CORK INSTITUTE OF TECHNOLOGY
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Computing@CIT

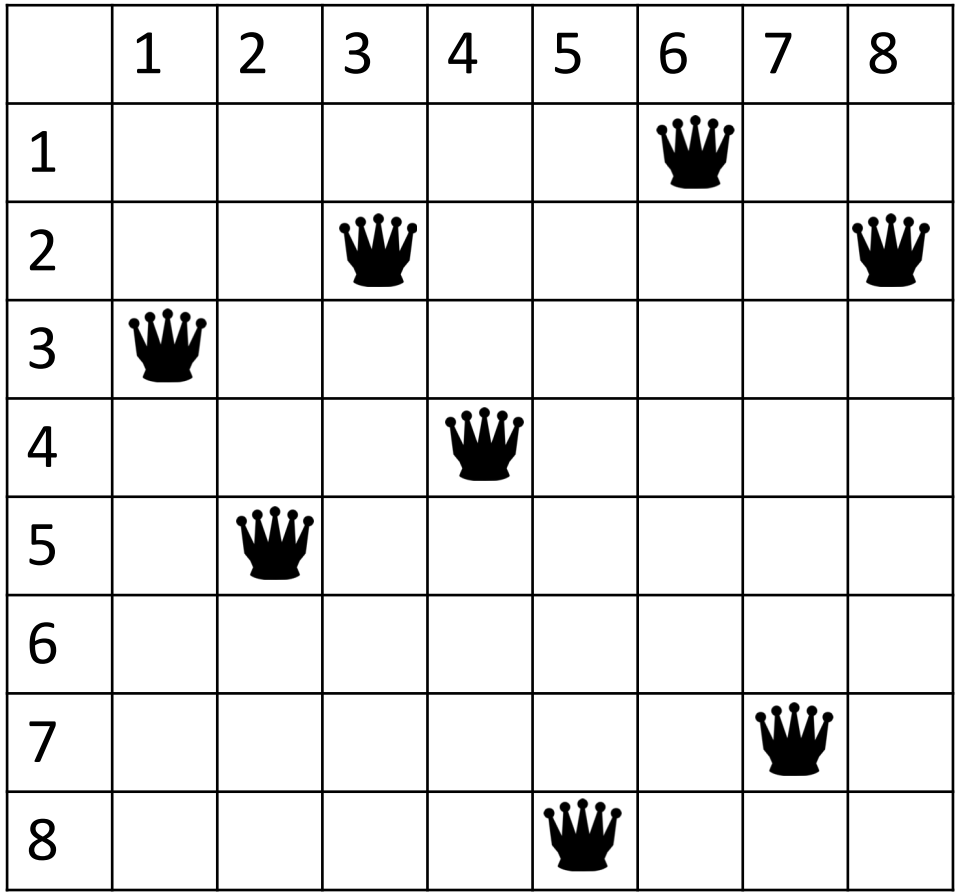
Metaheuristic Optimization

Genetic Algorithms

Dr. Diarmuid Grimes

Importance of modelling choices

Problem Representation: N-Queens



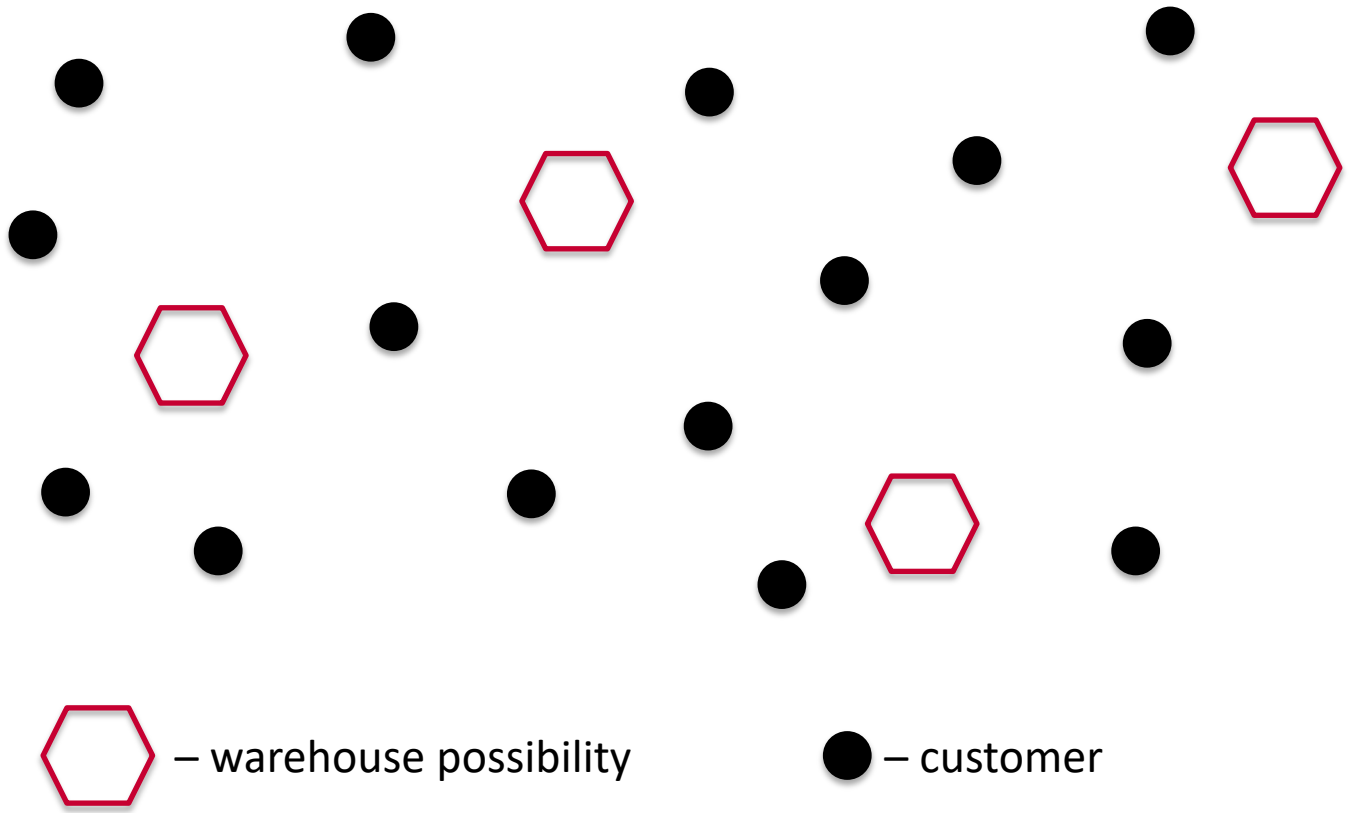
n variables

- Domain:
  - n^2?
  - n?

Both valid, what's the difference?

