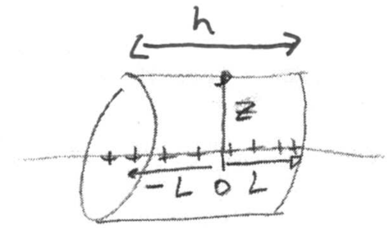
HW 1.3 a)

$Q = \lambda 2L$



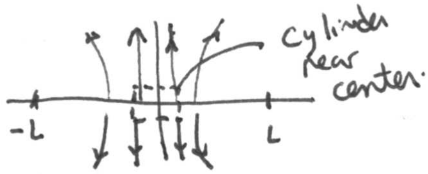
From Griffiths $\rightarrow$ $E(z) = \dfrac{k\, 2\lambda L}{z\sqrt{z^2+L^2}} = \boxed{\dfrac{kQ}{z\sqrt{z^2+L^2}}}$

Gauss Law $\rightarrow$ $E = \dfrac{Q}{\varepsilon_0 \text{ Area}}$ $\Rightarrow$ For cylinder.

$h \ll L$ for Gauss's law to apply

$\Phi = E\int_s dA = E\,(2\pi z\,(2L)) = \dfrac{Q}{\varepsilon_0}$ ✗

$\boxed{E(z) = \dfrac{Q}{4\pi\varepsilon_0\, z \cdot L}} = \dfrac{kQ}{z \cdot L}$



$E(z) = \dfrac{kQ}{z\sqrt{z^2+L}} = \dfrac{kQ}{z \cdot L \cdot \sqrt{1+z^2/L^2}}$; $E(z) = \text{Gauss Law} \cdot \dfrac{1}{\sqrt{1+\frac{z^2}{L^2}}}$ for $L=z$ $\dfrac{1}{\sqrt{2}}$ ✓

Do Taylor on $f(L) = \dfrac{1}{\sqrt{1+\frac{z^2}{L^2}}}$ Evaluating at $\frac{z}{L} = 0$

$f = z = L$

$f(L) = 1 - \frac{1}{2}\left(\frac{z}{L}\right)^2 + \frac{3}{4}\left(\frac{z}{L}\right)^4 - \frac{5}{16}\left(\frac{z}{L}\right)^6$ ✓

$f(L) = 1 - \frac{1}{2} + \frac{3}{8} - \frac{5}{16} \Rightarrow 0.56$ $\Rightarrow$ increasing the order to 10

$f(L) \approx 0.802$, and see how it aproaches $\frac{1}{\sqrt{2}}$ (exact for $z = L$) as the order increases.

From HW1.2 $\Rightarrow$ $E(z)$ for $\lambda = 1E^{-9}$ and $L = z = 1$

$E(z) = 12.7163$ N/C

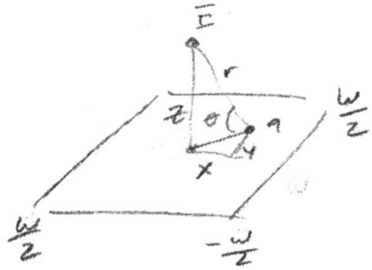
from Gauss Law $\Rightarrow$ $E(z) = 17.98$ with the same parameters

using the taylor aproximation of 0.802 $\Rightarrow$

$E(z) = 17.98 * 0.802 = 14.42$ N/C

This solution is still far off, even with an order 10 Taylor Series aproximation, but getting closer.

HW 1.36



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\sin\theta = \frac{z}{d} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$GL \Rightarrow E = \frac{\sigma}{2\epsilon_0}$

$$E = \frac{\sigma r}{4\pi\epsilon_0} \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\, dx\, dy = \frac{\sigma}{\pi\epsilon_0} \tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2} + z^2}}\right)$$ ok ✓

so do Taylor series on $2\pi \tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2}+z^2}}\right) = 2\pi \tan^{-1}\left(\frac{w^2}{4z^2}\frac{1}{\sqrt{1+\frac{1}{2}\frac{w^2}{z^2}}}\right)$

for order $2 \Rightarrow \frac{\pi}{2}\frac{w^2}{z^2} - \frac{\pi}{8}\left(\frac{w}{z}\right)^4 + \frac{7\pi}{192}\left(\frac{w}{z}\right)^6$ ✓

For $\sigma = -1E{-9}\,C$, $z = 1$ and $w = 1$

$$GL \to E = \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0} = \frac{1E^{-9}}{2\epsilon_0} \approx 56\ N/C$$

$$E_z(z) = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{w^2}{4z}\frac{1}{\sqrt{z^2+\frac{w^2}{2}}}\right) = 414\ N/C$$

