Misallocation of talent, teachers' human capital, and development in Brazil*

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Abstract

We study talent allocation in an economy where teachers play a crucial role in forming the workforce's human capital. We define a Roy model with an externality in the occupational choice because teachers' quantity and quality are essential to workers' human capital. Then, when individuals with more abilities choose to become teachers, all workforce benefits. However, in our model, individuals' occupational decisions are distorted by frictions in the labor and educational goods markets, which may cause a misallocation of talent and harm economic growth and development. After calibrating the model to the Brazilian economy, we find that removing the friction in the labor market increase the Brazilian income by 16.94%. Also, if we assume that the barriers to becoming a teacher across the Brazilian regions are equal to the state with the highest average teachers' human capital, GDP would increase by 87.85% due to externalities of the teacher's occupation.

Keywords: Misallocation; Human Capital; Labor; Externalities; Growth.

JEL: J20; J24; O11; O40.

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1 Introduction

Many studies analyze the hindrances to economic growth, and one relevant approach is that of resources misallocation. Misallocation of capital, credit, and talent has been pointed out as possible barriers to growth (Banerjee and Duflo, 2005; Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Hsieh et al., 2019). Misallocation of talent across occupations and sectors may be a consequence of race and gender discrimination, social norms and culture, and barriers to the human capital formation (Restuccia and Rogerson, 2017)[1]. In the present paper, we study the allocation of talent in an economy where individuals choose their occupation facing different barriers among professions, and teachers play an explicit role in the human capital formation of all workers.

Based on Hsieh et al. (2019) and Barros and Delalilbera (2018), we build a general equilibrium model where individuals choose consumption, time at school, investment in education, and the sector to work. We introduce two barriers that influence individuals' occupational choices, affecting talent allocation in the economy. First, we consider frictions in the labor market, which can be interpreted as the relative difficulty of finding a job in a given occupation and region. This barrier can result from social status or discrimination. The second barrier appears in the educational market. It is related to the costs of human capital formation in a given region and occupation.

In our model, the number of talented people with high human capital choosing an occupation decreases with higher barriers. Moreover, frictions in the teacher's occupation would harm the whole economy since it is essential to the human capital formation of all workers. Furthermore, following Eckstein and Zilcha (1994), we consider the quality of teachers as an input to human capital formation. Based on Gilpin and Kaganovich (2012) and Hatsor (2012), we also consider the number of teachers as input.

We calibrate our model to the Brazilian economy. Our baseline calibration shows a negative correlation between the barriers related to the teacher's occupation and the Brazilian states' per capita output, i.e., the barriers are lower in Brazil's less developed regions. As a result, teachers accumulate more human capital in poorer areas. However, more developed states are

[1]In the context of developing economies, Hnatkovska et al. (2012) show that the misallocation of talent in India comes from the caste system. In Brazil, Café (2018) shows an overqualification of workers in the public sector in relation to the private sector, especially when the evaluation in the public sector is not related to the worker's performance.

more productive due to higher Total Factor Productivity (TFP).

Barros and Delalilbera (2018) have also identified an inverse relationship between the relative wage of teachers and the Brazilian states' economic development. They point out that the occupational choice of workers with multiple skills is driven by labor market incentives (net wage) and the costs of investing in education. Our study differs from Barros and Delalilbera (2018) in two ways. First, in addition to considering that teachers' human capital is a source of positive externalities, we explicitly model the importance of the number of teachers in the workforce's human capital formation. Second, we use our model to study differentials in the relative workers' wages to better understand the relationship between market frictions and the misallocation of talent in Brazil.

We show that the frictions related to the teacher's occupation have a more relevant influence on the economy's output than those of other occupations. We run a series of counterfactual exercises, and we find that the complete removal of frictions in the Brazilian economy would generate an increase of 16.94% in GDP. Also, replicating the incentives of the state with the higher quality of teachers in all Brazilian states would increase the Brazilian average income by 87.85%. Furthermore, we calibrate our model with data from different periods to study the evolution of the allocation of talent in Brazil. We argue that the reduction of the barriers over time could be one of the drivers of absolute income convergence across the Brazilian states.[2]

Although we based our model on Hsieh et al. (2019), we are interested in understanding the impact of misallocation of talent in the teacher's occupations. In contrast, Hsieh et al. (2019) study the economic performance related to the reduction in gender and race discrimination over time in the United States. They find that between 20% and 40% of GDP per capita growth over the last five decades is due to declining occupational barriers, causing women and blacks to occupy highly qualified positions over time. Abdulla (2019) also investigates the misallocation of talent in Brazil and India. His results show that removing all frictions of the labor market and human capital accumulation in Brazil and India would increase average output by 22–52% and 38–53%, respectively. We extend the analysis of the above studies by modeling the tradeoff between quality and quantity of teachers in human capital formation.

Human capital is crucial for economic development by increasing labor productivity, besides facilitating innovation and diffusion of technology as in Romer (1990), Mankiw et al. (1992),

[2]See Ferreira (2000) and Ribeiro and Almeida (2012) for evidence of income convergence in Brazil.

Borensztein et al. (1998), and Benhabib and Spiegel (2005). We contribute to this literature by showing how regional disparities in the labor and educational markets can generate talent misallocation in the teacher's occupation and, in turn, affect aggregate human capital.

The recent literature has emphasized the relevance of education quality in economic growth. For example, Hanushek and Woessmann (2012) argue that Latin American countries lagged behind because of their students' poor performance in educational achievement. In addition, many studies point to the relevance of teachers in the students' learning process (Woessmann, 2016; Barros and Delalilbera, 2018; Hanushek et al., 2019). Indeed, Hanushek et al. (2019) find a robust and positive relationship between the teachers' cognitive skills and student performance measured by the Programme for International Student Assessment (PISA) scores. The cognitive skills of teachers are even more critical to students' performance than the cognitive skills of their parents (Hanushek et al., 2019).

Using the PISA's mathematics test score, Woessmann (2016) points to the relevance of teachers' quality measured by their relative wage and human capital on students' performance. Woessmann (2016) argues that higher teacher wages positively influence recruiting higher ability individuals into teaching. For Brazil, Menezes-Filho and Pazello (2007) find that the relative wage of teachers positively affects the proficiency of public school students. Machado and Scorzafave (2016) point out that wages may affect the decision of the most talented individuals to become teachers. In addition, after an individual becomes a teacher, the wages affect their effort in the classroom and the turnover rate. Several other studies also indicate that the ability of teachers is related to their relative wage, as Stoddard (2003), Lakdawalla (2006), and Bacolod (2007).

Tamura (2001) examines the role of education and the quality and quantity of teachers in economic growth and income convergence. Following Card and Krueger (1992a) and Card and Krueger (1992b), Tamura (2001) formulates a function of human capital formation, where teachers' quality and class size interact with private investment to produce human capital. Then, the author shows that human capital convergence across regions occurs if teachers' quality is relatively more important than class size in human capital production. He argues that poor school districts have relatively better teachers than wealthier districts, driving the income convergence observed in the data. We also consider teachers' quality and quantity to study income convergence across the Brazilian states. We find that income convergence is due to human capital convergence because teachers of poorer Brazilian states have a higher quality.

Besides this introduction, the present paper is organized as follows. Section 2 presents our general equilibrium model. Section 3 explains how this model is calibrated using data from the Brazilian economy. The calibration results, some stylized facts, and the counterfactual exercises are presented in Section 4. Section 5 presents the robustness checks of our main exercises. Finally, Section 6 brings our final remarks. Appendix A presents the descriptive statistics of the Brazilian economy, Appendix B shows the main parameters calibrated, Appendix C describes our strategy to calculate the educational expenditure in Brazil, and Appendix D brings the data on migration.

2 Model

We build a Roy model based on Hsieh et al. (2019) and Barros and Delalilbera (2018) and add a trade-off between quality versus quantity of teachers to study the impact of talent misallocation in an economy where teachers have a central role. In this section, we discuss the behavior of firms and workers, the model's main implications, and define the competitive equilibrium.

2.1 Firms

We consider a country divided into $R \in \mathbb{N}$ independent regions (states). There is a continuum of workers in each region choosing one of the $N \in \mathbb{N}$ occupations in the economy. A person born in region r can not work in a different region.[3] Many homogeneous competitive firms hire workers in all regions and occupations to produce a single product. The firm's production function is given by

$$Y = \sum_{r=1}^{R} \sum_{i=1}^{N} A_r H_{ir}, \tag{1}$$

where Y is output, A_r is Total Factor Productivity (TFP) of region r, and H_{ir} is the aggregate human capital of people working in occupation i at region r. Output can be consumed or

[3]In Appendix D we discuss about migration. Specifically, we argue that only a small fraction of the Brazilian population migrates. Therefore, our assumption of no migration is adherent to the data.

used as an educational good. The firm's problem is choosing labor in terms of efficient units (aggregate human capital) to maximize profit, taking wages (w_{ir}) of each occupation in each region as given.

$$\max_{H_{ir}\geq 0}\left[\sum_{r=1}^{R}\sum_{i=1}^{N}A_r H_{ir}-\sum_{r=1}^{R}\sum_{i=1}^{N}w_{ir}H_{ir}\right]. \tag{2}$$

The solution to the problem described above is simple. The demand for human capital is given by:

$$H_{ir}^{d}=\begin{cases}0 & \text{if } A_r < w_{ir}\\ x\in\mathbb{R}_+ & \text{if } A_r = w_{ir}\\ \infty & \text{if } A_r > w_{ir}\end{cases}. \tag{3}$$

2.2 Workers

Workers have idiosyncratic abilities for each occupation. In a world with multiple occupations, some workers can have a high talent for many occupations, some for only one, while others may lack the skills for any occupation in the economy. Individuals value consumption and leisure, which we model as the time not spent at school. Each worker is endowed with one unit of time to study or consume as leisure. The utility of a person is given by

$$U(c,s)=c^{\beta}(1-s), \tag{4}$$

where c represents consumption, s is time spent at school, and β is a parameter giving the relative importance of consumption to leisure.

We follow Hsieh et al. (2019) and introduce two frictions in our model. A person working in occupation i at region r is paid a net wage of $(1-\tau_{ir}^{w})w_{ir}$, where τ_{ir}^{w} is a barrier specific to occupation i and location r. One can interpret τ_{ir}^{w} as an unobserved cost (or benefit) of occupation i at region r. For example, it can represent social status or barriers to finding a job in a given occupation and region.

There is also friction in the educational market (τ_{ir}^{h}). We can think of this friction as representing forces affecting the cost of acquiring human capital in different occupations and regions. For example, τ_{ir}^{h} can indicate the difficulty of finding a good school or college or adequate training to work in a specific occupation. It can also represent the investments to develop skills required by certain occupations.

