

8.3.1) S. & F

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PHYS 513  
HW #8  
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Show that

$$V = V_+ e^{-j\beta z} + V_+ |\rho| e^{j(\theta_p + \beta z)}$$

can be written as a standing and travelling wave

Start with magnitude

$$\begin{aligned} |V|^2 &= \left( \underline{V_+} e^{-j\beta z} + \underline{V_+} |\rho| e^{j(\theta_p + \beta z)} \right)^* \\ &\quad \left( \underline{V_+}^* e^{j\beta z} + \underline{V_+}^* |\rho| e^{-j(\theta_p + \beta z)} \right) \\ &= |V_+|^2 \left( \frac{e^{-j\beta z} e^{j\beta z}}{e^{j\beta z} |\rho| e^{j(\theta_p + \beta z)}} + \frac{|\rho|^2 e^{j(\theta_p + \beta z)} e^{-j(\theta_p + \beta z)}}{|\rho| e^{j(\theta_p + \beta z)} e^{-j\beta z}} + \right. \\ &\quad \left. + |\rho| e^{j(\theta_p + 2\beta z)} e^{-j(\theta_p + 2\beta z)} \right) \\ &= |V_+|^2 \left( 1 + |\rho|^2 + |\rho| \left( e^{j(\theta_p + 2\beta z)} + e^{-j(\theta_p + 2\beta z)} \right) \right) \end{aligned}$$

$$a + bi + a - bi = 2a$$

$$|V|^2 = |V_+|^2 \left( 1 + |\rho|^2 + |\rho| 2 \operatorname{Re} \left( e^{j(\theta_p + 2\beta z)} \right) \right)$$

$$|V| = \sqrt{|V_+|^2 \left( 1 + |\rho|^2 + |\rho| 2 \operatorname{Re} \left( e^{j(\theta_p + 2\beta z)} \right) \right)}$$

$$|V| = \sqrt{|V_+|^2 (1 + |p|^2 + |p|^2 \cos(2(\omega t + \beta z) + \theta_p))}$$

$$\omega t = -\frac{\theta_p}{2} + \pi/4$$

$$= \sqrt{|V_+|^2 (1 + |p|^2 + |p|^2 \cos(2(-\frac{\theta_p}{2} + \frac{\pi}{4} + \beta z) + \theta_p))}$$

$$= \sqrt{|V_+|^2 (1 + |p|^2 + |p|^2 \cos(-\theta_p + \frac{\pi}{2} + 2\beta z + \theta_p))}$$

$$|V| = \sqrt{|V_+|^2 (1 + |p|^2 + 2|p| \cos(2\beta z + \frac{\pi}{2}))}$$