Taylor $\Rightarrow$ aproximates to $1.29 \Rightarrow$ GL · taylor $\Rightarrow 56 \cdot 1.29 = 72$

for $w = 2$ and $z = 1$

$E_z(z) = 1079\ NC$ and taylor $\Rightarrow 7.33 \Rightarrow$ GL · Taylor $= 410\ NC$

for $w = 3$ and $z = 1$

$E_z(z) = 1575\ NC$ and taylor $\Rightarrow 65 \Rightarrow$ GL · Taylor $= 3686$

✓ Can use Wolfram Alpha or

$$f(z') \simeq f(0) + z' \frac{\partial f}{\partial z'}\bigg|_{z'=0} + \ldots$$

could not find the series when evaluating for $\frac{z}{w}$:

$$f(z') = 2\pi \tan^{-1}\left(\frac{\sqrt{2}}{4\frac{z}{w}\sqrt{1+2\left(\frac{z}{w}\right)^2}}\right)$$

I may have solve this wrong..

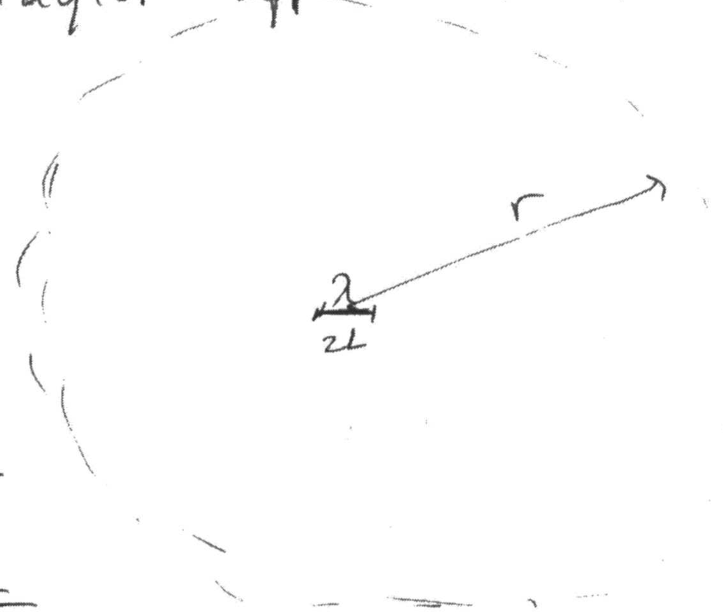
HW 1.3

Matthew Jackson
9/1/20
PHYS 513

a.) Show that $\vec{E}$ from a finite line of length $2L$ with charge density λ is the same as Gauss's when taking the Taylor approx of the $\vec{E}$ field.

$r >> L$

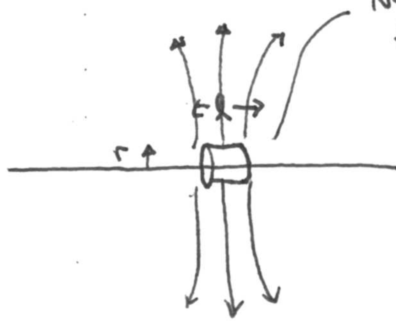
Gauss's Law



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{2\lambda L}{\epsilon_0}$$

$$E = \boxed{\frac{2\lambda L}{4\pi\epsilon_0 r^2}\hat{r}} \rightarrow \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$



Near center, lines nearly $\perp$ to curved surface ($r << 2L$)

$$\oint \vec{E} \cdot d\vec{a} = 2\lambda L/\epsilon_0$$

$$\int_{caps} \vec{E} \cdot d\vec{a} + \int_{side} \vec{E} \cdot d\vec{a} = 2\lambda L/\epsilon_0$$

$$E \, 2\pi r \, 2\cancel{L} = 2\lambda \cancel{L}/\epsilon_0$$

$$E = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}$$

1

Starting with E field from line charge

$$\vec{E}_z(z) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}\hat{z}$$ ← from Griffith's example 2.2

$$= \frac{2\lambda}{4\pi\epsilon_0}\frac{L}{z\cdot|z|\sqrt{1+\left(\frac{L^2}{z^2}\right)}}\hat{z}$$

condition from last problem is $z/L \ll 1$

$$= \frac{2\lambda}{4\pi\epsilon_0}\frac{L/z^2}{\sqrt{1+(L^2/z^2)}}\cdot\frac{L}{L}\hat{z}$$

$$K\frac{2\lambda \cancel{L}}{z\cancel{L}\sqrt{1+(z/L)^2}} \simeq$$

$$\underbrace{K\frac{2\lambda}{z}}_{\text{leading term}}\left(1-\frac{1}{2}\left(\frac{z}{L}\right)^2+\ldots\right)$$

$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 L}\frac{L^2/z^2}{\sqrt{1+(L^2/z^2)}}\cdot\hat{z}$$

define $f(x) = \frac{x}{\sqrt{1+x}}$ so $x = \frac{L^2}{z^2}$ & $\frac{L^2}{z^2} \ll 1$

$$f(x) = f(0) + x\cdot f'(0) + O(x^2)$$

$$f(0) = \frac{0}{\sqrt{1+0}} = 0$$

$$f'(x) = \frac{(x+2)}{2(1+x)^{3/2}} \longrightarrow f'(0) = \frac{\cancel{0}+2}{2(1+\cancel{0})^{3/2}} = 1$$