Following Tamura (2001) and Barros and Delalilbera (2018), we assume that the quality of teachers is a crucial input to human capital formation. As a contribution of the present paper, we also incorporate the number of teachers as an element to the workers' human capital formation. Therefore, the workers' human capital in each region is given by

$$h_{ir}(e,s) = T_r^{\varphi} s_i^{\phi_i} e_{ir}^{\eta}, \tag{5}$$

where e represents the consumption of educational goods, s is the time spent at school, η is the elasticity of the human capital with respect to consumption of educational goods, and $\phi_i > 0$ is the elasticity of human capital concerning the time spent at school. This parameter varies among occupations and generates differences in schooling. Finally, T_r represents the role of teachers in the workers' human capital formation. We set $T_r = p_{tr}^{\alpha} H_{tr}^{1-\alpha}$ where $\alpha \in (0,1)$, p_{tr} is the fraction of people working as teachers, and H_{tr} is the teachers' aggregate human capital. We use this functional form to incorporate the quality and quantity of teachers on the workers' human capital formation.[4]

Following McFadden (1974), Eaton and Kortum (2002), and Hsieh et al. (2019), abilities dispersion is modeled as a multivariate Fréchet distribution. Let ϵ_i be the ability of an individual in occupation i, then the distribution of abilities across occupations is:

$$F(\epsilon_1, \dots, \epsilon_N) = \exp\left[-\sum_{i=1}^{N} \epsilon_i^{-\theta}\right], \tag{6}$$

where θ governs the skill dispersion.

The individual decision is made into two steps. First, given the occupational choice i, for which the individual has an idiosyncratic ability ϵ_i, and taking wage w_{ir} as given, each worker chooses c, e, and s to solve the following problem:

$$\max_{c,s,e} c^{\beta}(1-s) \tag{7}$$

$$\text{s.t. } c = (1-\tau_{ir}^{w}) w_{ir} h_{ir}(e_{ir}, s_i)\epsilon_i - (1+\tau_{ir}^{h}) e_{ir},$$

[4]See Krueger (2003) and Lakdawalla (2006) for a discussion on teachers' quality and quantity.

Solving the problem above, we find the time spent on school and the purchased amount of educational goods:[5]

$$
\begin{aligned}
s_i^* &= \left(1 + \frac{1-\eta}{\beta\phi_i}\right)^{-1} && (8)\\
e_{ir}^*(\epsilon) &= \left[\eta \left(\frac{1-\tau_{ir}^w}{1+\tau_{ir}^h} w_{ir}\right) (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \left(1 + \frac{1-\eta}{\beta\phi_i}\right)^{-\phi_i} \epsilon_i\right]^{\kappa} && (9)
\end{aligned}
$$

where $\kappa = 1/(1-\eta)$.

A higher elasticity of human capital with respect to time for a given occupation (ϕ_i) leads to more time allocated to human capital accumulation. Individuals in occupations with a high ϕ_i acquire more schooling and have higher wages as compensation.

Using equations (8), (9) and the budget constraint into the utility function, we have the following indirect utility function for occupation i:

$$
D_{ir} = \left[\bar{\eta} \left(\frac{1-\tau_{ir}^w}{(1+\tau_{ir}^h)^{\eta}} w_{ir}\right) (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} (1-s_i)^{\frac{1}{\beta\kappa}} \epsilon_i\right]^{\beta\kappa} \quad (10)
$$

where $\bar{\eta} = \eta^{\eta}(1-\eta)^{1-\eta}$.

Therefore, the occupational choice problem reduces to picking the occupation that delivers the highest D_{ir}. Since talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution (McFadden, 1974). Proposition 1 states that the share of the workers in each occupation can be obtained by aggregating the individuals' optimal choices.

Proposition 1 (Occupational choice)**.** *Let p_{ir} be the fraction of workers in occupation i in region r. Then, aggregating the solution of individual's occupational choice problem across workers, we have:*

$$
p_{ir} = \frac{\tilde{w}_{ir}^{\theta}}{\sum_{j=1}^{N} \tilde{w}_{jr}^{\theta}} \quad (11)
$$

where

$$
\tilde{w}_{ir} = \bar{\eta} \left(\frac{1-\tau_{ir}^w}{(1+\tau_{ir}^h)^{\eta}} w_{ir}\right) (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} (1-s_i)^{\frac{1}{\beta\kappa}}
$$

[5]The complete solution of the model can be viewed in Online Appendix.

Proof. Let:

$$\tilde{w}_{ir} = \bar{\eta}\left(\frac{1-\tau_{ir}^{w}}{(1+\tau_{ir}^{h})^{\eta}} w_{ir}\right)(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}$$

Then, we can rewrite equation (10) as:

$$D_{ir} = [\tilde{w}_{ir}\epsilon_i]^{\beta\kappa}$$

Therefore, the problem solution of individual i living in region r involves picking the occupation with the highest value of $\tilde{w}_{ir}\epsilon_i$. Without loss of generality, consider the probability of an individual choosing occupation 1:

$$\begin{aligned} p_{ir} &= Pr(\tilde{w}_{1r}\epsilon_1 > \tilde{w}_{ir}\epsilon_i) \quad \forall\, i \neq 1 \\ &= Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}}\epsilon_1\right) \quad \forall\, i \neq 1 \\ &= \int F_1(\alpha_1\epsilon, \alpha_2\epsilon, ..., \alpha_N\epsilon) d\epsilon \end{aligned} \tag{12}$$

where F_1 represents the derivative of equation (6) with respect to its first argument, and $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$ for $i \in \{1, 2, ...N\}$. Taking the derivative of equation (6) with respect to ϵ_1, and evaluating in ϵ:

$$\begin{aligned} F_1 &= \theta\epsilon_1^{-\theta-1}\exp\left(-\epsilon_1\hat{Z}\right) \\ F_1(\epsilon) &= \theta\epsilon^{-\theta-1}\exp\left(-\epsilon\hat{Z}\right) \end{aligned}$$

where $\hat{Z} = \sum_{i=1}^{n} \alpha_i^{-\theta}$. Then, equation (12) can be written as:

$$\begin{aligned} p_{1r} &= \int \frac{\hat{Z}}{\hat{Z}}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right) d\epsilon \\ &= \frac{1}{\hat{Z}}\int \hat{Z}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right) d\epsilon \end{aligned}$$

This expression is the derivative of equation (6) with respect to ϵ. Hence:

$$\begin{aligned} p_{1r} &= \frac{1}{\hat{Z}} \int dF(\epsilon) \\ &= \frac{1}{\hat{Z}} \\ &= \frac{\tilde{w}_{1r}^{\theta}}{\sum_{i=1}^{N} \tilde{w}_{ir}^{\theta}} \end{aligned}$$

□

We can interpret $\tilde{w}_{ir}$ as a net reward of a person from region r and occupation i with average ability. Therefore, $\tilde{w}_{ir}$ is composed of wage per efficiency unit, schooling, teachers' human capital, and barriers. In this context, occupations with high w_i will attract more workers in all regions. On the other hand, differences in occupational choices are driven by frictions in the educational goods and labor markets. Therefore, the fraction of individuals choosing sector i is low when there are considerable barriers in human capital formation (τ^h is high) and in the labor market (τ^w is high). The following proposition defines the workers' human capital in each occupation in a given region.

Proposition 2 (Average quality of workers)**.** *For a given region, the human capital of workers in occupation i is:*

$$H_{ir} = p_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i | person\ choices\ i], \tag{13}$$

The average quality of workers is:

$$\mathbb{E}[h(e_{ir}, s_i)\epsilon_i | person\ choices\ i] = \bar{\Gamma} \left[\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}} w_{ir} \right)^{\eta} \tilde{h}_{ir} p_{ir}^{-\frac{1}{\theta}} \right]^{\kappa} \tag{14}$$

where $\bar{\Gamma} = \Gamma(1-\kappa/\theta)$ is related to the mean of the Fréchet distribution for abilities, $\tilde{h}_{ir} = [(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} \eta^{\eta}]^{\kappa}$ and $\kappa = 1/(1-\eta)$.

Proof. We have:

$$h(e_{ir}, s_i)\epsilon_i = (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \left[\eta \left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}} w_{ir} \right) (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi} \epsilon_i \right]^{\eta\kappa} s_i^{\phi_i} \epsilon_i \tag{15}$$

H_{ir} is the total labor supply in efficiency units of occupation i in region r. Then,

$$\begin{aligned}
H_{ir} &= p_{ir}\mathbb{E}\left\{(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}\left[\eta\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi_i}\epsilon_i\right]^{\eta\kappa}s_i^{\phi_i}\epsilon_i\middle|\text{person choices } i\right\} \\
&= p_{ir}\left\{(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}\left[\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)\eta(p_{tr}^{\alpha}H_{tr}^{1-\alpha})^{\varphi}s_i^{\phi_i}\right]^{\eta\kappa}s_i^{\phi_i}\mathbb{E}\left[\epsilon_i^{\kappa}\middle|\text{person choices } i\right]\right\} \\
&= p_{ir}\tilde{h}_{ir}\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}}w_{ir}\right)^{\eta\kappa}\mathbb{E}\left[\epsilon_i^{\kappa}\middle|\text{person choices } i\right] \qquad (16)
\end{aligned}$$

To calculate this last conditional expectation, we use the Fréchet distribution. We suppress the region index r because this calculation is similar in all regions. Let $y_i = \tilde{w}_i\epsilon_i$. Since we are maximizing y_i, it also has the extreme value distribution:

$$\begin{aligned}
\mathbf{Pr}\left(\operatorname*{Max}_{i} y_i < z\right) &= \mathbf{Pr}(\epsilon_i < z/\tilde{w}_i) \quad \forall i \\
&= F(z/\tilde{w}_1, ..., z/\tilde{w}_N) \\
&= \exp\left[-\sum_{i=1}^{N}(z/\tilde{w}_i)^{-\theta}\right] \\
&= \exp\left[-kz^{-\theta}\right]
\end{aligned}$$

where $k = \sum_i^N \tilde{w}_i^{\theta}$.

After some algebraic manipulation, we conclude that the distribution of ϵ^* (the workers' ability in their chosen occupation) has a Fréchet distribution:

$$G(x) = \mathbf{Pr}(\epsilon^* < x) = \exp\left[-k^* x^{-\theta}\right] \qquad (17)$$

where $k^* = \sum_{i=1}^{N}(\tilde{w}_i/\tilde{w}^*)^{\theta} = 1/p^*$.

Finally, we calculate the expectation of equation (16). Let i be the occupation an individual chooses, and λ a positive exponent.

$$\begin{aligned}
\mathbb{E}(\epsilon_i^{\lambda}) &= \int_0^{\infty}\epsilon_i^{\lambda}dG(\epsilon) \\
&= \int_0^{\infty}\theta\left(\frac{1}{p^*}\right)\epsilon^{(\lambda-\theta-1)}\exp\left[\left(\frac{1}{p^*}\right)\epsilon^{-\theta}\right]d\epsilon
\end{aligned}$$

We set $x = \left(\frac{1}{p^*}\right)\epsilon^{-\theta}$ and rewrite the last expression as:

$$
\begin{aligned}
\mathbb{E}(\epsilon_i^{\lambda}) &= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}} \int_0^{\infty} x^{-\frac{\lambda}{\theta}} \exp(-x)dx \\
&= \left(\frac{1}{p^*}\right)^{\frac{\lambda}{\theta}} \Gamma\left(1 - \frac{\lambda}{\theta}\right)
\end{aligned}
$$

Using this result in equation (16) completes the proof. □

This result points to a selection effect in the economy. In equation (14), the average quality in occupation i and region r is inversely related to the share of workers in that occupation (p_{ir}). If the friction is high in occupation i and region r, only the most qualified workers are selected. For example, in a region where it is easy to become teachers, their average human capital will be small (intensive margin). On the other hand, holding the average human capital constant, a higher share of workers in an occupation will result in higher aggregate human capital (extensive margin). The net effect depends on the parameters' values. If $\theta(1-\eta) > 1$, the extensive margin dominates. Otherwise, the intensive margin dominates. Next, we solve the model for the average wage in occupation i and region r.