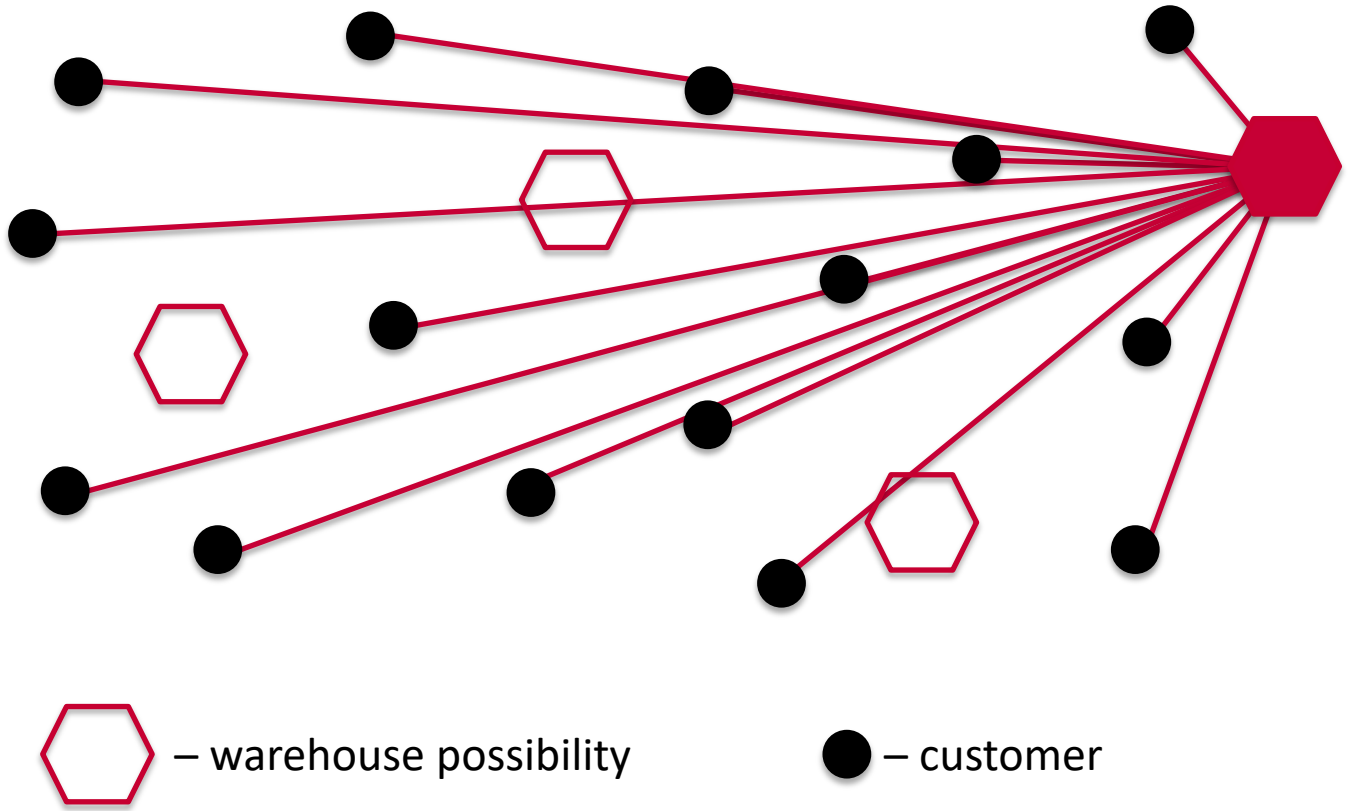
- d^n
- For 8-Queens there are 16,777,216 configurations.
- Or 281,474,976,710,656 configurations with n^2

Warehouse allocation problem



- Task: where to place warehouses?
- Cost for opening warehouse at position w: f_w
- Cost for transportation from w to c: $t_{w,c}$

Warehouse allocation problem



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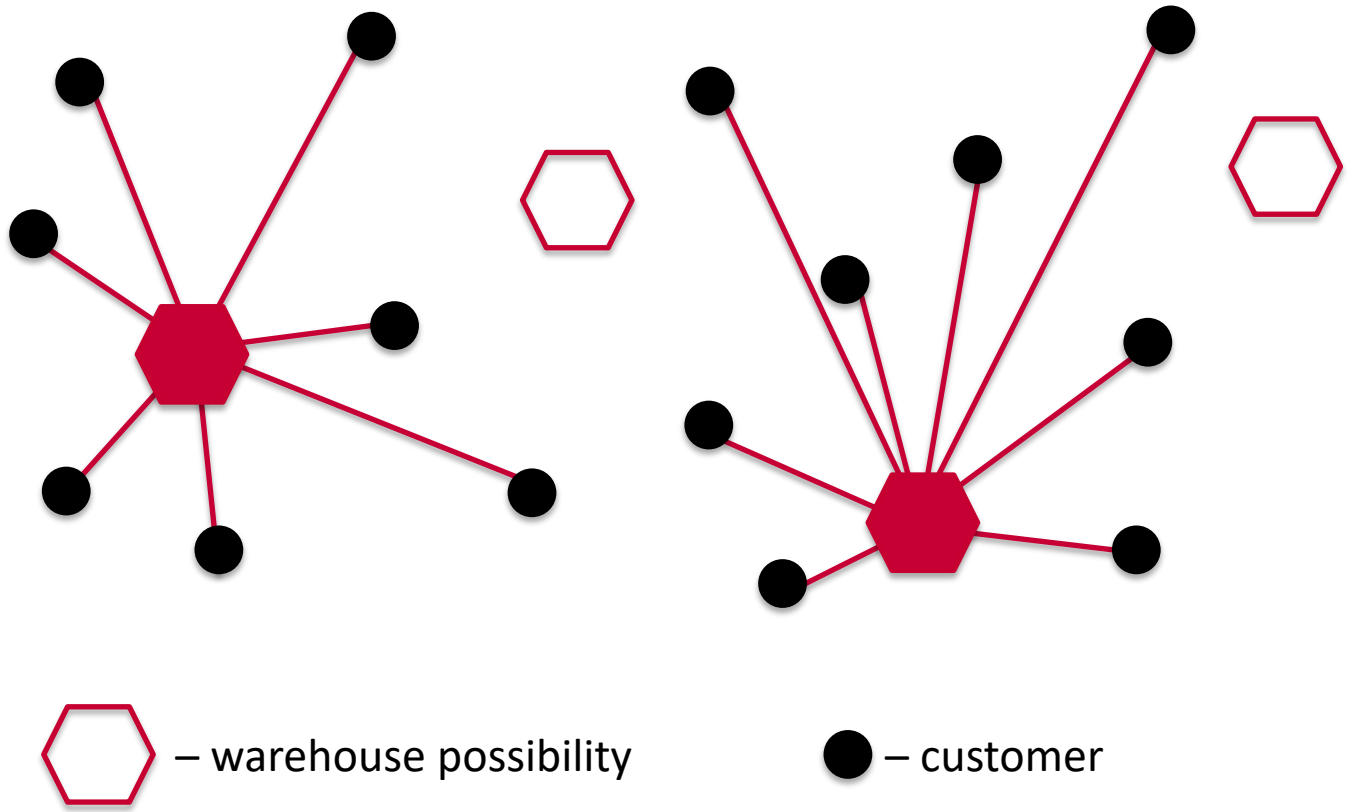
Warehouse allocation problem



- Task: where to place warehouses?
- Cost for opening warehouse at position w: f_w
- Cost for transportation from w to c: $t_{w,c}$

Importance of modeling choices!

