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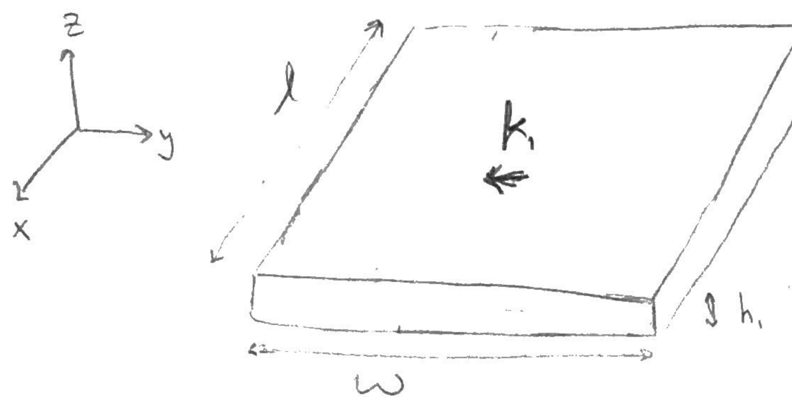
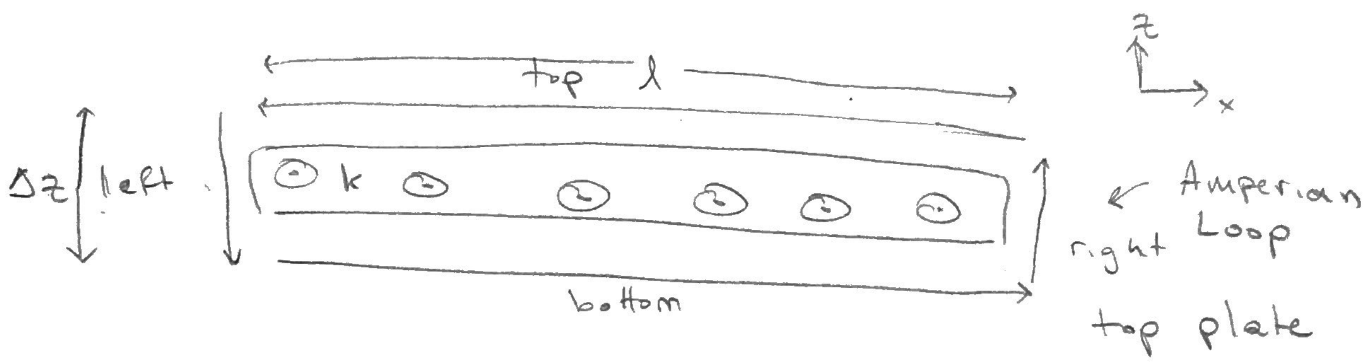
PHYS 513
HW # 9
November

9.1.1)





$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{a}$ (assume w is very large compared to Δz to ignore fringe fields)

$\oint \vec{B}\cdot d\vec{l} = \mu_0 k l$ ← total current enclosed

$B_{right}\cdot \Delta z + B_{left}\cdot \Delta z + B_{top}\cdot l + B_{bottom}\cdot l = \mu_0 k l$

assume $\Delta z \ll l$ such that $(B_{left} + B_{right})\Delta z \ll (B_{top} + B_{bottom})\cdot l$

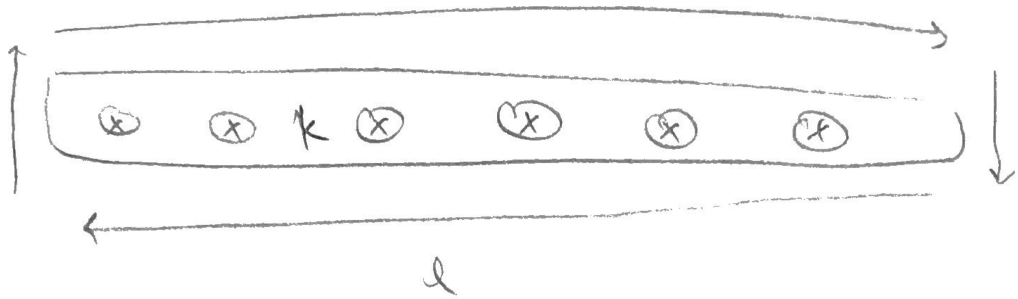
$(B_{top} + B_{bottom})\cancel{l} = \mu_0 k \cancel{l}$