Corollary 1 (Gross average wages)**.** *Let W_{ir} be the gross average wage in occupation i in region r. Then:*

$$
W_{ir} = w_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \bar{\Gamma}\eta \frac{(1-s_i)^{-1/\beta}}{(1-\tau_{ir}^{w})} \left(\sum_{i=1}^{N} \tilde{w}_{ir}^{\theta}\right)^{\frac{\kappa}{\theta}} \tag{18}
$$

This result is a consequence of Proposition 2. Equation (18) shows that gross average wage in a given region differs among occupations due to schooling and labor market frictions. Occupations with high workers' human capital have considerable gross average wages. From equation (3), we conclude that in equilibrium $A_r = w_{ir}$. Then, $\tilde{w}_{ir}$ is a function of A_r, and consequently, W_{ir} is a function of regional TFP. Therefore, frictions, average human capital, and TFP are important sources of regional variation in average wages. Finally, we use a standard definition of a competitive equilibrium.

2.3 Equilibrium

Definition 1 (Competitive equilibrium)**.** *A competitive equilibrium in this economy consists of:*

(i) Given an occupational choice, w_{ir}, and the idiosyncratic ability ϵ, each worker chooses c, e, s to maximize utility in equation (7).

(ii) Given market friction, w_{ir}, H_{it}, and ϵ, a worker chooses the occupation that maximizes D_{ir}.

(iii) A representative firm hires H_{ir} to maximize profits.

(iv) The occupational wage, w_{ir}, clears the labor market in each occupation and region.

(v) Total output is given by the production function in equation (1).

3 Empirical Investigation

In this section, we describe the calibration of the model to fit the Brazilian data. We consider data in two periods (2003 and 2015) [6] to evaluate the convergence of income and human capital across the Brazilian states.

Our calibration strategy consists of finding values for frictions and TFP to ensure that the competitive balance is consistent with the dataset of the Brazilian states for 2015. We use data from the National Household Sample Survey (PNAD) at the individual level for the following variables: years of schooling; work hours; gross earnings; and the share of workers into occupations. After some adjustments[7], we have a sample of 109,038 individuals on eight groups of occupation: 1) managers (except public sector); 2) sciences and arts; 3) middle-level technicians; 4) administrative service; 5) service-sector; 6) sellers and service providers; 7) agriculture; 8) goods and industrial production, services, and repairs-maintenance. We aggregate groups 4, 5, and 6 into the service sector. Finally, we separate the individuals working as teachers. Thus, we have the following occupational categories:

1. Managers (except public sector);
2. Professionals of sciences and arts (except teachers);

[6] We consider this period because there were changes in the Brazilian National Household Sample Survey (PNAD) methodology before 2003 and after 2015.

[7] We drop individuals with no occupation and those whose wages were less than 60% of the minimum wage. Therefore, we drop individuals that receive considerably less than the minimum wage. We also selected individuals between 25 and 65 years old. Concerning the occupations, we drop individuals with not well defined occupations and those in the army.

3. Middle-level technicians (except teachers);

4. Service sector;

5. Agriculture;

6. Goods and industrial production, services and repairs-maintenance;

7. Teachers.

The 26 Brazilian states[8] and the Federal District (DF) are considered in the empirical analysis. Thus, the dataset comprises seven occupations ($N = 7$) in twenty-seven regions ($R = 27$).

We split the parameters into three groups: preferences and technology parameters (η, θ, φ, β, α); the elasticity of human capital in relation to time spent at school (ϕ_i), frictions (τ_{ir}^{w}) and τ_{ir}^{h}); and TPF (A_r).

3.1 Preferences and technology parameters

The model's parameters define the functional forms of many equations, such as the distribution of abilities and the utility function. We set the first group of parameters (η, θ, φ, β, α) to evaluate income convergence using the mean of some statistics between 2003 and 2015. We follow Hsieh et al. (2019) to estimate the skill dispersion parameter (θ) and the elasticity of human capital to educational goods (η). We assume that wages within an occupation for a given region follow a Fréchet distribution, and it is shaped by θ and η in the following multiplicative form: $\theta(1-\eta)$. Therefore, wage dispersion depends on $1/\theta$ and $1/(1-\eta)$, and the coefficient of variation (CV) of wages within an occupation and region is given by:

$$CV = \frac{\Gamma\left(1 - \frac{2}{\theta(1-\eta)}\right)}{\left(\Gamma\left(1 - \frac{1}{\theta(1-\eta)}\right)\right)^2} - 1, \tag{19}$$

where γ represents the Gamma function.

[8] Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí (PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Next, we calculate the mean and variance of the exponent of these regression residuals and solve equation (19) for $\theta(1-\eta)$ using a root-finding algorithm. The result for 2003 is 2.39, for 2015 it is 2.00, and the average of the two years is 2.19.

We follow Hsieh et al. (2019) and calculate η as the ratio of educational expenditure to the labor compensation. The total amount of public and private educational expenditures as a proportion of GDP was 0.064 (2003) and 0.079 (2015), and its average was 0.072. The ratio of labor compensation to GDP was 0.53 (2003) and 0.58 (2015), and its average was 0.56[9]. Therefore, η is set to 0.129. Since we have $\theta(1-\eta)$ and η, it is easy to find θ, which is 2.52.

The remaining functional parameters of the model are specified in Table 1. We follow Tamura (2001) and set the parameters of teacher's role in human capital formation $\alpha = 0.31$, and $\varphi = 0.48$. Following Hsieh et al. (2019), we use $\beta = 0.231$. In section 5, we explore the robustness of our results for different values of α, β, θ, η, and φ.

Table 1: Baseline constant parameters

Parameters	Value	Description	Source
η	0.129	Elasticity of educational goods in the human capital function	Estimated using data from PNAD 2015 and 2003
φ	0.48	Elasticity of teacher's human capital in the human capital function	Tamura (2001)
θ	2.52	Dispersion of skills	Estimated using data from PNAD 2015 and 2003
α	0.31	Weight of the share of teachers in T_r	Tamura (2001)
β	0.231	Consumption preference	Hsieh et al. (2019)

3.2 Estimation of ϕ_i's

The elasticity of human capital to time spent at school for each occupation (ϕ_i's) compose the second group of parameters. First, we compute each occupation's average years of schooling and then the study hours. We assume that a typical individual studies six hours a day on weekdays, so the number of study hours in a year is $252 \times 6 = 1512$. Therefore, of the 8760 hours available in a year, the time studying represents 17.26%. We assume that the schooling period occurs in the first 25 years of the life cycle, so this is our model's upper bound of years of education. Therefore, we divide the average years of schooling from the dataset by 25 and multiply it by the studying timeshare in a year (0.1726). Finally, we use equation (8) to calculate the ϕ_i's[10]. Table 2 brings the results.

[9] Labour compensation as share of GDP comes from the Penn World Table 10.0.

[10] Equation (8) can be rewrite as $\phi_i = \dfrac{(1-\eta)s_i}{\beta(1-s_i)}$. Therefore, we substitute in this expression the time spent on education, calculated in the first step, and the remaining parameters.

Table 2: Descriptive statistics of years of schooling among occupations and implied ϕ

Occupation	Parameter ϕ_i	Schooling Statistics Mean	1° Quartile	Median	3° Quartile	Variance
Managers	0.28	11.77	11	11	15	3.36
Sciences and arts	0.35	13.98	15	15	15	2.39
Middle-level technicians	0.28	11.67	11	11	14	2.58
Service-sector	0.22	8.91	6	11	11	3.79
Agriculture	0.12	5.07	2	4	8	3.92
Industrial production and services	0.18	7.75	5	8	11	3.69
Teachers	0.35	14.13	14	15	15	1.69

Source: Elaborated by the authors with data from PNAD 2015. Each ϕ_i is computed using equation (8).

3.3 Calibration of $\tau' s$ and $A' s$

The remaining parameters, τ's and A's, are calibrated using the Method of Moments by minimizing the gap between the statistics of our model and the Brazilian data. In the calibration procedure, we use two groups of statistics for each occupation and region: the workers' share; and the average gross wage.

We calculate the average hourly wage using the PNAD microdata for each occupation in each region [11]. In our model, those statistics are described by equations (11) and (18). We use the First Order Conditions (FOC) of the firm's maximization problem, where $w_{i,r} = A_r \, \forall \, i, r$, to recover the equilibrium wage rate, which allows us to use equations (11) and (18) to compute the model's statistics that represents the competitive equilibrium.

The sum of the occupations' share in each region equals one, $\sum_{i=1}^{N} p_{ir} = 1$. Therefore, we have $(N-1)R$ independent statistics in each region. Thus, we assume that $\tau^h_{1r} = 0$, $\forall \, r$. Also, we assume that $\tau^w_{1r} = \tau^w_1 \quad , \forall \, r$, i.e, that frictions in occupation 1 are equal across regions. Also, we assume that the last region's TFP (A_R) is constant.

We define the following objective function to our numerical routine:

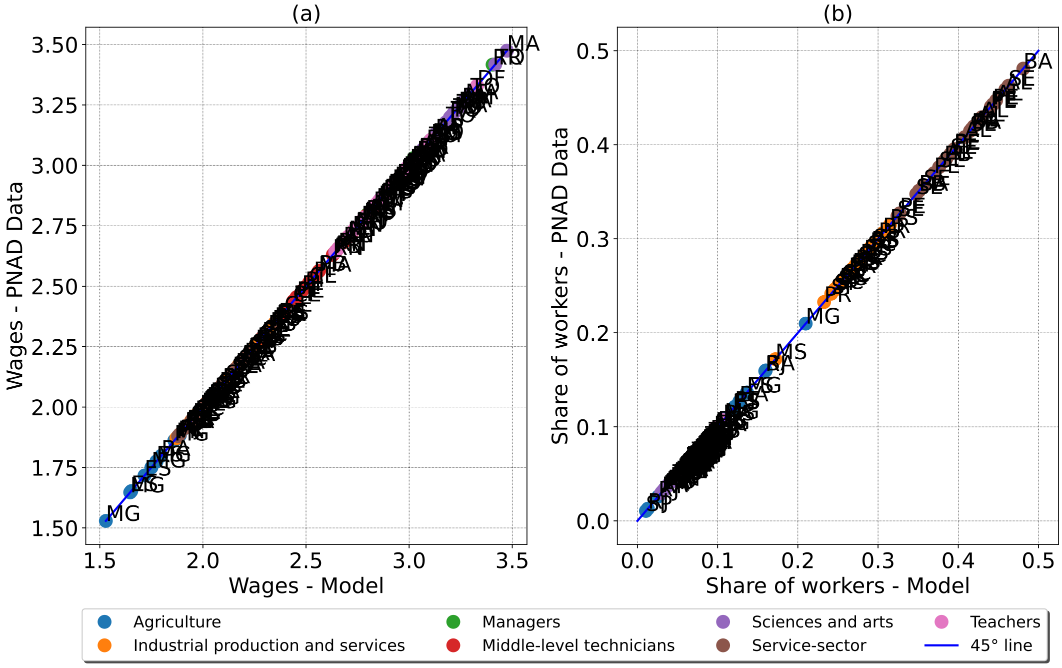
$$\mathcal{M} = \sum_{i=1,r=1}^{N,R} \left(\frac{W^M_{ir} - W^D_{ir}}{W^D_{ir}} \right)^2 + \sum_{i=1,r=1}^{N-1,R} \left(\frac{p^M_{ir} - p^D_{ir}}{p^D_{ir}} \right)^2 \tag{20}$$

where the superscripts M and D indicate the model and target statistics.[12] To minimize equation (20), we use the Nelder-Mead algorithm that finds $\mathcal{M} = 0.00092$, which we consider a small number because we have 378 different targets.

