**Topics:** Optimization Problems

Many real world applications depend on optimizing (minimizing/maximizing) a quantity.

**Steps to solving an optimization problem.**

1. Draw a picture.
2. Identify relevant knowns/unknowns and give them notation.
3. Write the desired quantity to be optimized in terms of **one** other variable.
4. If the input variable is on a *finite* interval use the **closed interval method**.
5. If the input variable is on an *infinite* interval use the **first derivative test**.

**Example 1.** *A farmer has* $2400ft$ *of fencing to create a rectangular field next to a straight river. How does he create a field of largest area?*

**Example 2.** *Find the point on the ellipse*

$$4x^2 + y^2 = 4$$

*farthest from the point* $(1, 0)$.

**Useful tip:** Some optimization problems do not have a finite interval of possible inputs. In this case we can use the first derivative test! Assume $c$ is a critical number of $f$. Then:

1. If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$ then
2. If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$ then

**Example 3.** *Find the dimensions of a rectangle with area* $1000m^2$ *whose perimeter is as small as possible.*

**Example 4.** *A retailer is selling 1200 tablets a week at* \$350 *each. The marketing department estimates that an additional 80 tablets will sell each week for every* \$10 *that the price is lowered. Find a function describing the* profit as a function of price. *Then, determine the price that will maximize the total profit.*

**Example 5.** *Let's figure out how long it takes for caffeine to be absorbed and released from the body. The following function models the concentration, $c$, of caffeine in the bloodstream at time $t$:*

$$c(t) = .2te^{-t/2}.$$

*Find when the caffeine in the bloodstream is highest.*