$B_{top} + B_{bottom} = \mu_0 k$

since only k is present $|B_{top}|$ and $|B_{bottom}|$ are same

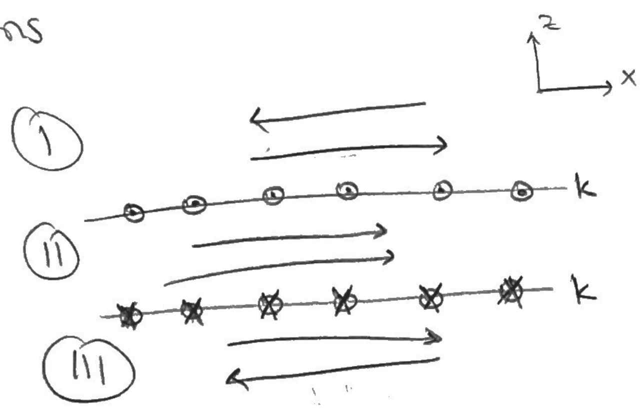
$\boxed{B = \frac{\mu k_0}{2}}$ $\quad B_{top} = \frac{\mu_0 k}{2}(-\hat{x}) \quad B_{bottom} = \frac{\mu_0 k}{2}(\hat{x})$

Same thing can be done with bottom plate



bottom and top are reversed here, but the magnitude's are the same

Since $|B| = \frac{\mu_0 k}{2}$ from each plate, this creates 3 regions



top arrow is from top plate

bottom arrow is from bottom plate

regions I and III cancel out

region II has magnetic field of $B = \mu_0 k (\hat{x})$

9.1.2) Using $\mathcal{E}_1 = -\frac{\partial \Phi_m}{dt}$ and $\mathcal{E}_1 = -L_1 \frac{\partial I}{dt}$, find Φ_m and then L_1 in terms of μ_0, l, and A_1, given as $A_1 = h_1 \omega$

$$\Phi_m = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ is only defined in the duct with $B = \mu_0 K \hat{x}$ (in l direction)

$$\Phi_m = \mu_0 K A_1$$

$$\frac{d\Phi_m}{dt} = \mu_0 A_1 \frac{\partial K_1}{dt}$$

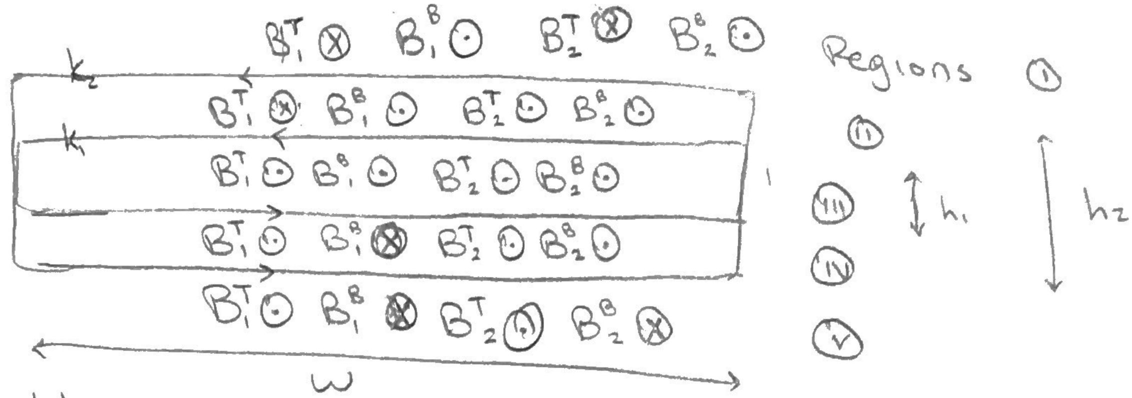
$$K l = I$$

$$I = \frac{K}{l}$$

$$\frac{\partial \Phi_m}{dt} = \mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t} = L \frac{\partial I}{dt}$$

$$\boxed{\begin{gathered} L_1 = \mu_0 \frac{A_1}{l} \\ \mathcal{E}_1 = -\mu_0 \frac{A_1}{l} \frac{\partial I}{dt} \end{gathered}}$$

9.1.3)