[11]The Appendix A brings the average hourly wage and the share of workers by occupation and region.
[12]We apply the logarithm in equation (20) to improve the algorithm's numerical stability.

Figure 1(a) brings the empirical data of average hourly wage (vertical axes) and the model estimated average hourly wage (horizontal axes). Figure 1(b) shows the empirical (vertical axes) and model estimated (horizontal axes) data for the share of workers in each occupation and region. The model has a good adjustment to the empirical data since the points are close to the 45° line. Appendix B brings the calibrated values of τ_{ir}^{w}, τ_{ir}^{h}, and A_r.

Figure 1: Model adjustment to data - wages and share of workers



4 Results

In this section, we present and discuss the numerical exercises' results. First, we compare our simulations' results with a set of stylized facts. In addition, we run a series of counterfactual exercises to see how sensitive the simulated GDP is to changes in the labor market and educational frictions. We also calibrated the model using data for 2003 and compared it to the previous calibration to analyze the income convergence process across Brazilian states. Finally, we examine the robustness of our results.

4.1 Comparing model results with a set of stylized facts

The calibrated model has a good fit for the GDP per worker, as shown in Figure 2 (a). Also, as expected, we see in Figure 2 (b) that the model's results indicate a positive relationship between GDP and TFP.

Figure 2: GDP - Model

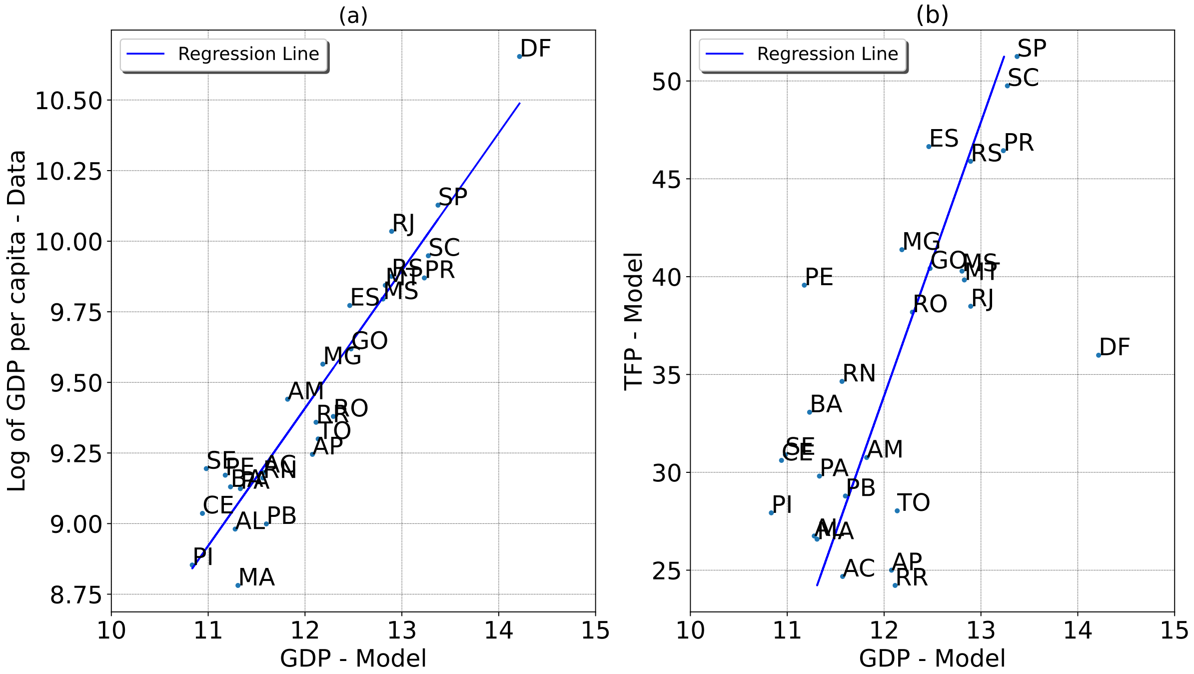
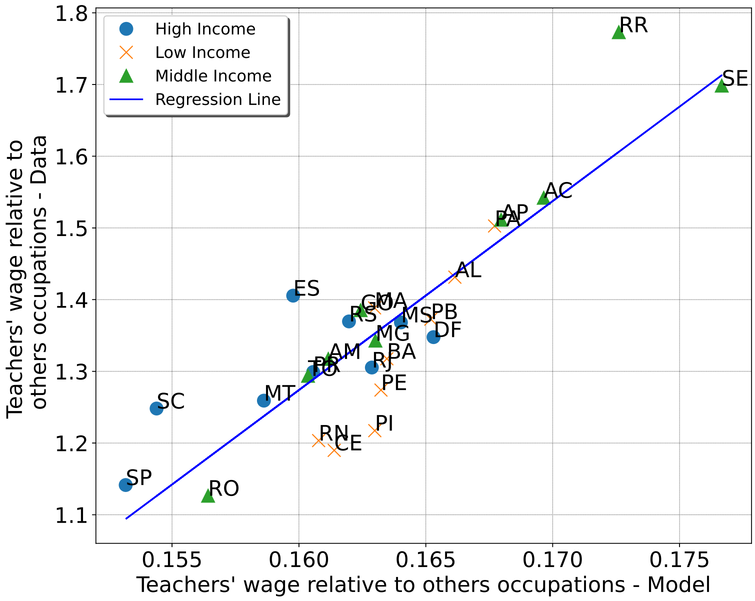


Figure 3 presents the model's results and data on teachers' wages relative to other occupations[13]. The model's results also display a good fit for relative wages. Furthermore, Figure 3 shows that, on average, teachers have a higher relative wage in low and middle-income states than in high-income states [14]. Barros and Delalilbera (2018) argue that one of the reasons for this is that the occupation of teachers is labor-intensive and barely affected by technological and structural changes. Therefore, in states with more advanced technologies, the relative teachers' wage is lower than in less developed states.

[13]In Brazil, the Law N 11.738 of 2008 regulates the national minimum wage for public teaching professionals in basic education. Nevertheless, Table A1 in Appendix A shows that there is dispersion in the hourly wage of teachers across regions.

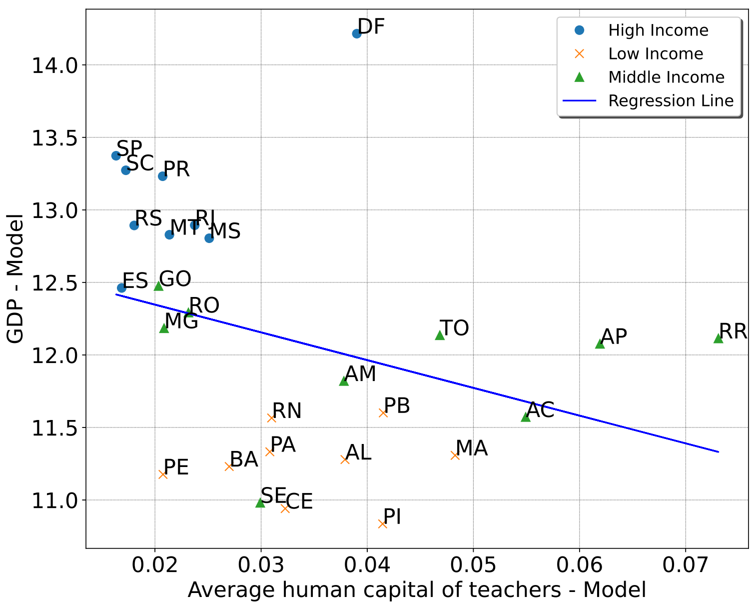
[14]We rank the 27 Brazilian states using 2015 GDP *per capita* data. The first nine are considered high-income, the middle nine are middle-income, and the last nine are low-income.

Figure 3: Teachers' relative wages



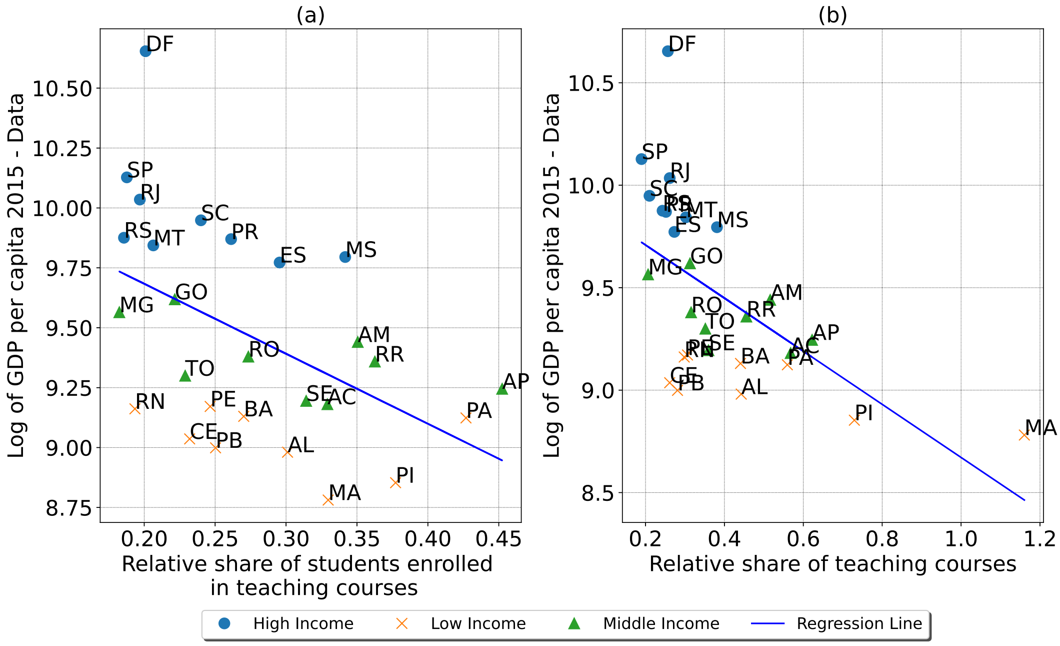
In poor states, the teaching occupation has relatively low labor and educational markets barriers. Then, it is easier to become a teacher in these states than in richer ones. In Figure 4, we see that a higher share of talented people chooses the teachers' occupation in low and middle-income states than in high-income states. Thus, the latter areas would have an even higher income if there were more incentives for talented people to become teachers.

Figure 4: GDP per worker and teachers' human capital



The last results are adherent to data. Indeed, Figure 5 (a) shows that, on average, more students are enrolled in teaching courses in the poorest states than in the wealthiest ones since the former have a greater offer of these courses, as shown in Figure 5 (b).

