CORK INSTITUTE OF TECHNOLOGY
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Metaheuristic Optimization

Lab 2: NP-Completeness

1. This problem concerns the proof of the NP-completeness of 3-SAT

   a) Convert the formula

   F = (x V p) Λ (-x V y V z V -p) Λ (-y V q V -z)

   into a 3SAT formula, using the construction/reduction

   F = (x V p V U_1) Λ (x V p V -U_1) Λ (-x V y V U_2) Λ (-U_2 V z V -p) Λ (-y V q V -z)

   b) Find a solution for the 3SAT instance of F and verify that it is a solution for the original problem.

   x = T, p = T, U_1 = T, y = T, z = F, U_2 = F, q = F

   (x V p V U_1) Λ (x V p V -U_1) Λ (-x V y V U_2) Λ (-U_2 Vz V -p) Λ(-y VqV -z)

   = (T V T V T) Λ (T V T V F) Λ (F V T V F) Λ (T V F V F) Λ (F V F V T)

   Every clause in 3SAT formula has at least one T literal given the solution

Plugging solution values into original formula:

(x V p) Λ (-x V y V z V -p) Λ (-y V q V -z)

= (T V T) Λ (F V T V F V F) Λ (F V F V T)

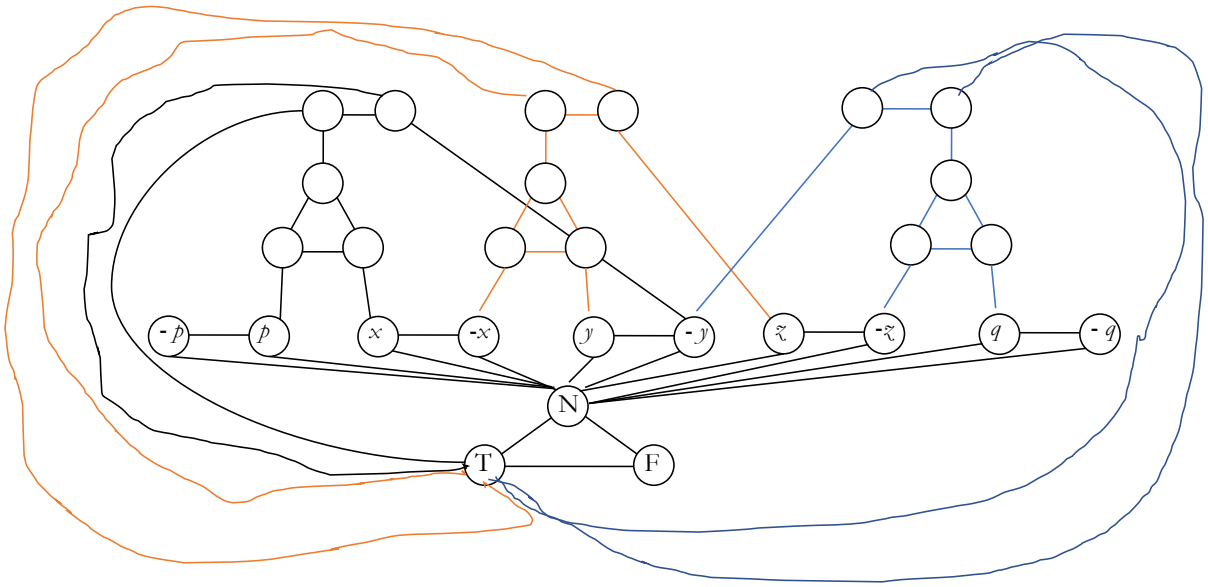
Every clause has at least one T literal

2. This problem concerns the proof of the NP-completeness of 3COL

a) Convert the 3SAT formula

F= (x V p V -y) Λ (-x V y V z) Λ (-y V q V -z)

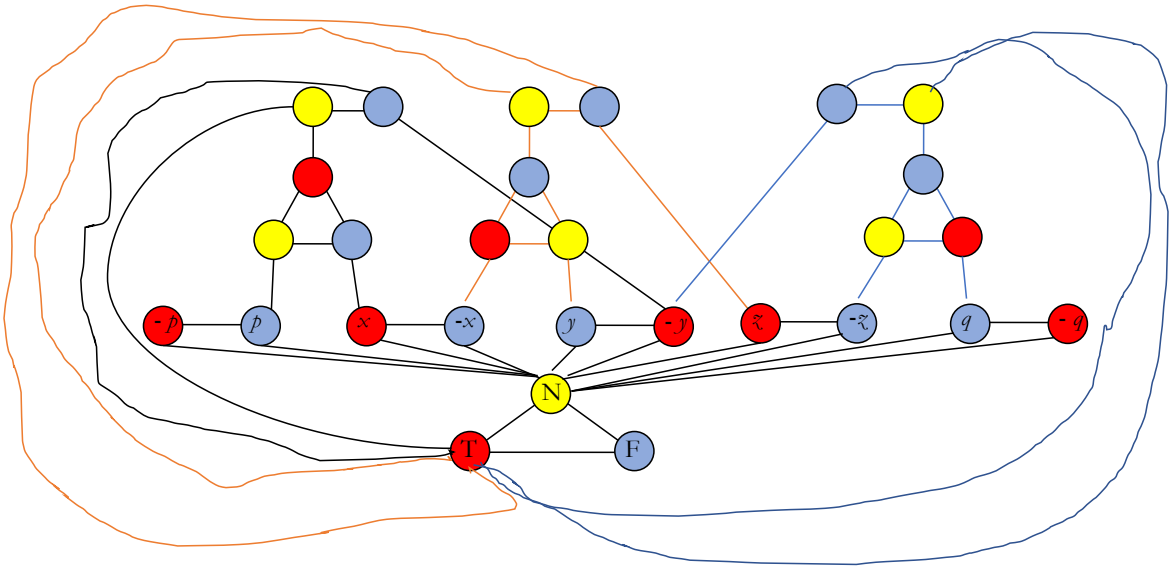
into a 3COL graph.



(x V p V -y) Λ (-x V y V z) Λ (-y V q V -z)

b) Find a solution for the 3COL instance of F and verify that it is a solution for the original problem.

Note a solution is an assignment of a color (T/F/Y) to every vertex added in the 3COL graph.



(x ∨ p ∨ -y) ∧ (-x ∨ y ∨ z) ∧ (-y ∨ q ∨ -z)

Solution for SAT nodes in graph is

p = F, x = T, y = F, z = T, q = F

Plugging values into SAT formula

(T ∨ p ∨ T) ∧ (F ∨ F ∨ T) ∧ (T ∨ F ∨ F)

All clauses have at least one T literal.