Matthew Jackson
PHYS 513
HW # 9
November 15, 2020

9.2.1) Given

$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta z} + \tilde{V}_n^- e^{j\beta z}$$

where

$$V_n(z,t) = Re\left[\tilde{V}_n(z)\, e^{j\omega t}\right]$$

similarly

$$\tilde{E}_n(z) = \tilde{E}_n^+ e^{-jk_n z} + \tilde{E}^- e^{jk_n z}$$

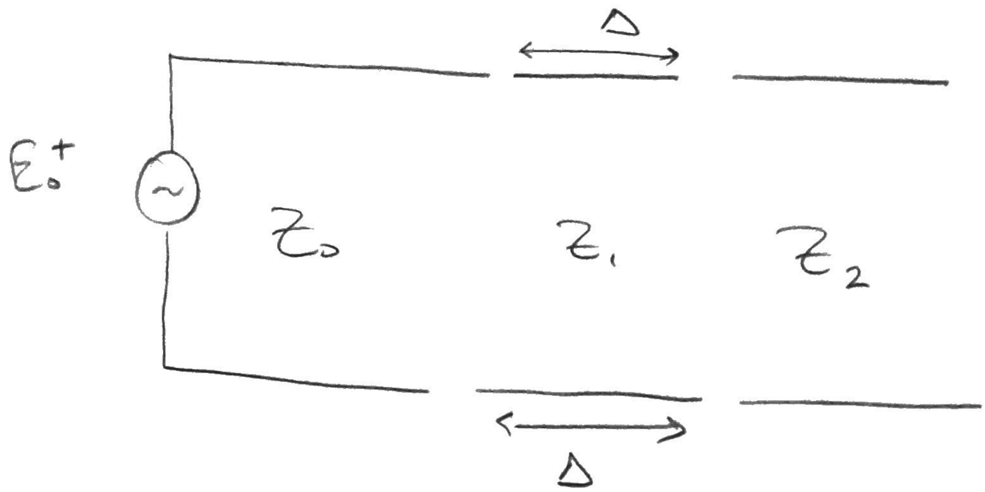
where

$$E_n(z,t) = Re\left[\tilde{E}_n(z)\, e^{j\omega t}\right]$$

Assume $\tilde{V}_0^+$ is known. At $z=0$ & $z=\Delta$, V & I are continous. Assume $V_2^- = 0$

find $\tilde{V}_0^-(z)$, $\tilde{V}_1^-(z)$, and $\tilde{V}_2^+(z)$. find $\tilde{V}_0^-(z) / \tilde{V}_0^+(z)$ and $\tilde{V}_1^+(z) / \tilde{V}_1^+(z)$

step 1 break the problem down



$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^L}{Z_L} \quad (\text{I}) \qquad Z_L = Z_1 + Z_2 \quad (\text{III})$$

$$V_0^+ + V_0^- = V_0^L \quad (\text{II})$$

$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$\frac{V_0^+ - V_0}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$Z_L(V_0^+ - V_0^-) = Z_0(V_0^+ + V_0^-)$$

$$V_0^+(Z_L - Z_0) = V_0^-(Z_0 + Z_L)$$

$$V_0^- = \frac{(Z_L - Z_0)}{(Z_0 + Z_L)} V_0^+$$

$$\boxed{V_o^- = \frac{(Z_1 + Z_2 - Z_o)}{(Z_1 + Z_2 + Z_o)} V_o^+}$$

Use (1) and (11) but (11) rearranged to $V_o^- = V_o^L - V_o^+$

$$\frac{V_o^+}{Z_o} - \frac{V_o^L - V_o^+}{Z_o} = \frac{V_o^L}{Z_L}$$

$$\frac{V_o^+ - V_o^L + (+V_o^+)}{Z_o} = \frac{V_o^L}{Z_L}$$

$$\frac{2V_o^+ - V_o^L}{Z_o} = \frac{V_o^L}{Z_L}$$

$$(2V_o^+ - V_o^L) Z_L = V_o^L Z_o$$

$$2V_o^+ Z_L = V_o^L (Z_o + Z_L)$$

$$V_o^L = \frac{2Z_L}{(Z_o + Z_L)} V_o^+$$

$$V_0^L = \frac{2(Z_1+Z_2)}{(Z_0+Z_1+Z_2)} V_0^+$$

$$V_0^L = \boxed{V_1^+ = \frac{2(Z_1+Z_2)}{(Z_0+Z_1+Z_2)} V_0^+}$$

Do the same for region
Start with (1) and (11)

$$\frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = \frac{V_1^+ + V_1^-}{Z_2} \qquad Z_L = Z_2$$

