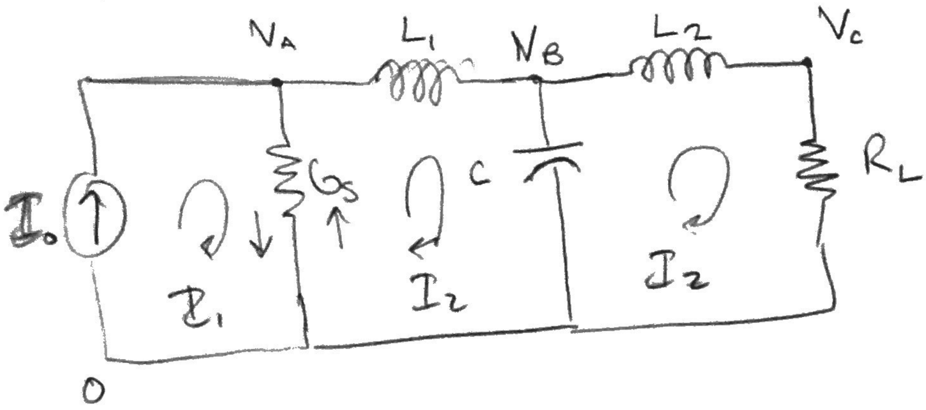
Matthew Jackson
PHYS 513
HW #7
October 20, 2020

7.2) 4.3b from book



First loop

Not possible with current source

Second loop

$$-\frac{(I_2-I_0)}{G_s} - L_1\frac{dI_2}{dt} - \frac{1}{C}\int (I_2-I_3)\,dt = 0$$

Third Loop

$$-\frac{1}{C}\int (I_3-I_2)\,dt - L_2\frac{dI_3}{dt} - R_L I_3 = 0$$

Nodes $\rightarrow \downarrow \rightarrow$

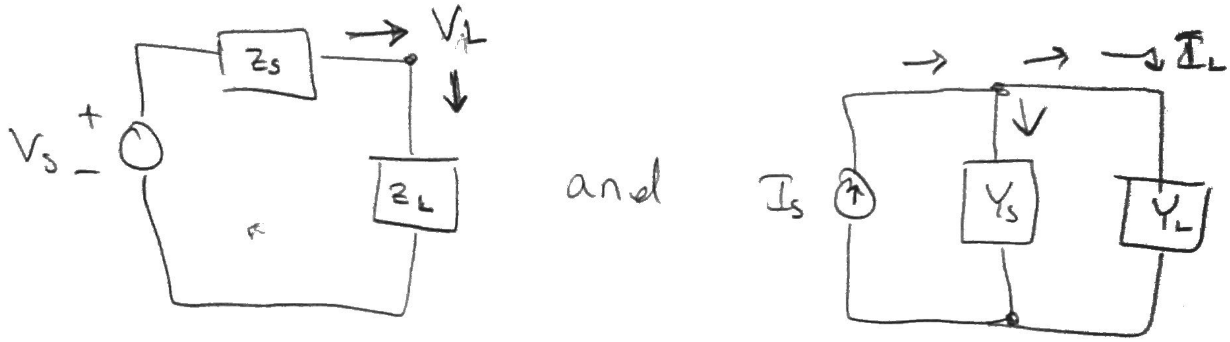
A: $I_o - V_A G_s - \frac{1}{L_1}\int (V_a - V_b)\,dt = 0$

B: $\frac{1}{L_1}\int (V_b - V_a)\,dt + \frac{1}{L_2}\int (V_b - V_c)\,dt + C\frac{dV}{dt} = 0$

C: $\frac{1}{L_2}\int (V_c - V_b)\,dt + \frac{V_c}{R_2} = 0$

7.2) 4.3c

Show that



produce the same current to and Voltage across the Load.

