10,3)

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PHYS 513
HW #10
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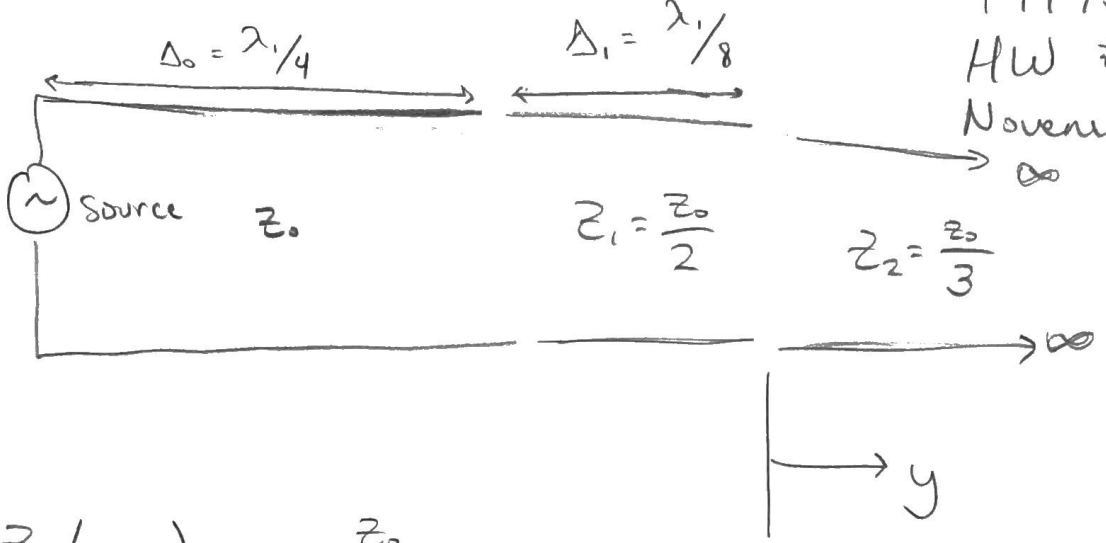

$$Z_2(y>0) = Z_2 = \frac{Z_0}{3}$$

$$Z_n(0) = Z_2 = Z_1 \frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}$$

$$Z_2(1-\tilde{\rho}_1(0)) = Z_1(1+\tilde{\rho}_1(0))$$

$$Z_2 - Z_1 = (Z_2+Z_1)\tilde{\rho}_1(0)$$

$$\tilde{\rho}_1(0) = \frac{Z_2-Z_1}{(Z_2+Z_1)} = \frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}$$

$$\tilde{\rho}_1(0) = \frac{\frac{Z_0/3}{Z_0/2} - 1}{\frac{Z_0/3}{Z_0/2} + 1} \rightarrow \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1}$$

$$\tilde{\rho}_1(0) = -\frac{1}{5}$$ (Also see slide 8)

$$\tilde{\rho}_1\left(\frac{-\lambda_1}{8}\right) = \tilde{\rho}_1(0)\, e^{2j\beta\left(\frac{-\lambda}{8}\right)} \qquad \beta = \frac{2\pi}{\lambda_1}$$

$$\tilde{\rho}_1\left(\frac{-\lambda}{8}\right) = \frac{-1}{5}\, e^{4\pi j \frac{1}{\lambda_1}\left(\frac{-\lambda}{8}\right)}$$

$$\tilde{\rho}_1\left(\frac{-\lambda}{8}\right) = \frac{-1}{5}\, e^{-\frac{\pi}{2}j} \qquad e^{-\frac{\pi}{2}j} = -j$$

$$\tilde{\rho}_1\left(\frac{-\lambda}{8}\right) = \frac{1}{5}j \quad \text{(Also see slide 9)}$$

$$z_{in}\left(\frac{-\lambda}{8}\right) = z_0 \frac{1+\tilde{\rho}_1\left(\frac{-\lambda_1}{8}\right)}{1-\tilde{\rho}_0\left(\frac{-\lambda_1}{8}\right)} = z_1 \left[\frac{1+\tilde{\rho}_1\left(-\lambda_1/8\right)}{1-\tilde{\rho}_1\left(-\lambda_1/8\right)}\right] \qquad \gamma$$

$$z_0\left[\frac{1+\tilde{\rho}_0\left(-\lambda_1/8\right)}{1-\tilde{\rho}_0\left(-\lambda/8\right)}\right] = z_1\gamma$$

$$z_0\left(1+\tilde{\rho}_0\left(-\lambda/8\right)\right) = z_1\gamma\left(1-\tilde{\rho}_0\left(-\lambda/8\right)\right)$$

$$\left(z_0+\gamma z_1\right)\tilde{\rho}_0\left(-\lambda/8\right) = z_1\gamma - z_0$$

$$\tilde{\rho}_0\left(-\lambda/8\right) = \frac{z_1\gamma - z_0}{z_1\gamma + z_0} \rightarrow \frac{\frac{z_1\gamma}{z_0} - 1}{\frac{z_1\gamma}{z_0} + 1}$$

$$\frac{Z_1}{Z_0}\gamma \rightarrow r + xj \qquad \gamma = \left[\frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)}\right]$$

$$\frac{Z_1}{Z_0}\left[\frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)}\right] = r + xj$$

$$\frac{Z_0}{Z}\left[\frac{1+\frac{1}{5}j}{1-\frac{1}{5}j}\right] = r + xj$$

$$\frac{1}{2}\left[\frac{(1+1/5j)(1+1/5j)}{(1-1/5j)(1+1/5j)}\right] \rightarrow 1^2 + 1/5j + 1/5j + (1/5j)^2$$

$$\rightarrow 1^2 + \cancel{1/5j} - \cancel{1/5j} - (1/5j)^2$$

$$\frac{1}{2}\left[\frac{\frac{25}{25} - \frac{1}{25} + \frac{2}{5}j}{1^2 + 1/25}\right] \rightarrow \frac{\frac{24}{25} + \frac{10}{25}j}{26/25}$$

$$\frac{1}{2}\ \frac{24+10j}{26}$$

$$\frac{12+5j}{26} = r + xj$$

$r = \frac{6}{13}$ $\quad x = \frac{5}{26}$ see smith chart slide 10

$r = 0.462$ $\quad x = 0.192$

$\tilde{\rho}_0(-\lambda_1/8) = .39 \angle 153$ (from Smith chart slide 10)

$\tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4) = 0.39\, e^{j\frac{153\pi}{180}}\, e^{-2j\frac{2\pi}{\lambda}\left(\frac{\lambda_0}{4}\right)}$

$\tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4) = 0.39\, e^{j\left(\frac{153\pi}{180} - \pi\right)}$

(see smith chart slide 11)

$$z_0(-\lambda_1/8 - \lambda_0/4) = z_0\left[\frac{1 + \tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4)}{1 - \tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4)}\right]$$

$$\boxed{z_0(-\lambda_1/8 - \lambda_0/4) = [1.85 - 0.77j]\, z_0}$$

(from smith chart slide 11)

→ logic for this step here

$\tilde{\rho}_0(y) = \rho_0(0)\, e^{2j\beta y} \rightarrow \tilde{\rho}_0(y_1 + y_2) = \rho_0(0)\, e^{j\beta(y_1+y_2)}$

$\tilde{\rho}_0(y_1 + y_2) = \rho_0(0)\, e^{j\beta(y_1)}\, e^{j\beta(y_2)}$

$\tilde{\rho}_0(y_1 + y_2) = \rho_0(y_1)\, e^{j\beta(y_2)}$