1 Reading mathematics

We will spend a lot of time building our ability to read mathematics in this class. Some tips I can share from the beginning:

1. Math takes time to read! It is very common to read math more slowly than you would a novel or other writing. Sometimes I have to re-read sections several times!
2. Math reading usually requires paper and pencil so that you can follow along on your own. This is a good way to check if you are understanding the reading.
3. It is a good idea to supplement readings with other resources: sometimes the textbook or online resources like Khan Academy have helpful explanations which may make an idea "click."

With that said, let's dive right in!

2 Course motivation

Linear algebra is possibly **the most important** collection of ideas and techniques in mathematics. It is used in almost all pure and applied mathematical fields and is central to physics, engineering, computer science. Some applications include:

1. data science/machine learning
2. image processing
3. cryptography
4. social media suggestions (aka how TikTok becomes addictive)

3 Systems of linear equations

Our course is is centered around systems of **linear equations.**

Definition: A *linear equation* is an equation which can be written in the form

$$a_1x_1 + a_2x_2 + \cdots a_nx_n = b,$$

where b and the coefficients a_i are real or complex numbers, and the unknown variables are the x_i. A collection of linear equations is called a *linear system.*

Example 1. *The equation*

$$2x_1 - \pi x_2 + 3x_3 - 4 = 0$$

is linear since we can write it as

$$2x_1 - \pi x_2 + 3x_3 = 4.$$

Example 2. *The system*

$$\begin{aligned} 2x_1 &= 4 \\ 5\sqrt{x_1} - x_3x_2 &= 0 \end{aligned}$$

is non-linear. Although the first equation is linear, the second equation is non-linear due to the square-root around the variable x_1, as well as the product x_3x_2. Neither is allowed! All unknown variables must appear by themselves with only a constant coefficient.

Example 3. *Do you think*

$$x_2 - 9 - 2x_3 = \sqrt{2}x_1 - 3x_1$$

is linear or non-linear?

4 Solutions of linear systems

A primary goal will be to **solve** systems of linear equations, which means to find values of $x_1,\ x_2,\ , \ldots, x_n$ which make all equations simultaneously true.

Example 4. *Consider the linear system*

$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3. \end{aligned}$$

A solution is $(x_1, x_2) = (3, 2)$. We can confirm this by plugging the values into both equations to make sure they are both simultaneously true:

$$\begin{aligned} (3) - 2(2) &= -1 \\ -(3) + 3(2) &= 3. \end{aligned}$$

Definition: We say that a system is *consistent* if it has at least one solution. If it has no solutions we say it is *inconsistent*.

Example 5. *Can you create a linear system of equations that is inconsistent?*