Topology Optimization (TopOpt)

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Outline

Physical Problem

How should material(s) be distributed within a prescribed domain Ω in order to obtain the best structural performance?

General problem

More formally: find the material distribution ρ that minimizes an objective F, subject to a volume constraint $G_0 \leq 0$ and (possibly) other constraints G_i.

$$\begin{aligned} \arg\min_{\rho} \quad & F = F(u(\rho), \rho) = \int_{\Omega} f(u(\rho), \rho) \mathrm{d}V \\ \text{s.t.} \quad & \int_{\Omega} \rho \mathrm{d}V - V_0 \leq 0, \\ & G_i(u(\rho), \rho) \leq 0, \end{aligned} \tag{1}$$

We would like ρ to take on 0 or 1 at each location. In G_i, we impose the physics we are interested in.

Outline

Approaches

Method: evaluate objective function (FEM) $\rightarrow$ make informed update to ρ $\rightarrow$ (ensure constraints $\rightarrow$) repeat.

- **Discrete**: Primarily, evolutionary algorithms (ESO/ BESO/ GESO) – some combination of addition/ removal of 1's based on the change in objective.
- **Density-based**: move ρ mass around based on some gradient, but $0 \leq \rho \leq 1$ is now continuous, not discrete
- **Shape optimization**: test out introduction of holes to ρ (Topological Derivative), or construct the material distribution's boundary (Level Set)

SIMP

Loosening to $0 \leq \rho \leq 1$ allows e.g. $\rho(e_i) = 0.5 \to$ we allow for (possibly) large "gray areas". How do we combat this?

- Solid Isotropic Material with Penalization (SIMP) approach.
- Use a power law $\rho^p E_0$ to relate density and material property. $p > 1$ will penalize intermediate values.
  - *One-Field SIMP*: Most widely-used is *sensitivity filtering*, wherein sensitivity[1] values are computed as weighted average with a radius.
  - *Two-Field SIMP*: Most widely-used is *density filtering*, where density values are computed as averages within a neighborhood. Can be written as the addt. constraint

$$-r^2 \Delta \overline{\rho} + \overline{\rho} = \rho \tag{2}$$

for a radius r and the average $\overline{\rho}$ within r.

[1] This is $\partial c / \partial \rho$, ie, the sensitivity of compliance to material density.

General issues to keep in mind

- *Mesh-independence*: Refining the mesh should lead to a higher resolution solution, not possibly a qualitatively different one.
- *Existence, Uniqueness, Convexity*: We have none of these in general.
- *Ensuring Convergence, Local Minima*: Often, significant "parameter engineering" is required. Addt., imposing certain sorts of constraints/ properties helps to ensure convergence.
- *Checkerboarding*: Classical issue, but now overcome with the penalization techniques.

Outline

1. Problem Formulation
2. Density-based TopOpt
3. Examples

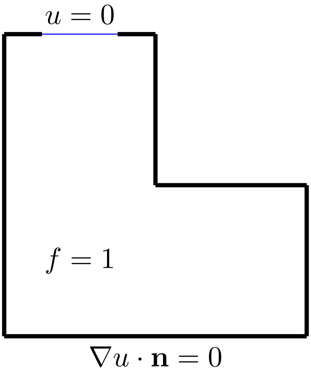
Thermal Compliance with EFEM

We would like to solve the thermal compliance problem

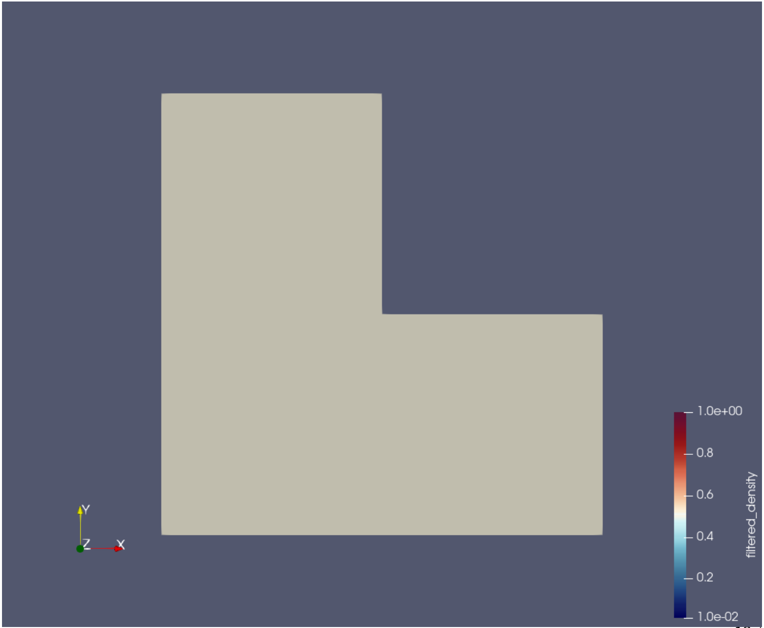
$$
\begin{aligned}
\min_{\rho} \quad & \int_{\Omega} f \cdot u \, \mathrm{d}x \\
\text{s.t.} \quad & \\
& \begin{cases}
-\nabla \cdot (r(\tilde{\rho}) \nabla u) = f & \text{in } \Omega \text{ and BCs,} \\
-\epsilon^2 \Delta \tilde{\rho} + \tilde{\rho} = \rho & \text{in } \Omega \text{ and Neumann BCs,} \\
0 \leq \rho \leq 1 & \text{in } \Omega, \\
\int_{\Omega} \rho \, \mathrm{d}x = \theta \cdot \mathrm{vol}(\Omega), &
\end{cases}
\end{aligned} \tag{3}
$$

Heated L-shaped plate with sink.

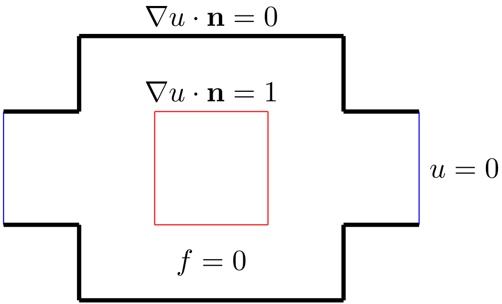
Figure: L-shaped Ω with a sink.



Heated L-shaped plate with sink.



Heatsink for square source.



Heatsink for square source.