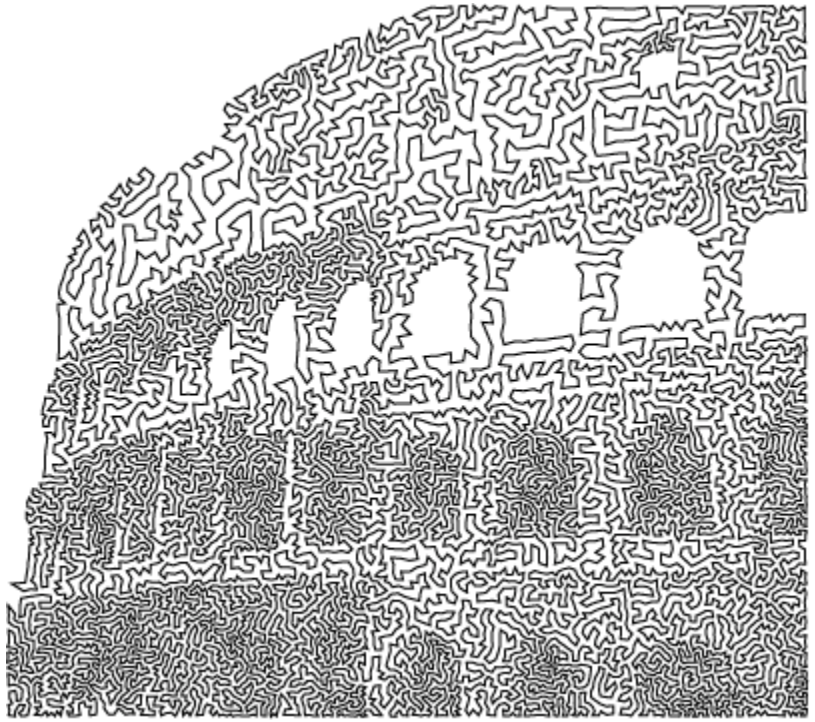
- Warehouse location problem: where to place warehouses? (input: f_w , $t_{w,c}$)

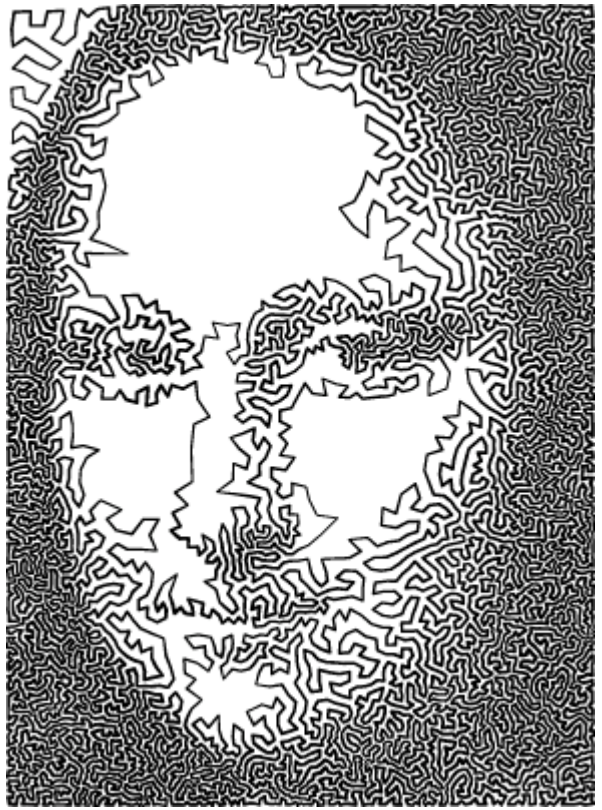
- Decision Variables:
  - $o_w \in \{0, 1\}$: whether warehouse w is open
  - $\alpha[c] \in \{1, \ldots, W\}$: the warehouse assigned to customer c

- Objective $\min \sum_{w \in W} f_w o_w + \sum_{c \in C} t_{a[c],c}$

- Key observations:
  - Once the warehouse locations have been chosen, the problem is easy
  - It's enough to assign customers to warehouses minimizing the transportation costs
  - **Note the difference between testing different values for the o_w variables and the $\alpha[c]$ variables**

TSP Art

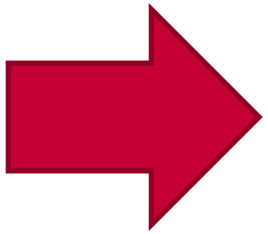


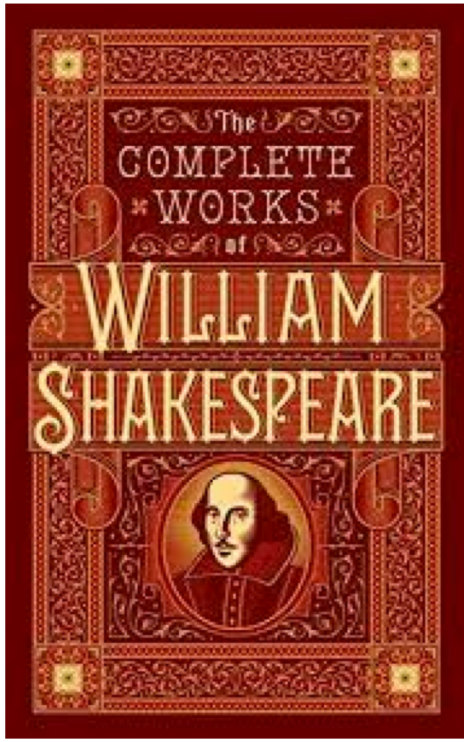


http://www.math.uwaterloo.ca/tsp/data/art/

Genetic Algorithms

Infinite Monkey Theorem





Given enough time, a hypothetical chimpanzee typing at random would, as part of its output, almost surely produce all of Shakespeare's plays.

Practical Theorem

- The probability of a monkey exactly typing a complete work such as Shakespeare's Hamlet is so tiny that the chance of it occurring during a period of time of the order of the age of the universe is minuscule, but not zero
- Example: typing 'banana'
  - Assume only 50 keys
  - Prob. Of each letter to be typed right is 1/50
  - Prob. Of that 'banana' is typed right is (1/50)^6 < 1 in 15 billion
  - Expected number of trials to write 'banana' = 15 billion

Monkeys, typewriters and Evolution

- Evolution being a randomized generate-and-test process present some similarities with the typewriter process

- However, natural selection can produce unlikely results. Could a monkey accidentally type the Hamlet line “To be or Not To Be”? The chances are virtually zero.

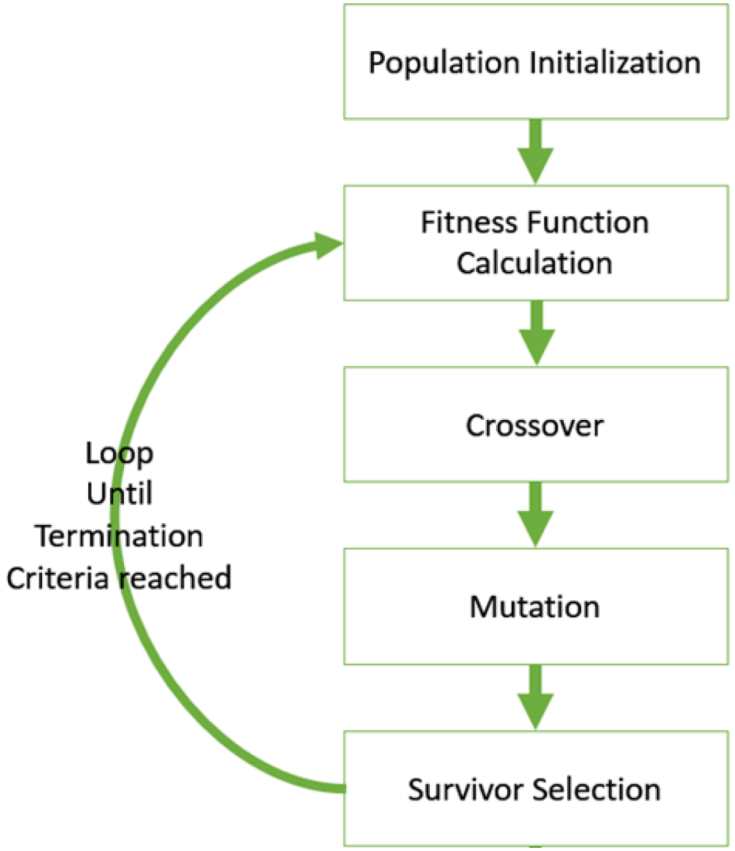
- How does an evolutionary algorithm work?

