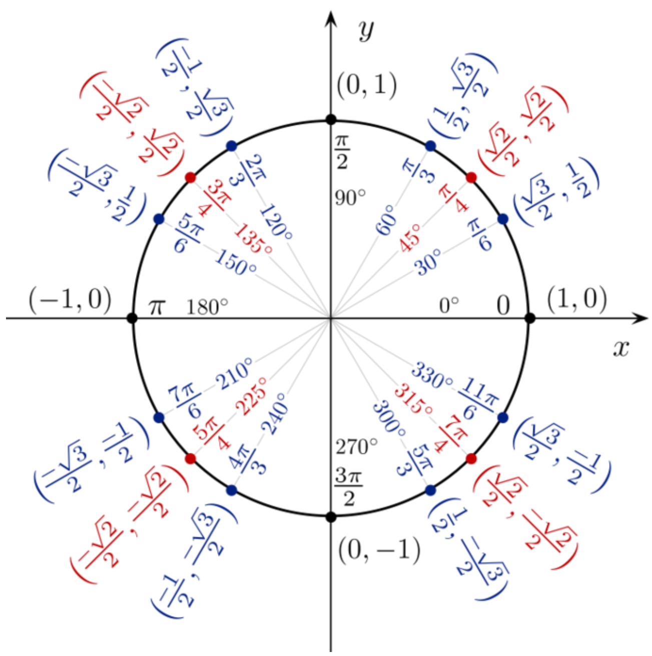
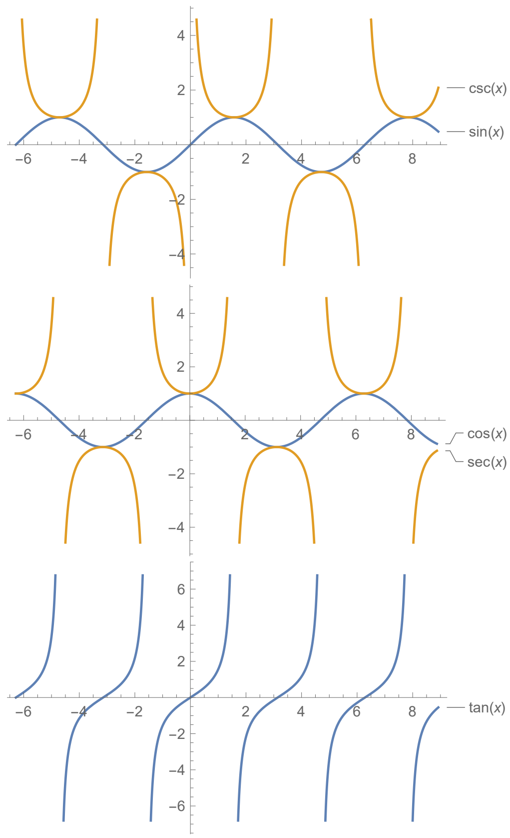
**Topics:** Derivatives of trigonometric functions, chain rule

**Unit circle:** $(\cos\theta, \sin\theta)$




**Trig identities**

$$\sin^2 x + \cos^2 x = 1$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

**Example 1.** *Find all $x$ in $[0, 2\pi)$ solving*

$$3\sin^2 x - 3 = 0$$

# Derivatives of Trigonometric Functions.

**Example 2.** *Using the graph of $\sin x$, graph $\frac{d}{dx}\sin x$.*

**Theorem.**

$$\frac{\mathrm{d}}{\mathrm{dx}} \sin x =$$
$$\frac{\mathrm{d}}{\mathrm{dx}} \cos x =$$

These two rules are enough to compute the derivatives of the remaining trigonometric functions!

**Example 3.** *Use the quotient rule to calculate*

$$\frac{d}{dx} \cot x.$$

**Table of Trigonometric Derivatives.**

$$\frac{\mathrm{d}}{\mathrm{dx}} \sin x = \qquad\qquad \frac{\mathrm{d}}{\mathrm{dx}} \cos x =$$

$$\frac{\mathrm{d}}{\mathrm{dx}} \sec x = \qquad\qquad \frac{\mathrm{d}}{\mathrm{dx}} \csc x =$$

$$\frac{\mathrm{d}}{\mathrm{dx}} \tan x = \qquad\qquad \frac{\mathrm{d}}{\mathrm{dx}} \cot x =$$

**Example 4.** *Differentiate*

$$f(x) = \frac{\sin x - x}{e^x \tan x}.$$

**Example 5.** *Find the equation of the tangent line to* $y = e^x \cos(x)$ *at* $(0, 1)$. *At which values of* $x$ *does it have a horizontal tangent?*

# The Chain Rule

**Example 6.** *Let $y = f(x)$ be a function so that $f'(0) = 2$. What is the derivative of $y = f(3x)$ at $x = 0$?* **Hint:** *Draw a sketch!*

**Take-away:** Derivatives of *compositions*

**The Chain Rule.** If $f$ and $g$ are differentiable functions then

$$(f \circ g)'(x) =$$

**Example 7.** *Compute the derivative of $F(x) = \sqrt{\sin x}$.*

**Example 8.** *Compute $f'(x)$ when*

$$f(x) = (2x + 1)^2$$

*both by using the chain rule and by expanding.*

**Example 9.** *Differentiate* $f(x) = \sin(\cos(\tan x))$.

# Chain rule boot camp

**Example 10.** *Find the derivative of*

$$g(x) = (x^3 + 2x^2 - 1)^5.$$

**Example 11.** *Find the derivative of*

$$k(r) = \sqrt{1 - 2r}.$$

**Example 12.** *Find the derivative of*

$$f(x) = \sin^2(e^{\tan(x)})$$

**Example 13.** *Find the derivative of*

$$f(x) = \frac{1}{(1 + \sec x)^2}.$$

**Example 14.** *Find the derivative of*

$$f(t) = \frac{\sin t}{\sqrt{t^2 + 1}}.$$

**Example 15.** *Find the derivative of*

$$y = \cos\left(\frac{1 - e^{2x}}{1 + e^x}\right).$$