Figure 5: Share of students in teaching courses, and share of teaching courses offered



Note: This figure was elaborated using data from the Higher Education Census of the 2015, provided by the National Institute of Educational Studies and Research Anísio Teixeira (INEP).

Our calibration is also adherent to other data findings. For example, in the model, agriculture has the highest average frictions in the educational market. In data, about 65% of workers in this occupation lived in rural areas in 2015 (PNAD), where it is more difficult to find and attend schools[15]. Furthermore, the quality of rural schools is lower than in urban areas[16]. On the other hand, at least 92% of the workers in other occupations live in urban areas.

4.2 Frictions and GDP

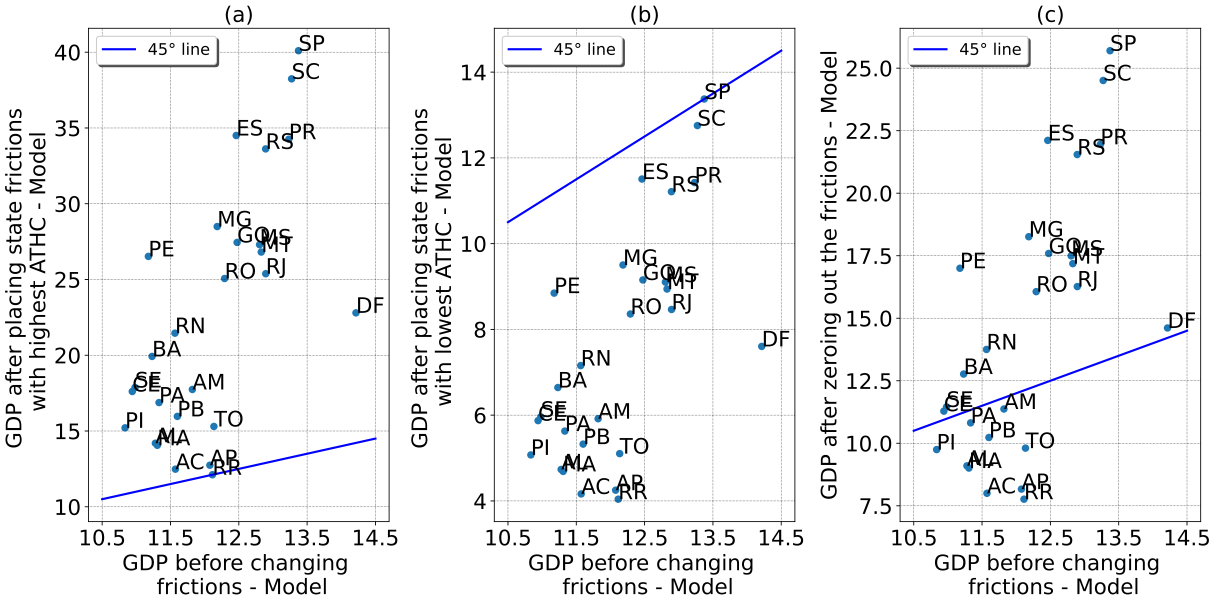
We conduct counterfactual exercises to check how sensitive the economy is to frictions. In the results of Figure 6 (a), we assume that all states have the same frictions, τ^w's and τ^h's, as the one with the highest Average Teachers Human Capital (ATHC), namely Roraima (RR). In this case, all states would have a higher GDP, the Brazilian GDP would increase by 87.85%, and the relative wage of teachers in all states would be equal to Roraima's relative wages. If the states had the same frictions as São Paulo (SP), the one with the lowest ATHC, the GDP of all states would decrease (Figure 6 (b)), and the Brazilian GDP would decline by 59.62%.

[15]See Appendix B for more details.

[16]See, for example, Williams (2005) and Zhang (2006).

Finally, 6 (c) brings a counterfactual exercise to eliminate all the friction, where the Brazilian economy would experience a 16.94% growth.

Figure 6: GDP before and after placing state barriers with the highest and lowest ATHC in all states



From this exercise, we can infer that labor misallocation is a relevant problem across the Brazilian states. When the teacher occupation experience reduction in barriers, its relative wage increases, leading to a reallocation of talent across occupations. Therefore, since there is a positive externality in becoming a teacher, a change in the frictions that induces more talented people to choose this occupation significantly affects regional GDP.

In the next exercise, we look at how market frictions in all occupations affect GDP *per capita*:

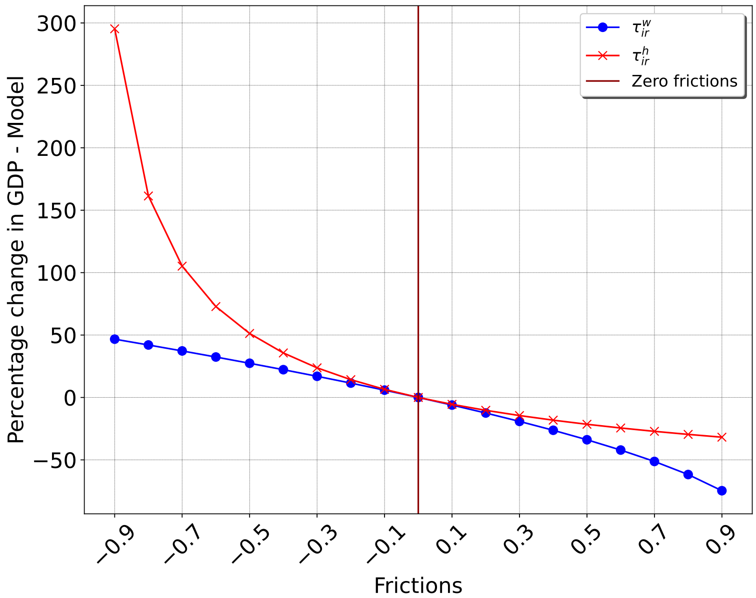
1. We calculate the GDP *per capita* without any frictions in the labor and educational markets.

2. We set the educational market frictions to zero and vary the labor market friction from −0.9 to 0.9.

3. We conduct a similar exercise, where the labor market frictions are set to zero, and we analyze the educational goods market frictions.

Frictions in the educational market act as a "price" and, thus, consumers can react to changes in those frictions by buying more or less of those goods. On the other hand, as the labor supply is inelastic, frictions in the labor market only affect the net wages. Therefore, changes in frictions regarding education have a more significant impact on GDP. Figure 7 shows the results

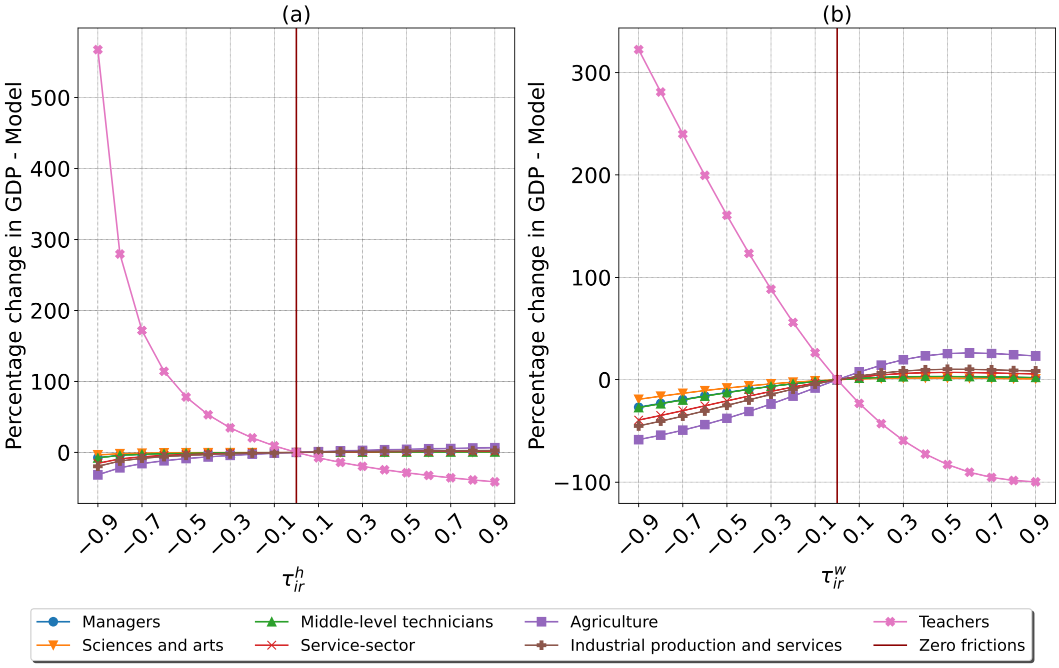
Figure 7: Increases in the frictions of all occupations and the percentage effects on GDP - Model



4.3 Teacher's human capital

In this section, instead of changing frictions for all occupations simultaneously, we change them one by one, keeping the other occupations' frictions at zero. Figures 8 shows that GDP is more sensitive to frictions in the teacher's career and mainly to frictions in the educational goods market. Therefore, a public policy should incentivize more qualified people to become teachers to promote GDP growth. It is noteworthy that reducing the frictions in other occupations may hurt economic development since it will diminish the incentive to become a teacher.

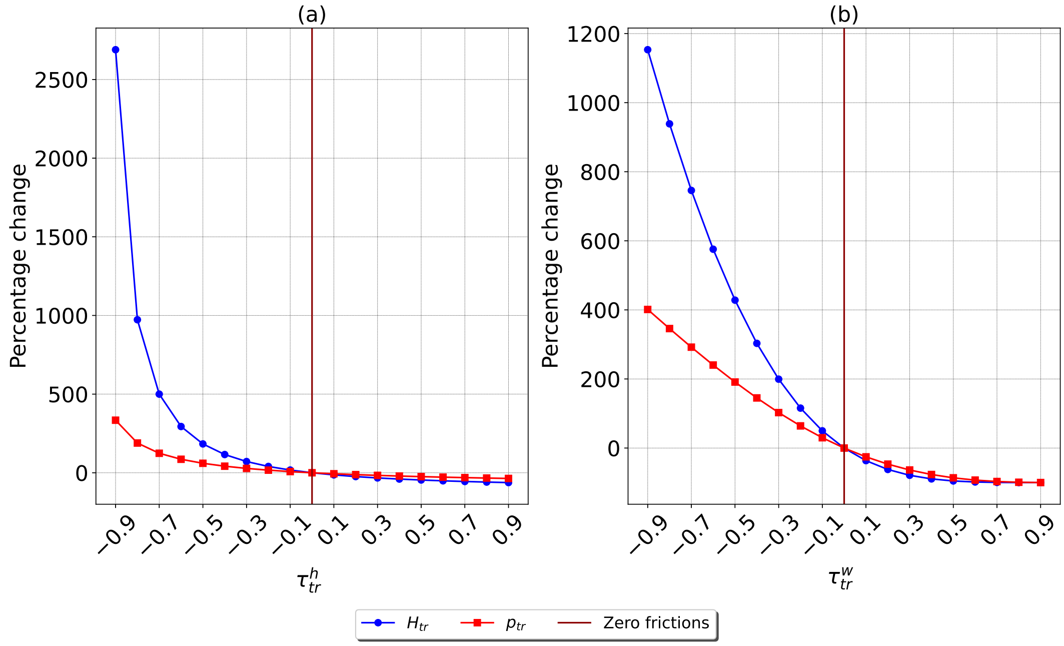
Figure 8: Increases in the frictions of each occupation and the percentage effects on GDP



In Figure 9, instead of measuring the sensitivity of GDP to frictions, we verify the effect of frictions on teachers' human capital and the proportion of teachers in the workforce.