Evolutionary Computation

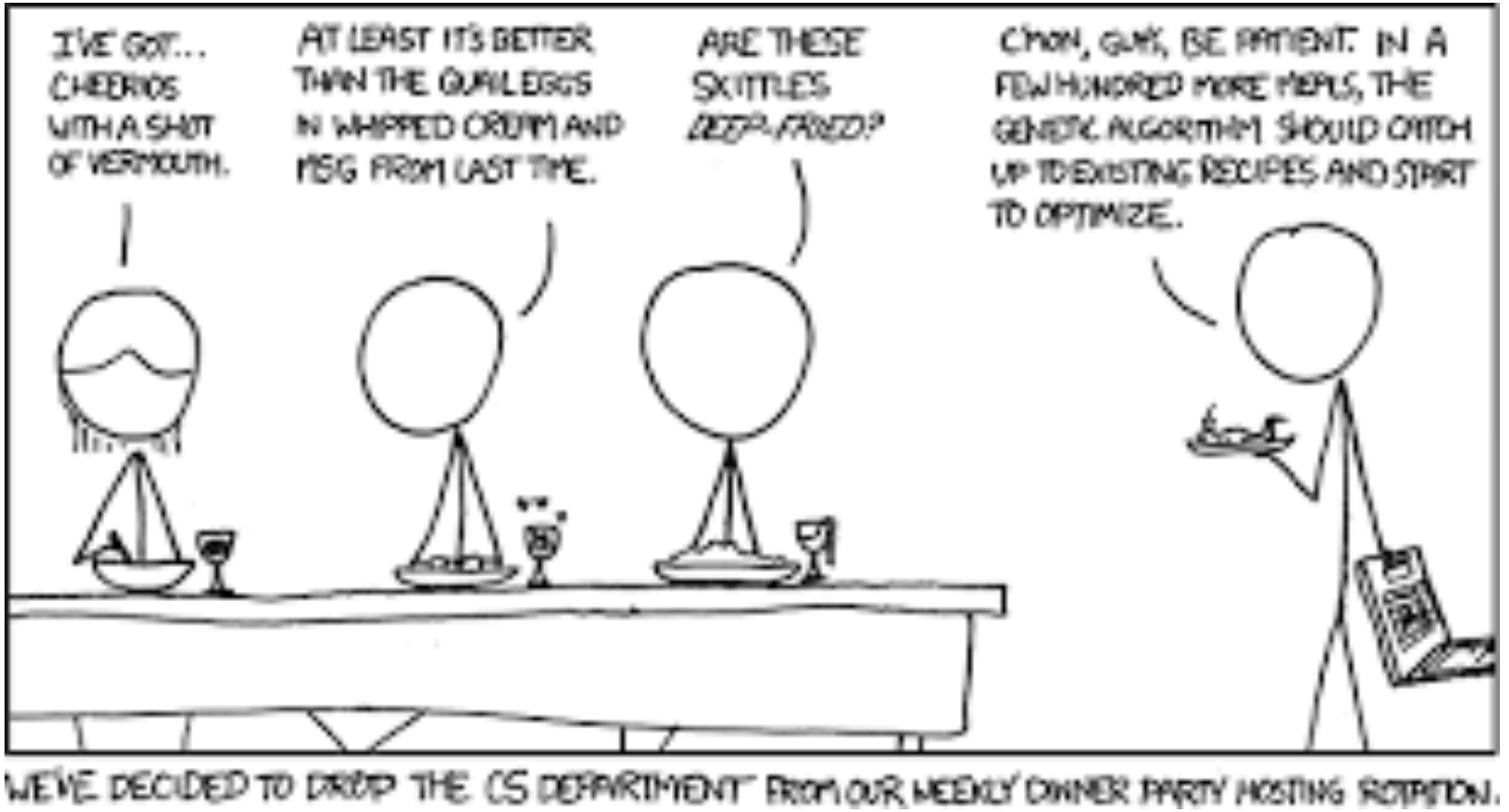
- There's a variety of classes of Evolutionary Algorithms

- Today, we will only look at the so-called **Simple Genetic Algorithm**

Evolutionary computation



Evolutionary computation



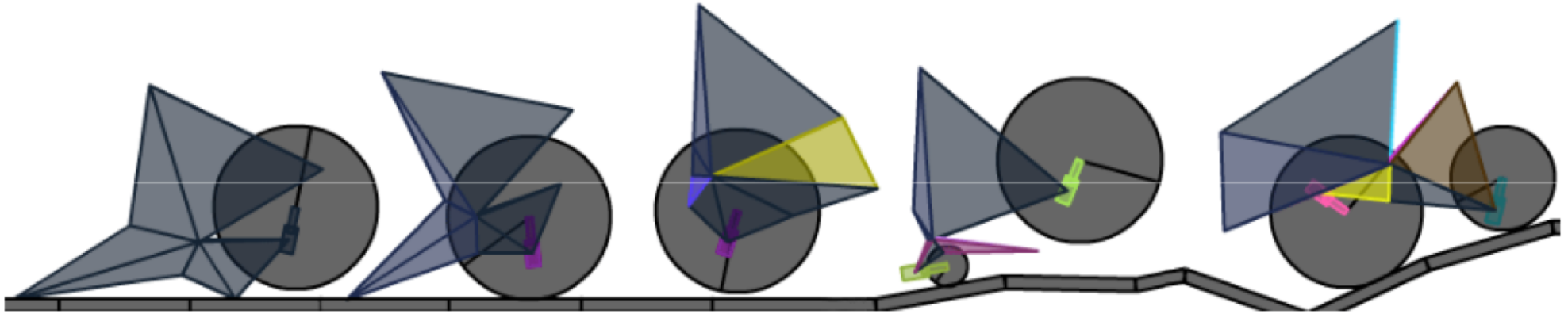
https://xkcd.com/720/

Characteristics of GAs

Flexible:	Applicable to different problems
Robust:	Can deal with noise and uncertainty
Adaptive:	Can deal with dynamic environments
Autonomous:	Without human intervention
Decentralized:	Without a central authority