Start with B_y^x where x is B from top (T) or bottom plate with current sheet from k_1 (1) or k_2 (2)

assuming $k_1 = k_2$

regions I and V have total B of

$B - B + B - B = 0$

Region II

$-B + B + B + B = 2B = \underline{\mu_0 k}$

Region III

$B + B + B + B = 4B = \underline{2\mu_0 k}$

Region IV

$B + B - B + B = 2B = \underline{\mu_0 k}$

Look at the areas encased by the inner current area is encased by w and h_1

$$\mathcal{E}_1 = -\frac{d\Phi_m}{dt}$$

$$\Phi_m = \int \vec{B}_{III} \cdot d\vec{A}$$

$$\Phi_m = 2\mu_0 k A_1$$

$$\Phi_m = 2\mu_0 \frac{A_1}{l} I$$

$$\mathcal{E}_1 = -2\mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t}$$

$$\mathcal{E}_2 = -\frac{\partial \Phi_m}{dt}$$

$$\Phi_m = \underbrace{\int \vec{B}_{II} \cdot d\vec{A}_{II}}_{\text{Region II}} + \underbrace{\int B_{III} \cdot d\vec{A}_{III}}_{\text{Region III (same as } \mathcal{E}_1)} + \underbrace{\int \vec{B}_{IV} \cdot d\vec{A}_{IV}}_{\text{Region IV}}$$

Region II and Region IV are symmetric so combine B_{II} and B_{IV}

$$\Phi_m = B_{(II)}\left(A_{(II)} + A_{(IV)}\right) + \mathcal{E}_1$$

$A_{(II)} + A_{(IV)}$ is the same as $A_{(II)} + A_{(III)} + A_{(IV)} - A_{(III)}$ which equals $wh_2 - wh_1 \Rightarrow w(h_2 - h_1)$

$$\Phi_m = \mu_0 k\, w(h_2 - h_1) + 2\mu_0 k\, wh_1$$

$$\Phi_m = (\mu_0 w h_2 + \mu_0 w h_1)\, k$$

$$\mathcal{E}_2 = -\frac{\partial \Phi_m}{\partial t}$$

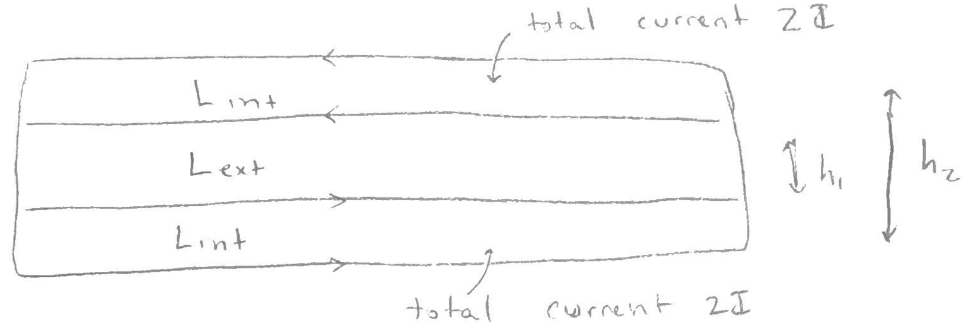
$$\mathcal{E}_2 = -\mu_0 \frac{(A_2 + A_1)}{l} \frac{\partial I}{\partial t}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -2\mu_0 \frac{A_1}{l}\frac{\partial I}{\partial t} - \mu_0 \frac{(A_2 + A_1)}{l}\frac{\partial I}{\partial t}$$

$$\boxed{\mathcal{E} = -\left(\mu_0 \frac{(A_2 + 3A_1)}{l}\right)\frac{\partial I}{\partial t} \qquad L = \mu_0 \frac{(A_2 + 3A_1)}{l}}$$

9.1.4)



Lext is given by the 2 currents which results in a magnetic field of $2\mu_0 k$

$$L_{ext} = \frac{1}{I_{tot}} \int \vec{B} \cdot d\vec{A}$$

$$L_{ext} = \frac{1}{2I} \int 2\mu_0 \frac{k}{l}\, dA$$

$$L_{ext} = \frac{1}{\cancel{2}(\cancel{k}l)} \cancel{2}\mu_0 \cancel{k}\, w h_1$$

$$L_{ext} = \frac{\mu_0 w h_1}{l} = \frac{\mu_0 A_1}{l}$$

$$L_{int} = \frac{1}{I_{tot}} \int \vec{B} \cdot d\vec{A}$$

$$L_{int} = \frac{1}{I_{tot}} \left(\mu_0 k\, w (h_2 - h_1) \right)$$ I_{tot} is one kl

$$L_{int} = \frac{1}{\cancel{k}l} \left(\mu_0 \cancel{k} w (h_2 - h_1) \right)$$

9.1.5) Flux linkages seem to be a difference between accounting for the way current will have a mutual inductance with another current in the same region. If you don't account for this, mutual inductance terms get left out.