Net Change Theorem: The integral of a rate of change is the net change: let s(t) be position and s'(t) = v(t) be velocity. Then

The displacement of an object is the net change of position:

The distance traveled is the total amount of distance moved:

Example 1. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$. Find the displacement and distance traveled from $1 \le t \le 4$.

Substitution: indefinite integrals

We learn to solve new antiderivatives using **substitution** or **u-substitution**.

Example 2. Compute

$$\int 2x\sqrt{1+x^2}dx.$$

Take-away:

Example 3.

$$\int x^3 \cos(x^4 + 2) dx.$$

Definite integrals

Example 4. Solve

$$\int_0^4 \sqrt{2x+1} dx.$$

Example 5. Solve

$$\int_{1}^{e} \frac{\ln x}{x} dx.$$

Example 6.

$$\int \sqrt{1+x^2}x^5dx.$$

Boot Camp

Example 7. Solve the following integrals:

Appetizers:

$$\int (3t-1)^{50}dt$$

$$\int_0^3 \frac{dx}{5x+1}$$

Entrees:

$$\int_0^{\pi/2} \cos x \sin(\sin(x)) dx$$

$$\int \tan x dx$$

Desserts:

$$\int \frac{x^9}{1+x^{20}} dx$$

$$\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$$

.