$$V_1^L = V_2^+$$

$$\frac{(V_1^+ - V_1^-)}{Z_1} = \frac{(V_1^+ + V_1^-)}{Z_2}$$

$$Z_2(V_1^+ - V_1^-) = Z_1(V_1^+ + V_1^-)$$

$$V_1^+(Z_2 - Z_1) = V_1^-(Z_1 + Z_2)$$

$$V_1^- = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} V_1^+$$

$$V_1^- = \frac{(Z_2 - Z_1)}{\cancel{(Z_1 + Z_2)}} \frac{2\cancel{(Z_1 + Z_2)}}{(Z_0 + Z_1 + Z_2)} V_0^+$$

$$\boxed{V_1^- = \frac{2(Z_2 - Z_1)}{(Z_0 + Z_1 + Z_2)} V_0^+}$$

Do the same as earlier with

$V_1^- = V_2^+ - V_1^+$ (sorry for changing notation it seemed easier with less sections)

$$\frac{V_1^+}{Z_1} - \frac{V_2^+ - V_1^+}{Z_1} = \frac{V_2^+}{Z_2}$$

$$\frac{V_1^+ - V_2^+ + (+V_1^+)}{Z_1} = \frac{V_2^+}{Z_2}$$

$$\frac{2V_1^+ - V_2^+}{Z_1} = \frac{V_2^+}{Z_2}$$

$$Z_2(2V_1^+ - V_2^+) = Z_1 V_2^+$$

$$2V_1^+ Z_2 = V_2^+(Z_1 + Z_2)$$

$$V_2^+ = \frac{2Z_2}{(Z_1+Z_2)} V_1^+$$

$$V_2^+ = \frac{2Z_2}{\cancel{(Z_1+Z_2)}} \frac{2\cancel{(Z_1+Z_2)}}{(Z_0+Z_1+Z_2)} V_0^+$$

$$\boxed{V_2^+ = \frac{4Z_2}{(Z_1+Z_2+Z_3)} V_0^+}$$

$$\boxed{\frac{V_0^-}{V_0^+} = \frac{(Z_1+Z_2-Z_0)}{(Z_1+Z_2+Z_0)}}$$

$$\boxed{\frac{V_1^-}{V_1^+} = \frac{(Z_2-Z_1)}{(Z_1+Z_2)}}$$

9.2.21) Show

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November 16, 2020

$$V(z,t) = V^+[\cos(\omega t - \beta z) + \rho\cos(\omega t + \beta z)]$$

can be written as two standing waves

$$V(z,t) = A\cos(\omega t)\cos(\beta z) + B\sin(\omega t)\sin(\beta z)$$

and find A + B

$$\cos(\omega t - \beta z) \to \cos(\mu - \nu)$$
$$\cos(\omega t + \beta z) \to \cos(\mu + \nu)$$

$$\cos(\mu - \nu) = \cos\mu\cos\nu + \sin\mu\sin\nu \quad (1)$$
$$\cos(\mu + \nu) = \cos\mu\cos\nu - \sin\mu\sin\nu \quad (11)$$

$$(1) + (11) = 2\cos\mu\cos\nu = \underline{\cos(\mu-\nu) + \cos(\mu+\nu)}$$
$$(1) - (11) = 2\sin\mu\sin\nu = \underline{\cos(\mu-\nu) - \cos(\mu+\nu)}$$

define γ such that

$$\gamma + (\gamma - \rho) = 1$$

$$V(z,t) = V^+\Big[\big(\gamma + (\gamma - \rho)\big)\cos(\mu - \nu) + \big(\gamma + (\rho - \gamma)\big)\cos(\mu + \nu)\Big]$$

$$= V^+\Big[\gamma\{\cos(\mu - \nu) + \cos(\mu + \nu)\} + \quad \leftarrow (1) + (11)$$

$$(\gamma - \rho)\{\cos(\mu - \nu) - \cos(\mu + \nu)\}\Big] \quad \leftarrow (1) - (11)$$

$$= V^+\Big[2\gamma \cos\mu \cos\nu + 2(\gamma - \rho)\sin\mu \sin\nu\Big]$$

Solve for γ

$$2\gamma - \rho = 1 \Rightarrow \gamma = \frac{\rho + 1}{2}$$

$$= V^+\Big[(\rho + 1)\cos\mu\cos\nu + (1 - \rho)\sin\mu \sin\nu\Big]$$

$$\boxed{V(z,t) = V^+\Big[(\rho + 1)\cos\omega t \cos\beta z + (1 - \rho)\sin\omega t \sin\beta z\Big]}$$

9.2.2.3) The plot generated from the code creates a standing wave with V_{max} and V_{min}. These V_{max} and V_{min} relate to the two standing waves found in part 9.2.1 by the relationship of the coefficients $(1-\rho)$ and $(1+\rho)$. If $\rho < 0$ $|(1-\rho)| \to (1+|\rho|)$ and $|(1+\rho)| \to (1-|\rho|)$. Therefore, the standing wave ratio is given by

$$S = \frac{1+|\rho|}{1-|\rho|}$$

$|\rho|$ can be determined from the plot of the ratios, and if the reflected wave is given, so can the sign.