Structure Design: Car Evolution

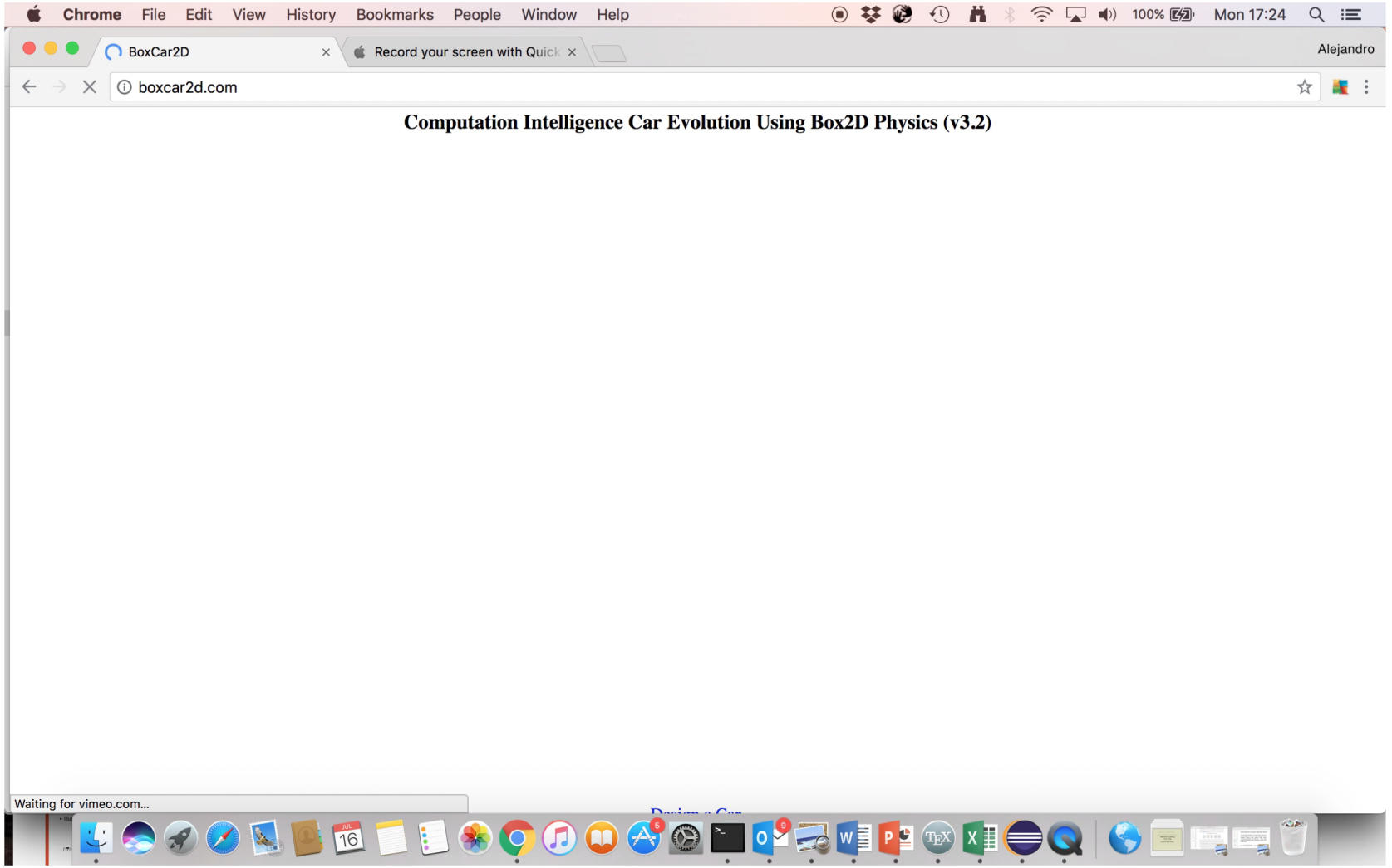
Problem: Design a car using polygons and wheels able to run on a terrain



Structure Design: Car Evolution

- A **candidate solution** is a set of polygons connected in a central point, and wheels attached to them.
- **Representation**: for each polygon there is a real vector (a “gene”) describing the shape of the polygon. For each wheel there is a value specifying its radius, location, and its centre
- **Fitness**: how far the car goes on the “terrain” when run
- **Mutation** and **Crossover**: variations on mutation and crossover for real vectors

Car Evolution Using Box2D



http://boxcar2d.com

Designing a car using Genetic Algorithms

Evolutionary Art: Mona Lisa Evolution

Problem: paint a replica of the Mona Lisa using only 50 semi transparent polygons

https://www.youtube.com/watch?v=S1ZPSbImvFE

Evolutionary Art: Mona Lisa Evolution

- A candidate solution is a set of 50 transparent polygons of various colors on the canvas
- Representation: for each polygon there is a real vector representing the shape, the location and the colour of the polygon
- Fitness (to minimize): sum of the differences in colour components (RGB) on each pixel between the phenotype and the target image
- Standard crossover and mutation on real vectors

https://www.youtube.com/watch?v=S1ZPSbImvFE

Application Areas

- Planning
  - Routing, Scheduling, Packing
- Design
  - Electronic Circuits, Neural Networks, Structure Design
- Simulation
  - Model economic interactions of competing firms in a market
- Identification
  - Fit a function to medical data to predict future value
- Control
  - Design a controller for gas turbine engine, design control system for mobile robots
- Classification
  - Game Playing, Diagnosis of heart disease, Detecting SPAM

What is a Genetic Algorithm (GA)?

- It is a search based on the mechanics of natural selection and genetics
- Often used to solve difficult optimization problems

Evolution

- Evolution can be seen as an optimization process where living creatures constantly adapt to their environment
  - The stronger survive and propagate their traits to future generations
  - The weaker die and their traits tend to disappear

- Darwin’s survival of the fittest

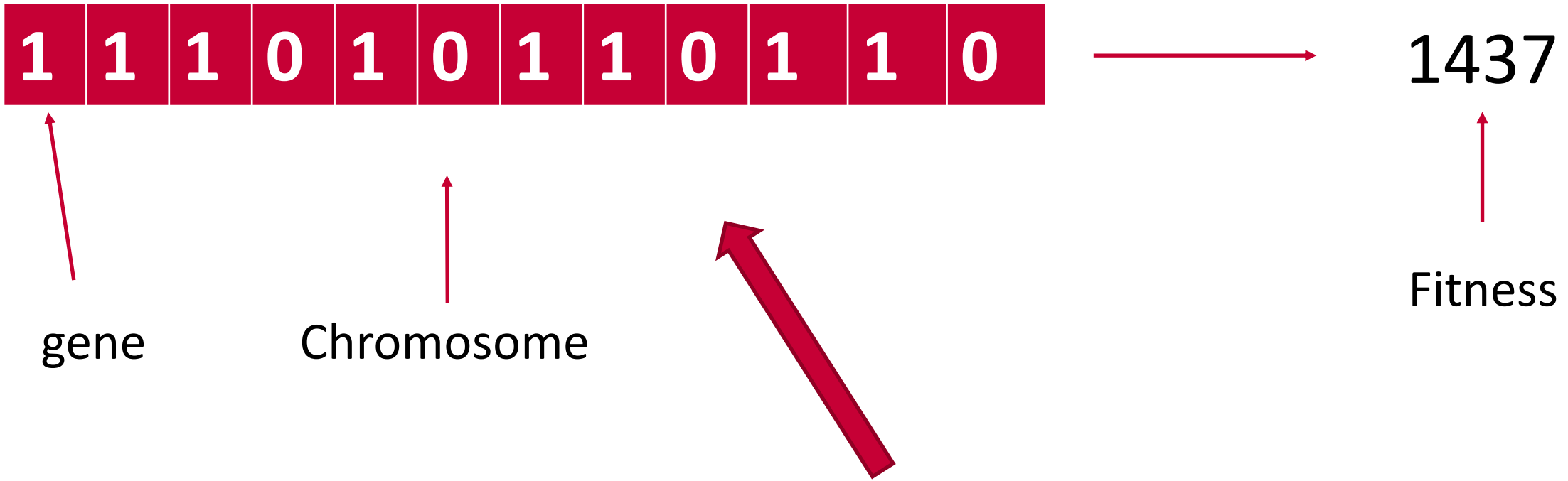
Two essential components

- Selection (choose *parent* solutions from the population)
- Variation (combine *parent* solutions to create *children* solutions which can be mutated further)
  - Recombination (or crossover)
  - Mutation

Encoding

- Problem solutions have to be encoded in some sort of structure (traditionally binary strings, but there are many other possibilities)
- We need a way of quantifying the value (fitness) of the given structure (solution)
  - Or given two solutions decide which is the better one
- The fitness value is obtained through an objective function

Terminology



Terminology

$$(x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_3) \wedge (x_5 \vee \overline{x_1})$$

Number of unsatisfied clauses

0	1	0	0	0

$x_1 = \mathbf{0}, x_2 = \mathbf{1}, x_3 = \mathbf{0}, x_4 = \mathbf{0}, x_5 = \mathbf{0}$ → Cost = 1

1	1	1	0	0

$x_1 = \mathbf{1}, x_2 = \mathbf{1}, x_3 = \mathbf{1}, x_4 = \mathbf{0}, x_5 = \mathbf{0}$ → Cost = 1

Gene/ Variable

Chromosome/ Assignment

Fitness

A Simple GA

Generate a population with N random individuals.

