Learning Objectives:

1. Use the characteristic equation to find the eigenvalues of a matrix.
2. Define when two matrices are similar and describe the relationship between their eigenvalues.

1 Characteristic polynomial

Recall: So far we have seen how to find eigenvectors if we are given eigenvalues. So far we do not know how to find the eigenvalues themselves.

Example 1. *How do we find the eigenvalues of a matrix A?*

Example 2. *Find the eigenvalues of*

$$A = \begin{pmatrix} 2 & 1 & 3 & -1 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Example 3. *Find the eigenvalues of and eigenvectors of*

$$A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}.$$

Example 4. *Find the eigenvalues of*

$$A = \begin{pmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{pmatrix}.$$

2 Similar matrices

In practice, it is often hard to compute eigenvalues for large matrices. Instead we utilize **similar matrices** to simplify the calculation.

Definition: Two matrices A and B are **similar** if

Remark: If A is similar to B then $B = P^{-1}AP$, so A is similar to B if and only if B is similar to A.

Theorem: If $n \times n$ matrices A and B are similar, then they have the same characteristic polynomials and eigenvalues.

Example 5. T/F: *If A and B are similar then* $\det A = \det B$.

Example 6. T/F: *The matrices*

$$\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

are similar.