Learning Objectives

1. Calculate determinants using row operations

2. Explain and apply properties of determinants

1 Calculating determinants using row operations

Theorem: Let A be a square matrix.

a. If B is produced by adding a multiple of one row to another, then $\det A = \det B$.

b. If B is produced by interchanging two rows, then $\det B = -\det A$.

c. If B is produced by scaling one row by k then $\det B = k \cdot \det A$.

Example 1. *Compute*

$$\begin{vmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{vmatrix}.$$

Notice that using this method, $\det A \neq 0$ if and only if it has n pivots...

Theorem: A square matrix A has $\det A = 0$ if and only if

2 Properties of determinants

Example 2. *One of the following statements is false! Determine which one is false, and aim to prove the true ones using geometric and/or algebraic proofs!*

1. $\det(A^T) = \det(A)$.
2. $\det(AB) = \det(A)\det(B)$
3. $\det(A^{-1}) = \frac{1}{\det(A)}$
4. $\det(A + B) = \det(A) + \det(B)$

Example 3. *Compute*

$$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 0 & 2 & 2 \end{vmatrix}.$$

We can combine all our techniques to calculate determinants: cofactor expansion, row operations, or column operations.

Example 4. *Compute*

$$\begin{vmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ -2 & -5 & 4 & -2 \end{vmatrix}.$$

3 Boot camp

Example 5. *What is*

$$\begin{vmatrix} 3 & -1 & 2 & -5 \\ 0 & 5 & -3 & -6 \\ -6 & 7 & -7 & 4 \\ -5 & -8 & 0 & 9 \end{vmatrix}?$$

Example 6. *Compute*

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}.$$

Example 7. *Suppose that A is a square matrix satisfying $A^2 = I$. Prove that* $\det A = \pm 1$.