The teacher's occupation becomes more attractive with lower barriers. Therefore, more people choose this carrier (extensive margin). On the other hand, people with low idiosyncratic skills may decide to become teachers, resulting in a lower average quality of teachers (intensive margin). Although this trade-off between quality and quantity of teachers is present in our model, a lower τ^h generates a lower "price" of educational goods (see equation (9)). Then, all workers who choose to become teachers invest more in human capital, compensating for the first effect. Thus, the net result of a reduction in barriers is an increase in the average quality of teachers (intensive margin). It is essential to mention that all these effects are amplified due to teachers' externality on the whole workforce.

Figure 9: Increases in frictions and percentual effects on H_{tr} and p_{tr}



Note: H_{tr} = Human capital of teachers, p_{tr} = proportion of workers in teacher occupation.

4.4 Income convergence

Barro and Sala-i Martin (1992) state that absolute income convergence occurs when poor economies grow faster than more prosperous ones in per capita terms. Hence, the income gap between poor and rich regions tends to narrow over time. In this sense, using data from the model calibrated for 2015 and 2003, we test whether there is income convergence among the Brazilian states by estimating the following equation via OLS:

$$\frac{1}{T}\log\left(\frac{Y_{r,2015}}{Y_{r,2003}}\right) = a + b\log(Y_{r,2003}) + \epsilon_r \tag{21}$$

where $Y_{r,2015}$ and $Y_{r,2003}$ represent the 2015 and 2003 GDP of the region r, T is the number of periods, a and b are constants, and ϵ_r is the error term. A negative b supports the convergence hypothesis.

The results in Table 3 corroborate the hypothesis of absolute income convergence among the Brazilian states since b is negative and statistically significant. We calculate the speed of convergence of this economy, which is $\beta_s = 7.58\%$ [17]. This result can be interpreted using the

[17] The speed of convergence is given by: $\beta_s = -\frac{\log(Tb+1)}{T}$.

half-life concept, which is the time required to reduce the income gap by half. The half-life is given by $HL = \log(2)/\beta_s$, and it is 9.14 years.

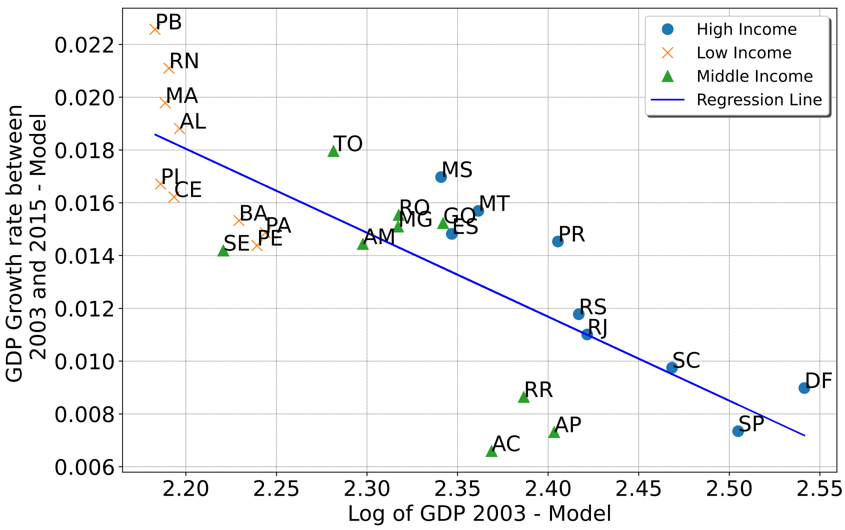
Table 3: OLS to verify the absolute convergence of income between Brazilian states between 2003 and 2015

	$\frac{1}{T}\log\left(\frac{Y_{r,2015}}{Y_{r,2003}}\right)$
a	0.0880***
	(0.0114)
b	-0.0318***
	(0.0049)
R-squared	0.6284
R-squared Adj.	0.6136

Source: Search results.
Notes: Standard errors in parentheses. Single (*), double (**) and triple (***) asterisk denote statistical significance at 10%, 5% and 1%, respectively.

In Figure 10, the result of absolute income convergence is more evident. The low-income states such as Paraíba (PB), Rio Grande do Norte (RN), Maranhão (MA), Alagoas (AL), Piauí (PI), and Ceará (CE) had experienced fast income growth in the period. On the other hand, high-income states such as the Federal District (DF), São Paulo (SP), Santa Catarina (SC), Rio de Janeiro (RJ), and Rio Grande do Sul (RS) had lower growth rates.

Figure 10: Growth rate from 2003 to 2015 and Log of GDP 2003



The absolute convergence of income can be explained by the reduction of educational market frictions and TFP increase from 2003 to 2015[18]. Our results indicate that educational frictions have reduced more sharply in the poorest states and Rio de Janeiro. Furthermore, on average, the teacher's occupation had experienced a reduction in frictions. However, the most significant reduction in frictions occurred in agriculture, probably due to the increase in the average years of education of the Brazilian agricultural workers in the period.

On average, we also notice a slight increase in the labor market barriers from 2003 to 2015. However, as we saw in Figure 7, increases in the labor market barriers have a smaller effect on GDP when compared to reductions in the educational market's frictions.

5 Robustness check

In Table 4, we analyze the robustness of our results. In this robustness check, we repeat the counterfactual exercise of Section 4.2, where we set frictions, τ^w and τ^h, to match those of the states with the highest and lowest Average Teachers Human Capital (ATHC).

In our counterfactual exercise, when we change the educational goods elasticity in the human capital function η using the frictions of the state with the highest ATHC (Roraima), GDP increases substantially from 2.71% ($\eta = 0.05$) to 87.85% ($\eta = 0.129$), and up to 162.36% ($\eta = 0.25$). The results are analogous when we use the frictions of the state with the lowest ATHC (São Paulo). Among all parameters, GDP is more sensitive to changes in η and φ.

In the following two lines of Table 4, we analyze the sensitivity to skill dispersion, θ. In the second column, we see that when skill dispersion is $\theta = 2$, GDP is 57.95% higher, 87.85% in the baseline scenario, and 90.41% when $\theta = 3$. In the third column, GDP is more sensitive to changes in θ. In other words, in an economy with greater friction, the dispersion of skills impacts the GDP more. The parameter β also affects GDP positively. Appendix β brings the calibrated frictions and TFP for 2003.

A critical parameter in the human capital function is the one measuring the trade-off between quantity vs. quality of teachers. By raising α, we increase the importance of the number of teachers and, consequently, decrease their quality (teachers' average human capital) [19]. When $\alpha = 0.2$, GDP is 107.55% higher, and it is 57.87% higher for $\alpha = 0.6$.

[18] Appendix B brings the calibrated frictions and TFP for 2003.

[19] Recall that $T_r = p_{tr}^{\alpha} H_{tr}^{(1-\alpha)}$. Where H_{tr} is the average of human capital of teachers and p_{tr} is the proportion of teachers, in region r.

Table 4: Robustness check for constant parameters

Parameter	GDP variation (Largest ATHC)	GDP variation (Lowest ATHC)	GDP variation (Zero Frictions)
$\eta = 0.05$	2.71%	-14.07%	9.31%
$\eta = 0.25$	162.36%	-108.50%	20.22%
$\theta = 2.0$	57.95%	-48.50%	21.19%
$\theta = 3.0$	90.41%	-60.59%	16.68%
$\beta = 0.1$	80.46%	-52.38%	-0.29%
$\beta = 0.3$	88.52%	-61.37%	18.35%
$\alpha = 0.2$	107.55%	-73.20%	18.29%
$\alpha = 0.6$	57.87%	-38.79%	14.41%
$\varphi = 0.1$	8.44%	-16.06%	20.20%
$\varphi = 0.6$	162.13%	-99.39%	13.19%
Benchmark	87.85%	-59.62%	16.94%

Source: Search results.
Notes: ATHC is Average teacher human capital.
Recall that our baseline values are $\eta = 0.129$, $\beta = 0.231$, $\theta = 2.52$, $\varphi = 0.48$ and $\alpha = 0.31$.

Finally, we analyze the robustness of our results with respect to the teacher's contribution to the human capital formation, φ. As can be seen, a reduction in this parameter substantially reduces GDP.

6 Final Remarks

In this paper, we build a Roy model to study the influence of frictions on the labor and educational markets in Brazil. In addition, we incorporate a function where teachers play a vital role in the workforce's human capital accumulation. After calibrating the model to the Brazilian data, we find a negative correlation between the barriers related to the teacher's occupation and GDP in the Brazilian states. Next, we show that raising the teacher occupation's attractiveness increases GDP. When more individuals with higher idiosyncratic ability choose the teaching career, they directly impact the workforce's productivity.

Furthermore, comparing the calibrated model for 2015, the absolute income convergence was mainly due to reducing frictions related to the teacher's career because it increases the entire economy's average human capital and productivity. In this sense, policymakers should increase incentives for this occupation to attract more talented people.

Overcoming the frictions in the real economy is not trivial. However, suitable design of public policies to incentivize more talented people to become teachers is crucial to spur economic growth. For example, creating a carrier path for teachers related to performance and wages similar to the teachers' qualifications in relation to other occupations, a work environment fomenting interaction among teachers and investments in their training, or a good retirement plan for this class of workers generate incentives for this profession. Studies to understand the attraction of workers to the teaching occupation and their qualifications are crucial to foment economic growth and development.