WHILE not happy enough with the solution quality DO

- Compute the fitness of every individual in the population
- Select better individuals
- Do crossover between pairs of selected individuals with probability P_c
- Mutate each gene with probability P_m

END WHILE

Large Random Component:

- **Initial Population**
- **Selection**
- **Crossover**
- **Mutation**

One iteration of the GA

	Generation at time t
1	$Sol_{1,t}$
2	$Sol_{2,t}$
N-1	$Sol_{N-1,t}$
N	$Sol_{N,t}$

	Generation at time t+1
1	$Sol_{1,t+1}$
2	$Sol_{2,t+1}$
N-1	$Sol_{N-1,t+1}$
N	$Sol_{N,t+1}$

Selection, Recombination, and Mutation

Selection

- Simulates survival of the fittest
  - Bias based on fitness of chromosomes
- The stronger have a better opportunity to reproduce
- The weaker tend to disappear from the population
- Various methods to implement this operator (roulette wheel, ranking, tournament, and many others)

Example: binary tournament selection

- Pick a pair of individuals from the population
- The winner survives, the loser dies
- Repeat K times (K is the number of parents)

Example: Binary Tournament For a Maximisation Problem

individual	Fitness
A	8
B	2
C	4
D	5

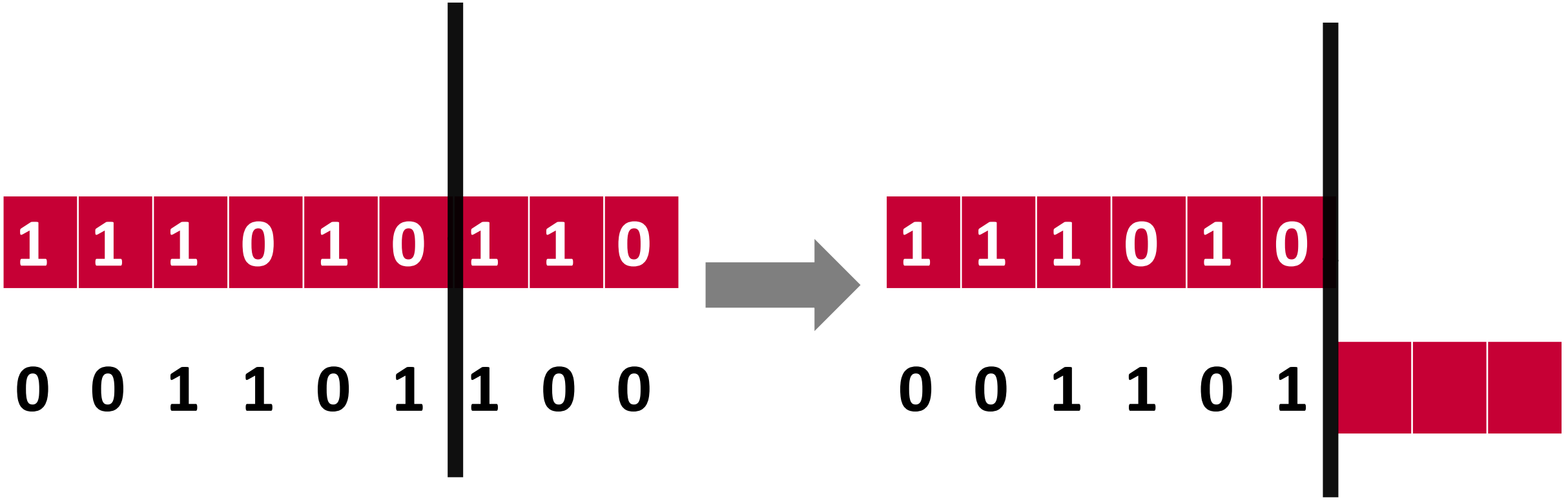
4 tournaments:	(A, D),	(B, C),	(A, B),	(C, D)
4 winners:	A,	C,	A,	D

Crossover

- Recombine 2 individuals (father and mother) to obtain two new individuals (the children)

- In the previous example, **A** recombines with **C**, and **A** recombines with **D**

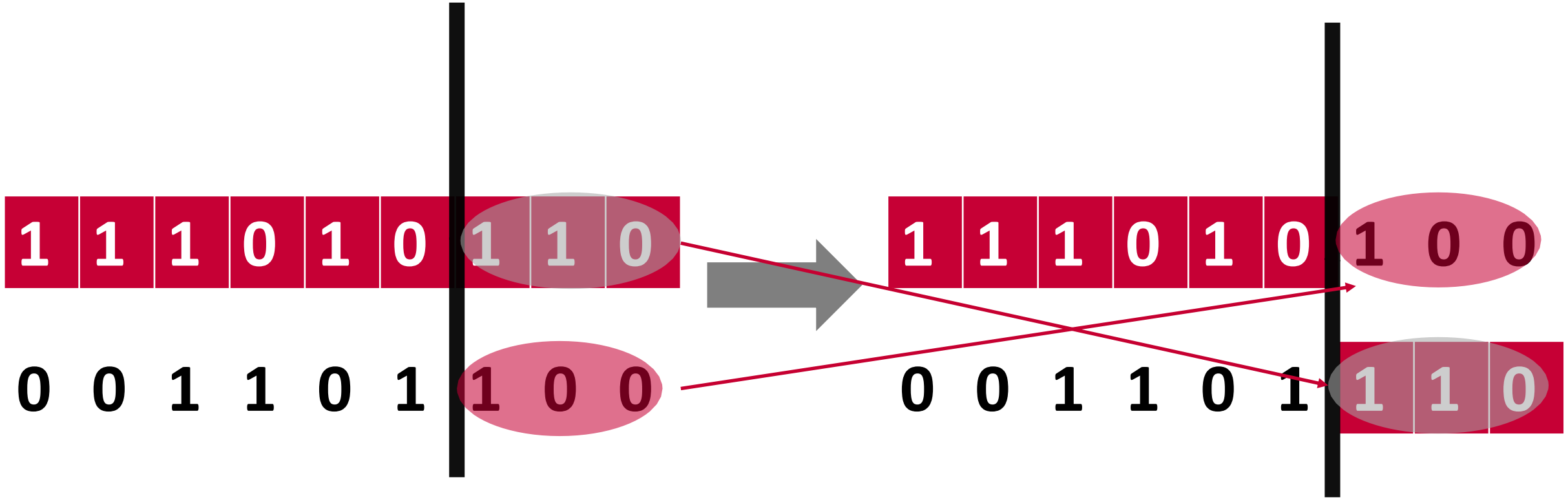
Example of crossover



Before recombination

After recombination

Example of crossover



Before recombination

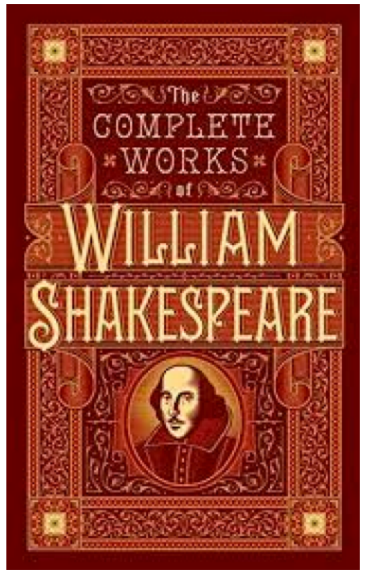
After recombination

Mutation

- With probability P_m, flip a gene from 0 to 1, or from 1 to 0
- In traditional GAs, this operator is typically used with a low probability