$$\frac{V_s - V_L}{Z_s} - \frac{V_L}{Z_L} = 0$$

$$\frac{V_s}{Z_s} - V_L\left(\frac{1}{Z_s} + \frac{1}{Z_L}\right) = 0$$

$$\frac{V_s}{Z_s} = V_L\left(\frac{1}{Z_s} + \frac{1}{Z_L}\right)$$

$$\frac{V_s}{Z_s} = V_L\left(\frac{Z_L}{Z_s Z_L} + \frac{Z_s}{Z_L Z_s}\right)$$

$$\frac{V_s}{Z_s} = V_L\left(\frac{Z_L + Z_s}{Z_s Z_L}\right)$$

$$V_L = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)$$

Check with Norton circuit

Check currents into node

$$I_s - V_L Y_s - V_L Y_L = 0$$

$$I_s - \frac{V_L}{Z_s} - \frac{V_L}{Z_L} = 0$$

$$I_s = \frac{V_L}{Z_s} + \frac{V_L}{Z_L}$$

$$I_s = \frac{Z_L V_L + Z_s V_L}{Z_s Z_L}$$

$$I_s Z_s Z_L = V_L (Z_L + Z_s)$$

$$V_s \rightarrow \frac{I_s Z_s Z_L}{(Z_L + Z_s)} = V_L$$

$$V_L = \frac{V_s Z_L}{(Z_L + Z_s)} \quad \leftarrow \text{same voltage}$$

Given that Z_L and V_L are the same I_L should also be the same

7.2) 4.3 d

V_s and Z_s are constant. Show that Power is maximized when $Z_L = Z_s^*$

$$P = VI \rightarrow Re[\tilde{V} e^{i\omega t}]\, Re[\tilde{I} e^{i\omega t}]$$

$$P = Re[(V_r + iV_i)(\cos(\omega t) + i\sin(\omega t))] \times$$

$$Re[(I_r + iI_i)(\cos(\omega t) + i\sin(\omega t))]$$

$$Re[(V_r + iV_i)(\cos(\omega t) + i\sin(\omega t))] =$$

$$Re[V_r\cos(\omega t) + iV_r\sin(\omega t) + iV_i\cos(\omega t) - V_i\sin(\omega t)]$$

same with I

$$Re[(V_r + iV_i)(\cos(\omega t) + i\sin(\omega t))] = V_r\cos(\omega t) + V_i\sin(\omega t)$$

$$P = (V_r\cos(\omega t) - V_i\sin(\omega t))(I_r\cos(\omega t) - I_i\sin(\omega t))$$

$$P = V_r I_r\cos^2(\omega t) + V_i I_i\sin^2(\omega t) - (V_r I_i + V_i I_r)\cos(\omega t)\sin(\omega t)$$

$$\bar{P} = \frac{1}{2}(V_r I_r + V_i I_i) = \frac{1}{2} Re[VI^*]$$

$$\bar{P} = \frac{1}{2} Re[VI^*] \qquad V = IZ$$

$$\bar{P} = \frac{1}{2} Re[\overline{VI^*}]$$

Maximize $\overline{P_L} = \frac{1}{2} Re[\overline{V_L I^*}]$

$$V_L = I Z_L$$

$$\overline{P_L} = \frac{1}{2} |I|^2 \; Re[Z_L]$$

$$\overline{P_L} = \frac{1}{2} \left(\frac{|V_s|}{|Z_s + Z_L|} \right)^2 Re[Z_L]$$

$$\bar{P} = \frac{1}{2} \left(\frac{|V_s|}{|Z_{sR} + iZ_{si} + Z_{LR} + iZ_{Li}|} \right)^2 Re[Z_L]$$

$Re[Z_s + Z_L]^2 + Im[Z_s + Z_L]^2$ ← get this to zero with $Im[Z_L] = -Im[Z_s]$

$$\bar{P} = \frac{1}{2} \left(\frac{|V_s|^2 \, Re[Z_L]}{(Re[Z_s] + Re[Z_L])^2} \right)$$ take ~~derivative~~ derivative wrt $Re[Z_L]$

$u = Re[Z_L]$ $\qquad v = Re[Z_s]$

$$\frac{\partial}{\partial u} \bar{P} = \frac{\partial}{\partial u} \left(\frac{1}{2} \frac{|V_s|^2 u}{(u+v)^2} \right) = 0$$

$$\frac{\partial \bar{P}}{\partial u} = \left(\frac{1}{(u+v)^2} + \frac{-2u}{(u+v)^3} \right) \cancel{\frac{1}{2} |V_s|^2} = 0$$

$$\frac{(u+v) - 2u}{\cancel{(u+v)^3}} = 0$$ only care about numerator

$\cancel{u}+v-\cancel{2}u=0$

$v=u$ ← sub back in

$Re[Z_s] = Re[Z_L]$

$(Im[Z_L] = -Im[Z_s])$

Plug in $Z_L = Z_s^*$

This should be the local max because if $Z_L \ll Z_s$ then $\bar{P} \to 0$ and if $Z_L \gg Z_s$ $\bar{P} \propto \frac{1}{Z_L}$ which goes to 0.

$\boxed{\therefore\ P \text{ is maximized when } Z_L = Z_s^*}$