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Appendix A Descriptive Statistics

Table A1: Descriptive statistics of teachers' hourly wages by state

State	Relative Wage	Mean	1° Quartile	Median	3° Quartile	Variance	Income Group
AC	1.57	17.53	9.72	14.29	23.81	10.46	Middle Income
AL	1.45	16.04	9.38	13.91	20.37	9.65	Low Income
AM	1.30	16.13	9.52	14.29	20.24	8.63	Middle Income
AP	1.48	19.77	13.17	17.80	23.53	10.03	Middle Income
BA	1.33	15.40	8.33	11.90	17.86	11.97	Low Income
CE	1.21	14.04	8.33	11.90	15.87	11.17	Low Income
DF	1.49	27.93	13.69	23.81	35.71	17.42	High Income
ES	1.29	18.16	9.68	15.01	21.33	12.99	High Income
GO	1.42	19.30	10.19	14.29	22.55	15.18	Middle Income
MA	1.30	15.93	8.93	11.90	20.40	12.50	Low Income
MG	1.37	18.42	9.52	14.29	21.65	13.78	Middle Income
MS	1.45	21.84	11.11	17.86	26.19	16.22	High Income
MT	1.31	18.93	11.90	17.06	21.43	10.58	High Income
PA	1.50	17.81	9.38	14.29	21.71	13.82	Low Income
PB	1.37	17.13	9.04	12.50	21.60	12.55	Low Income
PE	1.29	14.99	7.28	11.43	19.05	11.57	Low Income
PI	1.28	14.24	9.52	13.10	15.67	7.72	Low Income
PR	1.36	21.04	11.90	17.27	23.81	14.86	High Income
RJ	1.33	20.00	9.52	15.87	23.81	15.05	High Income
RN	1.24	15.53	7.37	11.90	18.45	13.25	Low Income
RO	1.25	16.21	10.39	14.07	17.86	10.30	Middle Income
RR	1.62	22.34	9.72	20.22	29.17	14.21	Middle Income
RS	1.38	20.40	10.84	15.16	23.81	15.12	High Income
SC	1.21	18.15	11.90	14.88	20.83	11.36	High Income
SE	1.70	19.61	9.40	16.67	26.19	13.52	Middle Income
SP	1.13	18.61	9.52	14.88	23.15	14.07	High Income
TO	1.30	17.32	9.38	14.58	19.05	13.09	Middle Income

Source: Elaborated by the authors with data from PNAD 2015.
Notes: Relative wage is the average hourly wage of teachers divided by the average hourly wage of other six occupations. Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Distrito Federal(DF), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí (PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Table A2: Logarithm of average hourly wages by occupation and state

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	2.87	2.86	2.19	2.00	2.04	2.06	2.86
AL	2.68	2.99	2.29	1.95	2.06	1.95	2.77
AM	2.97	3.05	2.46	2.02	1.91	2.07	2.78
AP	3.11	3.13	2.50	2.08	1.86	2.10	2.98
BA	2.81	3.10	2.37	1.92	1.78	2.02	2.73
CE	2.83	3.13	2.44	1.92	1.53	1.87	2.64
DF	3.42	3.47	2.97	2.36	2.24	2.36	3.33
ES	2.92	3.21	2.74	2.06	2.03	2.27	2.90
GO	2.97	3.02	2.64	2.14	2.24	2.26	2.96
MA	3.10	2.99	2.44	1.99	1.75	1.95	2.77
MG	2.92	3.20	2.63	2.04	2.00	2.17	2.91
MS	3.05	3.27	2.70	2.14	2.29	2.26	3.08
MT	2.93	3.22	2.56	2.16	2.37	2.36	2.94
PA	2.94	2.92	2.48	1.96	2.00	1.99	2.88
PB	2.80	3.24	2.45	1.95	1.91	2.01	2.84
PE	2.91	3.07	2.36	1.89	1.72	1.94	2.71
PI	2.92	3.00	2.29	1.88	1.65	1.92	2.66
PR	3.10	3.23	2.79	2.23	2.26	2.32	3.05
RJ	3.03	3.42	2.65	2.15	1.88	2.27	3.00
RN	2.97	3.13	2.51	2.00	1.77	1.92	2.74
RO	2.89	3.00	2.56	2.08	2.20	2.28	2.79
RR	3.04	3.26	2.66	2.03	1.80	2.11	3.11
RS	3.05	3.24	2.68	2.19	2.22	2.22	3.02
SC	3.01	3.18	2.73	2.30	2.33	2.34	2.90
SE	2.92	3.02	2.39	1.91	1.65	1.97	2.98
SP	3.25	3.30	2.83	2.22	2.22	2.35	2.92
TO	2.90	3.20	2.54	2.10	2.01	2.19	2.85

Source: Elaborated by the authors with data from PNAD 2015.

Table A3: Average years of schooling by occupation and state

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	9.82	10.82	10.69	8.71	4.26	6.79	14.40
AL	10.33	13.24	11.91	7.83	3.70	6.20	14.13
AM	11.48	13.48	11.24	9.09	4.39	8.40	14.20
AP	10.82	13.50	11.84	8.88	4.92	7.38	13.95
BA	11.09	13.51	11.27	8.80	3.52	7.19	13.69
CE	10.57	13.45	11.46	8.65	3.69	7.44	14.19
DF	12.76	14.18	11.88	9.55	5.61	7.90	14.47
ES	11.07	13.85	11.83	8.82	5.82	7.91	14.59
GO	11.69	13.43	11.47	8.77	5.85	7.73	14.45
MA	11.35	13.62	11.27	8.60	4.32	7.07	13.36
MG	11.57	13.97	11.54	8.62	5.02	7.41	14.11
MS	11.71	13.98	11.48	8.58	5.39	7.42	14.11
MT	11.08	13.79	11.37	9.14	5.71	7.54	14.39
PA	10.82	12.58	10.68	8.63	4.05	7.01	14.09
PB	11.88	13.82	10.81	8.51	3.41	6.29	14.32
PE	11.09	14.06	11.50	8.58	4.38	7.02	14.13
PI	11.03	13.35	11.37	8.21	4.05	6.13	14.27
PR	11.93	13.81	11.79	9.01	6.32	8.02	14.29
RJ	11.99	14.17	11.76	9.02	5.29	8.16	14.02
RN	10.47	13.57	11.13	8.85	3.67	7.09	14.03
RO	10.08	14.03	10.69	8.78	5.45	7.03	14.28
RR	10.83	12.95	12.05	9.34	4.83	7.23	14.16
RS	11.66	13.92	11.73	8.98	6.10	7.75	14.42
SC	11.60	13.69	11.57	9.20	6.57	8.18	14.37
SE	11.07	13.68	11.11	8.53	3.27	6.30	14.20
SP	12.48	14.27	12.02	9.18	6.46	8.38	14.09
TO	10.40	13.34	11.34	8.98	5.13	8.05	14.20

Source: Elaborated by the authors with data from PNAD 2015.

Table A4: Share of workers in each occupation by state

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	0.04	0.03	0.05	0.42	0.13	0.25	0.09
AL	0.04	0.04	0.07	0.41	0.12	0.25	0.07
AM	0.05	0.05	0.08	0.38	0.07	0.29	0.08
AP	0.04	0.03	0.07	0.44	0.07	0.25	0.10
BA	0.05	0.04	0.06	0.45	0.08	0.26	0.06
CE	0.05	0.03	0.06	0.44	0.05	0.31	0.07
DF	0.06	0.10	0.10	0.48	0.01	0.17	0.08
ES	0.07	0.06	0.07	0.35	0.12	0.29	0.05
GO	0.05	0.05	0.06	0.42	0.08	0.29	0.05
MA	0.04	0.04	0.06	0.37	0.14	0.27	0.09
MG	0.06	0.06	0.06	0.39	0.10	0.28	0.06
MS	0.06	0.06	0.05	0.38	0.12	0.27	0.06
MT	0.05	0.05	0.06	0.35	0.16	0.28	0.05
PA	0.03	0.04	0.05	0.44	0.11	0.27	0.06
PB	0.05	0.05	0.07	0.42	0.08	0.25	0.08
PE	0.05	0.06	0.07	0.46	0.05	0.25	0.06
PI	0.04	0.03	0.05	0.40	0.10	0.30	0.08
PR	0.08	0.07	0.07	0.36	0.07	0.29	0.06
RJ	0.05	0.08	0.08	0.46	0.01	0.25	0.06
RN	0.06	0.05	0.07	0.42	0.06	0.26	0.07
RO	0.06	0.04	0.05	0.35	0.16	0.28	0.06
RR	0.05	0.03	0.07	0.39	0.11	0.24	0.11
RS	0.06	0.08	0.08	0.39	0.06	0.28	0.05
SC	0.08	0.06	0.07	0.32	0.09	0.31	0.06
SE	0.04	0.04	0.05	0.43	0.14	0.25	0.06
SP	0.07	0.09	0.08	0.41	0.03	0.27	0.05
TO	0.05	0.04	0.05	0.33	0.21	0.23	0.09

Source: Elaborated by the authors with data from PNAD 2015.

Appendix B Calibrated $\tau's$ and $A's$ to 2015 and 2003

Table B1: Labor market frictions τ_{ir}^{w} - 2015

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	**Mean friction by state**	Income Level
AC	0.56	0.53	0.43	0.43	0.50	0.47	0.53	0.49	Middle Income
AL	0.56	0.58	0.49	0.45	0.54	0.47	0.55	0.52	Low Income
AM	0.56	0.54	0.48	0.42	0.45	0.45	0.50	0.49	Middle Income
AP	0.56	0.53	0.46	0.41	0.41	0.43	0.51	0.47	Middle Income
BA	0.56	0.57	0.48	0.42	0.44	0.47	0.52	0.50	Low Income
CE	0.56	0.58	0.50	0.41	0.35	0.42	0.50	0.47	Low Income
DF	0.56	0.54	0.50	0.43	0.47	0.45	0.52	0.50	High Income
ES	0.56	0.57	0.54	0.44	0.49	0.51	0.53	0.52	High Income
GO	0.56	0.54	0.51	0.45	0.53	0.50	0.53	0.52	Middle Income
MA	0.56	0.51	0.45	0.38	0.38	0.39	0.47	0.45	Low Income
MG	0.56	0.57	0.52	0.43	0.49	0.49	0.53	0.51	Middle Income
MS	0.56	0.56	0.51	0.43	0.53	0.48	0.53	0.52	High Income
MT	0.56	0.57	0.50	0.46	0.56	0.53	0.53	0.53	High Income
PA	0.56	0.53	0.49	0.41	0.48	0.44	0.52	0.49	Low Income
PB	0.56	0.60	0.51	0.43	0.49	0.47	0.54	0.51	Low Income
PE	0.56	0.56	0.47	0.39	0.40	0.43	0.50	0.47	Low Income
PI	0.56	0.54	0.45	0.38	0.38	0.42	0.48	0.46	Low Income
PR	0.56	0.55	0.52	0.45	0.52	0.49	0.52	0.52	High Income
RJ	0.56	0.59	0.50	0.44	0.44	0.49	0.53	0.51	High Income
RN	0.56	0.56	0.49	0.41	0.41	0.41	0.49	0.48	Low Income
RO	0.56	0.55	0.51	0.45	0.54	0.52	0.51	0.52	Middle Income
RR	0.56	0.56	0.50	0.41	0.41	0.45	0.54	0.49	Middle Income
RS	0.56	0.56	0.51	0.45	0.52	0.48	0.52	0.51	High Income
SC	0.56	0.56	0.52	0.48	0.54	0.51	0.51	0.53	High Income
SE	0.56	0.55	0.47	0.39	0.38	0.43	0.54	0.48	Middle Income
SP	0.56	0.54	0.50	0.42	0.48	0.47	0.48	0.49	High Income
TO	0.56	0.58	0.50	0.45	0.49	0.49	0.52	0.51	Middle Income
Mean by occupation	0.56	0.56	0.49	0.43	0.47	0.46	0.52		

Source: Search results. Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Distrito Federal(DF), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí (PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Table B2: Education market frictions τ_{ir}^{h} - 2015