Gene 7 was mutated

Demo



Target → “To be or not to be”

Fitness → percentage of correct symbols

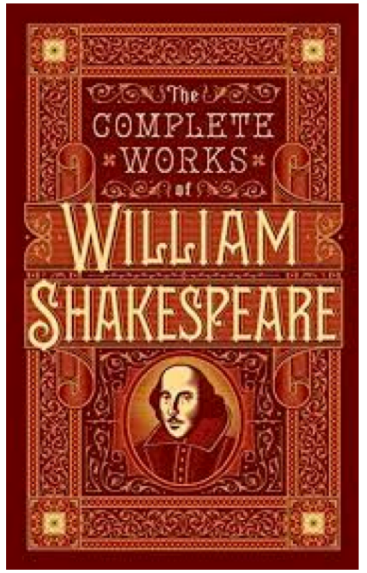
cP +es3;*,t+z GR. → 0.11

c	P		+	e	s	3	;	*	,	t	,	+	z		G	R	.

Correct symbol: Fitness → 1/size(To be or not to be) → 2/18 → 0.11

Demo

Target → “To be or not to be”
Fitness → percentage of correct symbols
nr5**b**kRB/O.?6,zbnDt→ 0.05



n	r	5	b	k	R	B	/	O	.	?	6	,	z		n	D	t

Correct symbol: Fitness → 1/size(To be or not to be) → 1/18 → 0.05

T	o		b	e		o	r		n	o	t		t	o		b	e

Example demo

```
class Individual:
    def _init_(self, num):
        self.fitness = -1
        self.genes = []
        self.genSize = num
        for i in range(0, num):
            self.genes.append( chr(random.randint(32, 128)) )

    def getPhrase(self):
        return "".join(str(x) for x in self.genes)

    def getFitness(self):
        return self.fitness

    def computeFitness(self, target):
        score = 0.0
        for i in range(0, len(self.genes)):
            if self.genes[i] == target[i]:
                score+=1
        self.fitness = score/len(target)
```

Chromosome, e.g., [HeLlo World]

Random values

percentage of correct characters

Example demo

```
class GA:

    #Crossover
    def crossover(self, ind1, ind2):
        child = Individual(self.genSize)
        splitPoint = random.randint(0, self.genSize)
        child.genes[0:splitPoint] = ind1.genes[0:splitPoint]
        child.genes[splitPoint:self.genSize] = ind2.genes[splitPoint:self.genSize]
        return child

    #Mutation
    def mutate(self, ind):
        for i in range(0, self.genSize):
            if(random.random() < self.mutationRate):
                ind.genes[i] = chr(random.randint(32, 128))
```

1-point crossover
One part from one parent and rest from the other

Mutation rate

Random modifications for a given gene

Example demo

```
def GAStep(self):
    matingPool = []
    for ind_i in self.population:
            elementsInPool = int(ind_i.getFitness() * 100)
            for i in range(0, elementsInPool):
                    matingPool.append(ind_i)

    for ind_i in range(0, len(self.population)):
        indexPartnerA = random.randint(0, len(matingPool)-1)
        indexPartnerB = random.randint(0, len(matingPool)-1)
        partnerA = matingPool[indexPartnerA]
        partnerB = matingPool[indexPartnerB]

        child = self.crossover(partnerA, partnerB)
        self.mutate(child)
        child.computeFitness(self.target)

        self.population[ind_i] = child
        if child.getFitness() > self.best.getFitness():
            self.best = child
```

Mating Pool
Candidate Individuals

Selection

Update fitness

Example demo

```
def search(self):
    i=0
    while i < self.maxIterations and self.best.getFitness() < 1:
            self.GAStep()
            i+=1
    print ("i: ",i)
```

1. Selection
2. Crossover
3. Mutation

Demo

Why do they work?

- We can make an analogy with the way humans are creative and innovative (Goldberg)
- Humans are creative and innovative when they combine notations that work well in some other context
- Likewise, GAs can be creative when combining pieces of a good solution, with pieces of another good solution

Components of a GA

- Initialization
  - Usually at random, but can use prior knowledge
- Selection
  - Give preference to better solutions
- Replacement
  - Combine original population with newly created solutions
- Variation
  - Create new solutions through crossover and mutation

Variation operators in GAs

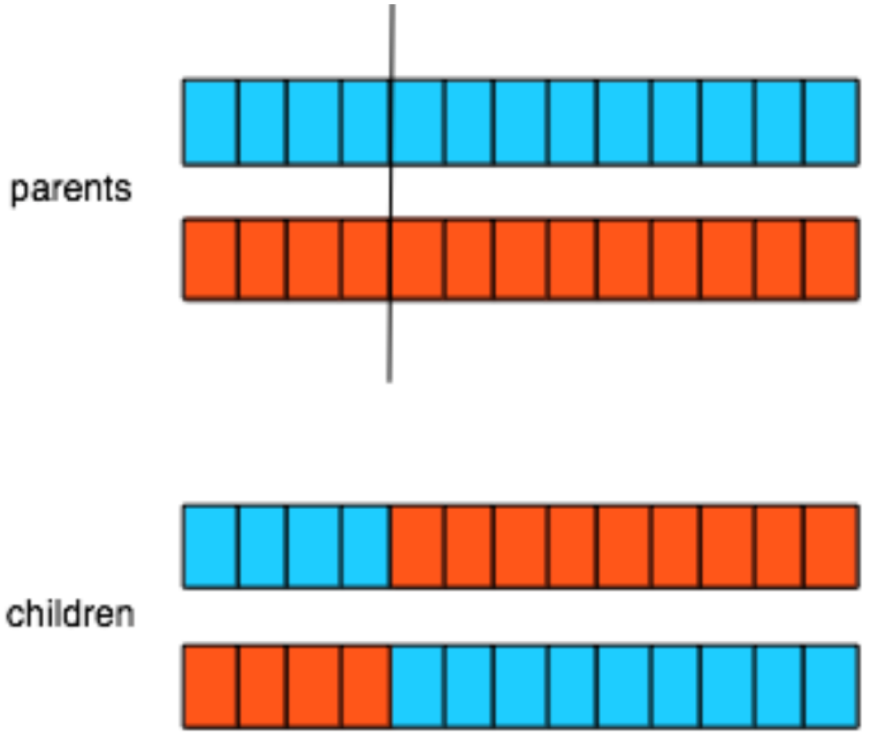
- Let’s look at some of the commonly used variation operators for Gas
  - For binary strings
  - For real-valued vectors and permutations

