104: Introduction to Analysis

Nir Elber

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Theme 1
A Chapter Name

1.1 A segment or subsection 1

With posets, we are interested in admission of combinatorial equivalence w.r.t. the structure of the object.

Definition 1.1 (Poset). A *poset (partially ordered set)* is a set P with a binary relation $\leq$ such that $\leq$ is

1. Reflexive (for sake of completeness, $x \leq x$)
2. Transitive ($x \leq y$ and $y \leq z \implies x \leq z$)
3. Antisymmetric ($x \leq y$ and $y \leq x \implies x = y$)

Remark 1.2. Note on notation: we define $[n] = \{1, \ldots, n\}$

Example 1.3 (Boolean Algebra/ Boolean Lattice B_n). $\mathcal{P}([n])$ with a relation defined by $S \leq T$ iff $S \subseteq T$ (ordered by inclusion) makes a poset.

Example 1.4 (D_n). Given integer n, D_n is defined on all divisors of n with relation

$$i \leq j \text{ iff } i|j$$

Example 1.5 ($L_n(q)$). All subspaces of $U_n(q)$ (n-dimensional vector spaces over $\mathbb{F}(q)$) ordered under inclusion forms a poset.

Example 1.6 (Π_n). All partitions of $[n]$ ordered by refinement, ie, $\pi \leq \sigma$ if every block of π is contained in a block of σ, forms a poset.

Definition 1.7 (Cover Relation). Denoted by $\prec$; "x is covered by y" ie $x \prec y$ if $x < y$ and there does not exist a z such that $x < z < y$.

Another point of interest is the *Hasse Diagram* – to be defined properly later on – with which we graph the relations in a poset. From this, we derived colloquial definitions of *chain* and *anti-chain*; respectively, a subset which is totally ordered vs. a subset in which no two elements are comparable. Respectively, the Hasse diagrams for these end up looking like a collection of vertices along a line and a collection of vertices skewed about, with no edges between them.

List of Definitions