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PHYS 513
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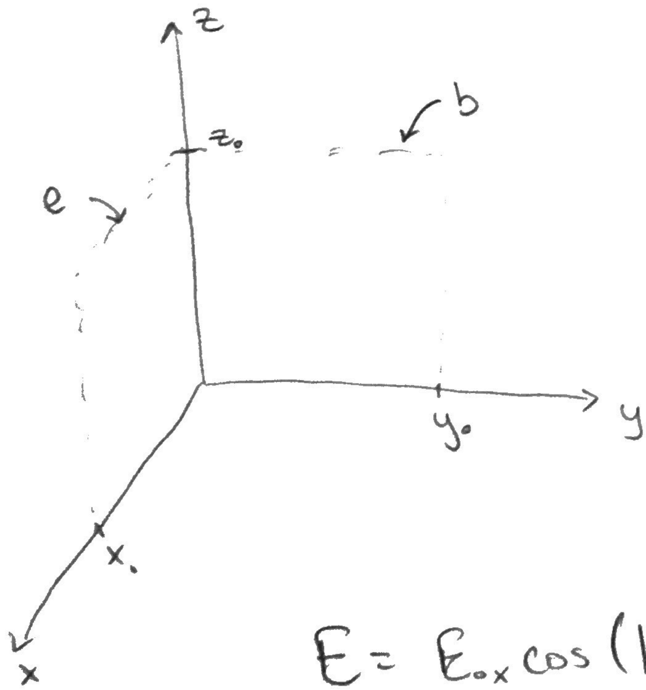
HW # 6

6.1) Faraday's law

$$\oint \mathbf{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$$

Generalized Ampere's law ($J=0$)

$$\oint \mathbf{B} \cdot d\vec{l} = \frac{1}{c^2} \frac{\partial \Phi_E}{\partial t}$$



$$E = E_{0x} \cos(k_z z - \omega t)\,\hat{x}$$

6.1.1) Find the magnetic field $\mathbb{B}$ that must exist using $\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$

$$\nabla \times \mathbb{E} = \nabla \times \left(E_{0x} \cos(k_z z - \omega t) \hat{x} \right)$$

$$\begin{matrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_x & 0 & 0 \end{matrix}$$

$$\langle 0, \partial_z E_x - 0, 0 - \cancel{\partial_y E_x}^{0} \rangle$$

$$\nabla \times \mathbb{E} = -k_z E_{0x} \sin(k_z z - \omega t) \hat{y}$$ ← plug in

$$-\frac{\partial B}{\partial t} = -k_z E_{0x} \sin(k_z z - \omega t) \hat{y}$$

$$\int + dB = \int + k_z E_{0x} \sin(k_z z - \omega t) \hat{y} \, dt$$

$$\mathbb{B} = k_z E_{0x} \int \sin(k_z z - \omega t) \hat{y} \, dt$$

$$\phi = k_z z - \omega t$$

$$d\phi = -\omega dt \rightarrow dt = \frac{-d\phi}{\omega}$$

$$\mathbb{B} = \frac{k_z E_{ox}}{\omega} \int -\sin\phi \, d\phi \, \hat{y}$$

$$\mathbb{B} = \frac{k_z E_{ox}}{\omega} \cos\phi \, \hat{y}$$

$$\boxed{\mathbb{B} = \frac{k_z E_{ox}}{\omega} \cos(k_z z - \omega t) \, \hat{y}}$$

6.1.2) Show that $\vec{E}$ satisfies Faraday's Law in the form of $\oint \mathbb{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{dt}$ along the rectangle e