Commonly used crossover operators for binary strings

- One-point, two-point, *k*-point crossover
- Uniform crossover

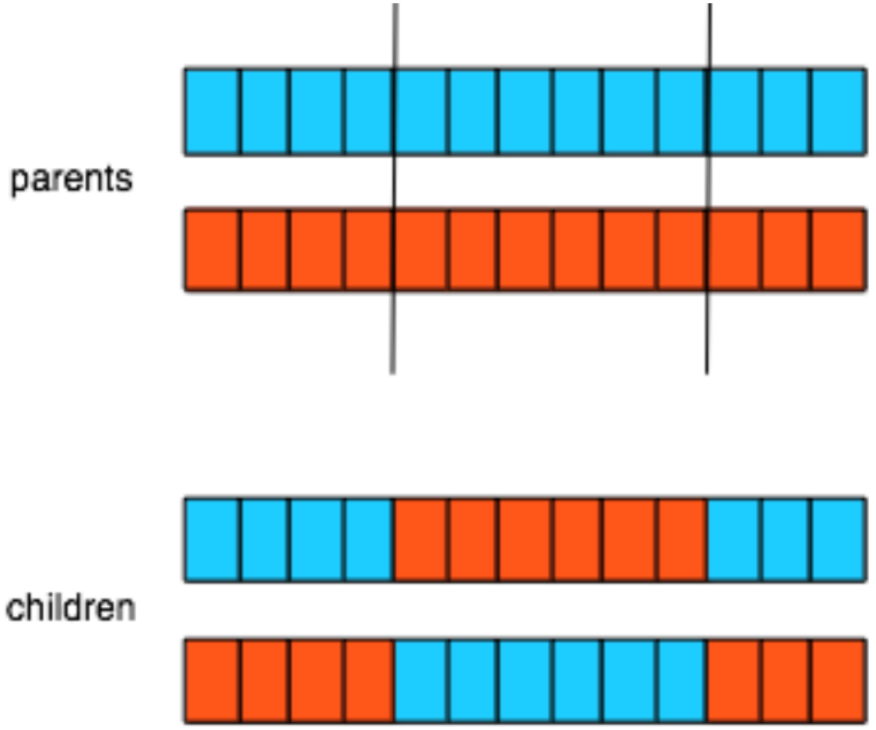
One-point crossover

- A crossing position is chosen at random and the two parents exchange all their bits after that position (like we did in our example by hand)



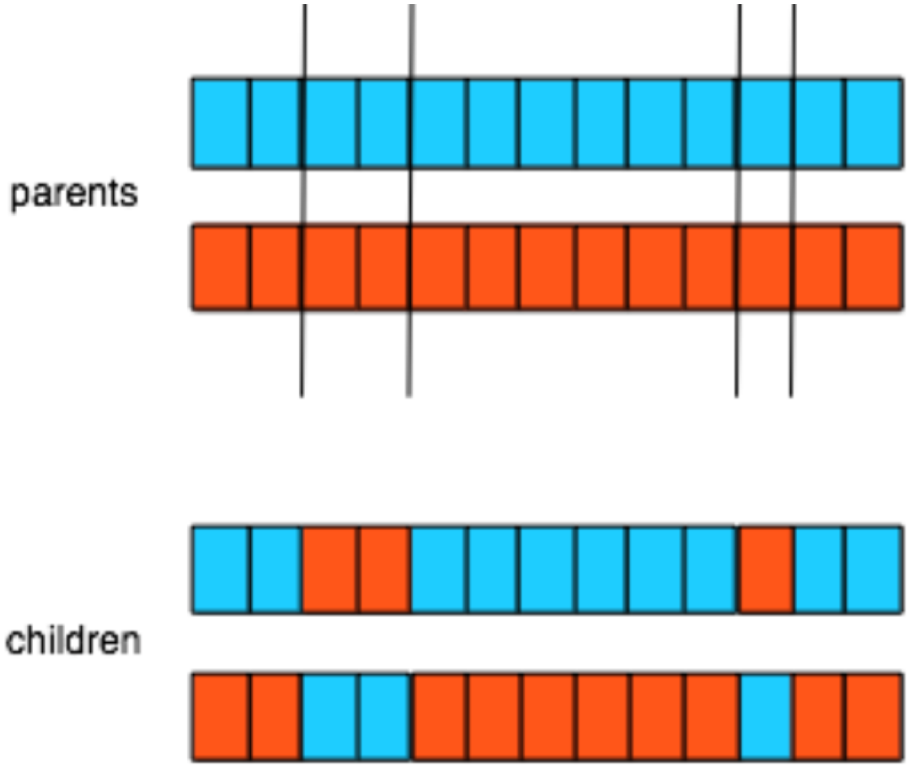
Two-point crossover

- Two crossing positions are chosen at random and the two parents exchange all their bits between the two locations



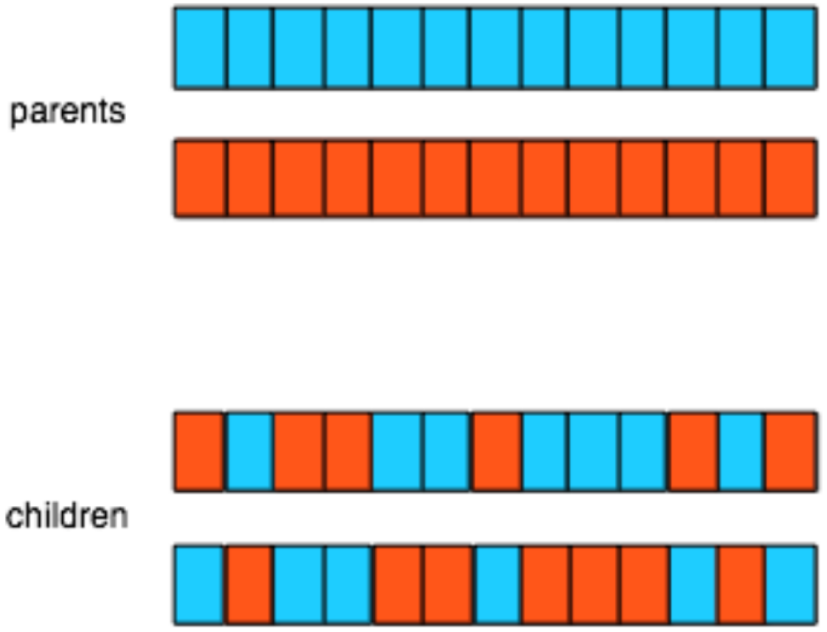
k-point crossover

- Can generalize method to k crossing positions. Here is an example with k=4



Uniform crossover

- For each position, exchange genes with a given probability (typically 0.5). Here's an example with '1011001000101'
  - 1 → exchange
  - 0 → don't exchange



k-point vs. Uniform crossover

- 1-point, 2-point, …, k-point crossover establish an implicit linkage among genes (string positions)
  - Positions close to each other are likely to be treated together (either exchanged or not)

- Uniform crossover treats every position independently
  - It is independent of the ordering of the genes

Crossover or Mutation?

- Decade long debate: which one is better / necessary / main background
- Answer (at least, rather wide agreement):
  - It depends on the problem, but
  - In general, it is good to have both
  - Both have another role
  - Mutation-only-EA is possible, xover-only-EA would not work
- Adding crossover to mutation-only GA makes the GA less sensitive to the parameter Pm

Crossover or Mutation?

- **Exploration / Diversification**: Discovering promising areas in the search space, i.e. gaining information on the problem
- **Exploitation / Intensification**: Optimizing within a promising area, i.e. using information
- There is co-operation AND competition between them
- Crossover explores, it makes a big jump to an area somewhere ”in between” two parent areas
- Mutation is exploitative, it creates random small diversions, thereby staying near (in the area of) the parent

Crossover or Mutation?

- Only crossover can combine information from two parents
- Only mutation can introduce new information (alleles)
- Crossover does not change the allele frequencies of the population (thought experiment: 50% 0's on first bit in the population, ?% after performing crossovers)
- To hit the optimum you often need a 'lucky' mutation

How do GAs differ from other techniques?

- Work with a population of solutions, rather than a single solution

- Use probabilistic rather than deterministic mechanisms

When should we use GA?

- When other simpler methods are not good enough
- When we don’t have much knowledge about the problem that we are trying to solve
- … and when the search space is very large (making complete enumeration unfeasible).