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	**Mean friction by state**	Income Level
AC	0	0.09	1.83	−0.70	1.72	−0.35	−0.72	0.27	Middle Income
AL	0	−0.52	−0.12	−0.77	0.92	−0.48	−0.69	−0.24	Low Income
AM	0	−0.22	0.21	−0.55	9.96	−0.30	−0.47	1.23	Middle Income
AP	0	0.14	0.43	−0.67	11.87	−0.18	−0.70	1.56	Middle Income
BA	0	−0.46	0.29	−0.69	7.44	−0.34	−0.45	0.83	Low Income
CE	0	−0.25	0.32	−0.68	33.70	−0.29	−0.46	4.62	Low Income
DF	0	−0.62	−0.10	−0.60	170.40	0.95	−0.43	24.23	High Income
ES	0	−0.42	0.34	−0.35	4.88	−0.31	0.12	0.61	High Income
GO	0	−0.16	0.36	−0.68	3.88	−0.48	−0.26	0.38	Middle Income
MA	0	−0.03	0.59	−0.57	4.48	−0.15	−0.60	0.53	Low Income
MG	0	−0.51	0.44	−0.49	6.16	−0.24	−0.16	0.74	Middle Income
MS	0	−0.32	1.11	−0.45	2.91	−0.15	−0.26	0.41	High Income
MT	0	−0.38	0.70	−0.59	0.62	−0.53	−0.27	−0.06	High Income
PA	0	−0.27	0.12	−0.77	1.93	−0.47	−0.62	−0.01	Low Income
PB	0	−0.57	0.19	−0.62	6.90	−0.19	−0.60	0.73	Low Income
PE	0	−0.43	0.56	−0.54	26.38	0.13	−0.09	3.72	Low Income
PI	0	0.17	1.17	−0.56	9.07	−0.28	−0.54	1.29	Low Income
PR	0	−0.25	0.58	−0.31	9.78	−0.05	0.11	1.41	High Income
RJ	0	−0.76	0.03	−0.70	129.70	−0.34	−0.33	18.23	High Income
RN	0	−0.24	0.51	−0.52	20.34	0.30	−0.27	2.88	Low Income
RO	0	0.10	0.79	−0.54	1.11	−0.49	−0.22	0.11	Middle Income
RR	0	0.17	0.22	−0.52	8.05	−0.04	−0.75	1.02	Middle Income
RS	0	−0.55	0.20	−0.53	11.59	−0.17	0.01	1.51	High Income
SC	0	−0.13	0.79	−0.32	6.14	−0.21	0.43	0.96	High Income
SE	0	−0.36	0.36	−0.71	3.96	−0.32	−0.63	0.33	Middle Income
SP	0	−0.49	0.46	−0.39	51.65	0.08	0.61	7.42	High Income
TO	0	−0.29	0.79	−0.54	0.93	−0.27	−0.60	0.00	Middle Income
Mean by occupation	0	−0.28	0.49	−0.57	20.24	−0.19	−0.33		

Source: Search results.

Table B3: Total productivity factors - 2015

State	A_r	State	A_r	State	A_r
AC	24.68	MA	26.59	RJ	38.49
AL	26.75	MG	41.38	RN	34.65
AM	30.76	MS	40.28	RO	38.19
AP	25	MT	39.83	RR	24.22
BA	33.07	PA	29.81	RS	45.89
CE	30.61	PB	28.79	SC	49.75
DF	35.99	PE	39.56	SE	30.90
ES	46.65	PI	27.93	SP	51.25
GO	40.42	PR	46.44	TO	28.03

Source: Search results.

Notes: Recall that in our model TFP is equal across occupations. The average of TFP is 35.4.

Table B4: Labor market frictions τ_{ir}^{w} - 2003

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	**Mean friction by state**	Income Level
AC	−0.05	−0.06	−0.28	−0.72	−0.98	−0.68	−0.36	−0.45	Middle Income
AL	−0.05	0.10	−0.34	−0.76	−0.94	−0.50	−0.38	−0.41	Low Income
AM	−0.05	−0.12	−0.65	−0.82	−0.92	−0.75	−0.47	−0.54	Middle Income
AP	−0.05	−0.15	−0.36	−0.68	−0.74	−0.45	−0.31	−0.39	Middle Income
BA	−0.05	−0.08	−0.22	−0.69	−0.77	−0.54	−0.42	−0.40	Low Income
CE	−0.05	−0.06	−0.22	−0.67	−1.00	−0.68	−0.38	−0.44	Low Income
DF	−0.05	−0.02	−0.16	−0.50	−0.22	−0.45	−0.16	−0.22	High Income
ES	−0.05	−0.15	−0.19	−0.58	−0.53	−0.50	−0.28	−0.33	High Income
GO	−0.05	−0.05	−0.15	−0.57	−0.38	−0.48	−0.30	−0.28	Middle Income
MA	−0.05	−0.01	−0.39	−0.63	−0.61	−0.54	−0.28	−0.36	Low Income
MG	−0.05	−0.06	−0.20	−0.61	−0.58	−0.45	−0.20	−0.31	Middle Income
MS	−0.05	0.02	−0.18	−0.50	−0.14	−0.52	−0.28	−0.24	High Income
MT	−0.05	−0.06	−0.30	−0.64	−0.41	−0.47	−0.36	−0.33	High Income
PA	−0.05	−0.02	−0.25	−0.64	−0.38	−0.54	−0.25	−0.30	Low Income
PB	−0.05	−0.06	−0.25	−0.74	−0.87	−0.76	−0.34	−0.44	Low Income
PE	−0.05	−0.07	−0.29	−0.67	−0.93	−0.64	−0.41	−0.44	Low Income
PI	−0.05	0.02	−0.39	−0.74	−1.00	−0.93	−0.50	−0.51	Low Income
PR	−0.05	−0.10	−0.21	−0.54	−0.29	−0.41	−0.22	−0.26	High Income
RJ	−0.05	−0.04	−0.19	−0.53	−0.92	−0.38	−0.13	−0.32	High Income
RN	−0.05	0.01	−0.22	−0.61	−1.00	−0.53	−0.23	−0.38	Low Income
RO	−0.05	0.01	−0.15	−0.58	−0.18	−0.40	−0.16	−0.22	Middle Income
RR	−0.05	−0.18	−0.27	−0.60	−0.61	−0.61	−0.29	−0.37	Middle Income
RS	−0.05	−0.04	−0.18	−0.50	−0.34	−0.44	−0.19	−0.25	High Income
SC	−0.05	−0.05	−0.10	−0.36	−0.14	−0.31	−0.19	−0.17	High Income
SE	−0.05	0.04	−0.27	−0.61	−0.78	−0.57	−0.41	−0.38	Middle Income
SP	−0.05	−0.11	−0.19	−0.49	−0.45	−0.39	−0.25	−0.28	High Income
TO	−0.05	0.01	−0.18	−0.72	−0.56	−0.46	−0.38	−0.34	Middle Income
Mean by occupation	−0.05	−0.05	−0.25	−0.62	−0.62	−0.53	−0.30		

Source: Search results.

Table B5: Education market frictions τ_{ir}^{h} - 2003

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	**Mean friction by state**	Income Level
AC	0	0.49	2.93	0.23	155.73	3.97	0.03	23.34	Middle Income
AL	0	−0.19	1.75	0.73	14.16	1.50	−0.15	2.54	Low Income
AM	0	0.71	3.83	0.24	205.14	1.32	0.80	30.29	Middle Income
AP	0	−0.00	0.49	−0.15	77.86	−0.16	−0.68	11.05	Middle Income
BA	0	0.33	1.24	0.34	16.33	1.66	0.86	2.97	Low Income
CE	0	−0.11	0.82	−0.17	51.68	0.99	0.13	7.62	Low Income
DF	0	−0.58	0.32	−0.02	253.03	3.08	−0.20	36.52	High Income
ES	0	0.37	1.60	0.46	12.16	1.39	0.92	2.41	High Income
GO	0	0.27	1.01	0.19	10.03	1.01	1.14	1.95	Middle Income
MA	0	−0.06	2.00	−0.05	5.79	0.68	−0.40	1.14	Low Income
MG	0	0.05	1.46	0.39	23.15	0.93	0.26	3.75	Middle Income
MS	0	0.78	3.69	0.35	7.17	2.07	1.26	2.19	High Income
MT	0	0.31	3.40	1.29	5.16	1.41	1.18	1.82	High Income
PA	0	−0.34	0.82	−0.26	28.28	0.52	0.03	4.15	Low Income
PB	0	0.38	1.39	0.56	28.44	3.18	−0.16	4.83	Low Income
PE	0	−0.19	0.93	0.00	54.79	2.18	0.45	8.31	Low Income
PI	0	−0.13	1.17	0.29	27.12	5.69	−0.22	4.84	Low Income
PR	0	0.21	1.20	0.55	14.87	1.27	0.83	2.70	High Income
RJ	0	−0.63	0.28	−0.33	360.08	0.31	−0.18	51.36	High Income
RN	0	−0.45	1.66	−0.32	28.81	0.43	−0.55	4.23	Low Income
RO	0	1.20	1.11	0.00	7.43	0.29	−0.28	1.39	Middle Income
RR	0	0.13	4.32	0.38	50.15	3.63	−0.41	8.32	Middle Income
RS	0	−0.08	0.86	0.39	21.53	1.06	0.78	3.51	High Income
SC	0	0.78	1.27	0.56	8.81	0.76	1.24	1.92	High Income
SE	0	−0.19	1.01	−0.13	16.92	0.81	−0.21	2.60	Middle Income
SP	0	−0.24	1.11	0.20	81.56	1.02	1.25	12.13	High Income
TO	0	0.30	1.87	1.62	13.17	2.16	0.37	2.79	Middle Income
Mean by occupation	0	0.12	1.61	0.27	58.49	1.60	0.30		

Source: Search results.

Table B6: Total productivity factors - 2003

State	A_r	State	A_r	State	A_r
AC	25	MA	19.65	RJ	28.93
AL	20.95	MG	31.16	RN	18.05
AM	31.24	MS	36.08	RO	24.47
AP	15.60	MT	34.85	RR	19.02
BA	30.05	PA	27.16	RS	37.54
CE	24.95	PB	21.09	SC	39.91
DF	26.92	PE	26.77	SE	19.57
ES	35.74	PI	17.88	SP	40.71
GO	36.73	PR	36.03	TO	25.16

Source: Search results.
Notes: Recall that in our model TFP is equal across occupations. The average of TFP is 27.82.

Appendix C Public and private spending on education

Using table 49 from the Family Budget Survey (POF)[20] we estimate private expenditures on education in Brazil for 2003, 2009 and 2018. In 2003 these expenditures were around R$ 32.4 billion, in 2009 around R$ 40.5 billion and in 2018 about R$ 145.4 billion. As a percentage of GDP, private spending on education is 1.8, 1.2 and 2.0, respectively, on average 1.7.

[20]Details about the POF can be seen on the Brazilian Institute of Geography and Statistics (IBGE) website.

Public spending on education as a percentage of GDP is provided by the National Institute of Educational Studies and Research Anísio Teixeira (INEP). In 2003 it was 4.6 and in 2015, 6.2. So we have that public and private spending on education, as a share of GDP, in Brazil, is 6.4 in 2003 and 7.9 in 2015.

Appendix D Migration between states

Using PNAD microdata from 2015, we found that on average 20.36% of workers migrated to another state or country. In Table D1 we show the share of workers who migrated to another state or country by occupation. As can be seen, only a small portion of workers migrated from their home state to another. Therefore, it makes sense to assume in the theoretical model that workers do not migrate.

Table D1: Share of workers who migrated and did not migrate to another state or country

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
Migrated	0.21	0.18	0.19	0.22	0.20	0.22	0.15
Not Migrated	0.79	0.82	0.81	0.78	0.80	0.78	0.85

Source: Elaborated by the authors with data from PNAD 2015.