← wolfram alpha

$$f(x) = x + O(x^2)$$

$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 L}f(x)\hat{z} \rightarrow \frac{2\lambda}{4\pi\epsilon_0 L}\cdot x\,\hat{z}$$

(2)

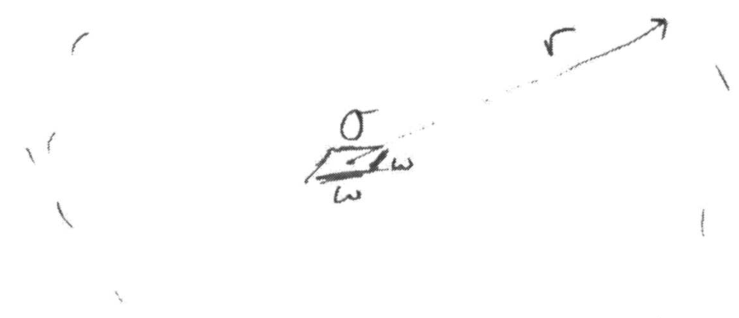
$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 \cancel{L}}\frac{L^{\cancel{2}}}{z^2} \rightarrow \boxed{\vec{E}_z(z) = \frac{2\lambda L}{4\pi\epsilon_0 z}\hat{z}}$$

units

b.) Show that E from sheet charge of sides w and w with charge density σ is the same as Gauss's law when taking the Taylor approx of the E field.

Gauss's Law

$r >> w$



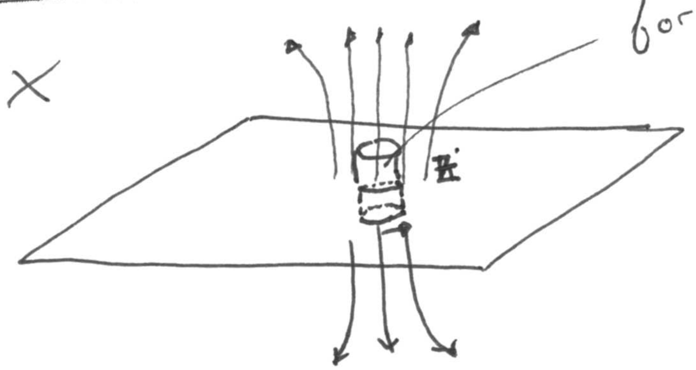
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\sigma w^2}{\epsilon_0}$$

$$\vec{E} = \boxed{\frac{\sigma w^2}{4\pi\epsilon_0 r^2}\hat{r}} \rightarrow \frac{q}{4\pi\epsilon_0 r^2}$$

$\frac{r}{L} << 1$

for $z/L << 1$ field lines nearly $\perp$.



$$E = \frac{\sigma}{\epsilon_0}\underbrace{\frac{z}{|z|}}_{\text{gets sign correct for above \& below}}\hat{z}$$

Start $\vec{E}_z(z) = \frac{\sigma_0}{\pi\epsilon_0} \tan^{-1}\left[\frac{W^2}{4z} \frac{1}{\sqrt{z^2+\frac{W^2}{2}}}\right]\hat{z}$

$$= \frac{\sigma}{\pi\epsilon_0} \tan^{-1}\left[\frac{w^2}{4z} \frac{1}{z\sqrt{1+\frac{w^2}{2z^2}}}\right]\hat{z}$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} \tan^{-1}\left[\frac{1}{2} \frac{w^2}{2z^2} \frac{1}{\sqrt{1+\frac{w^2}{2z^2}}}\right]\hat{z}$$

define $f(x) = \tan^{-1}\left[\frac{1}{2} \frac{x}{\sqrt{1+x}}\right]$

so $x = \frac{w^2}{2z^2}$ & $\frac{w^2}{2z^2} << 1$

$$f(x) = f(0) + x \cdot f'(0) + O(x^2)$$

$$f(0) = \tan^{-1}\left[\frac{1}{2}\frac{0}{0}\right] = 0$$

$f'(x) = \frac{1}{(x+2)\sqrt{1+x}}$ ← wolfram alpha $\quad f'(0) = \frac{1}{(0+2)\sqrt{1+0}} = \frac{1}{2}$

$$f(x) = 0 + x\frac{1}{2} + O(x^2)$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} f(x)\hat{z} \rightarrow \frac{\sigma}{\pi\epsilon_0} \cdot \frac{1}{2} x \hat{z}$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} \frac{1}{2} \frac{w^2}{2z^2} \hat{z}$$

$$\boxed{\vec{E}_z(z) = \frac{w^2\sigma}{4\pi\epsilon_0 z^2}\hat{z}}$$

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