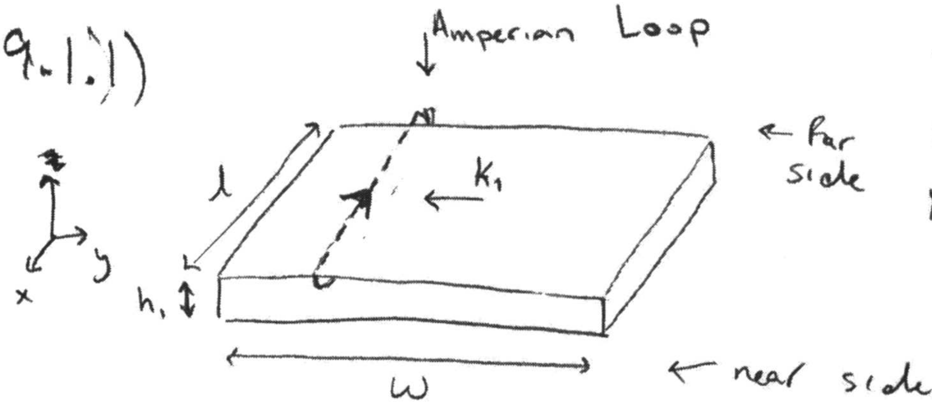
9.1.1)



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November 16, 2020

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \rightarrow \int_0^l K_1 \cdot dl$ for the top plate

Amperian loop is rectangle

$h_1 << w$ and $h_1 << l$

$\oint \vec{B} \cdot d\vec{l} = \cancel{B_{near}} \cdot h_1 + B_{up} \cdot l - \cancel{B_{far}} \cdot h_1 + B_{bottom} \cdot l$

assume $h_1 \rightarrow 0$ compared to l

$\oint \vec{B} \cdot d\vec{l} = B_{bottom} \cdot l - B_{top} \cdot l$

$B_{bottom} \cdot \cancel{l} - B_{top} \cdot \cancel{l} = -\mu_0 k \cdot \cancel{l}$

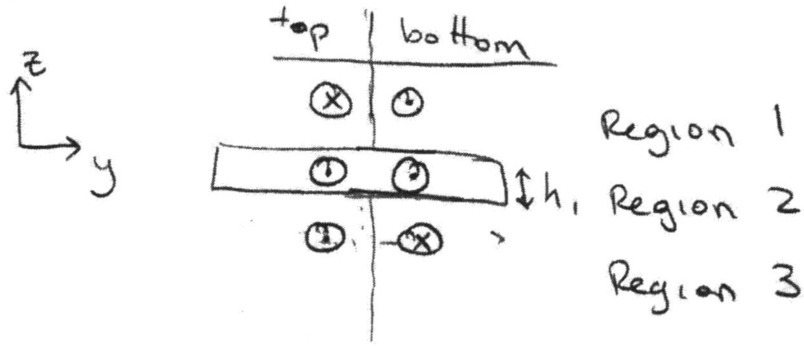
$B_{bottom} - B_{top} = -\mu_0 k$

$B_{top} - B_{bottom} = \mu_0 k$ top plate

the $-\mu_0 k$ becomes $\mu_0 k$ for the bottom plate

$B_{bottom} - B_{top} = \mu_0 k$ bottom plate look at the fields along

zy plane



Region 1 $\quad B_{top} - B_{top} = 0$ ✓

Region 2 $\quad B_{bottom} + B_{top} \neq 0$

Region 3 $\quad B_{bottom} - B_{bottom} = 0$ ✓

Given that $B_{bottom} - B_{top} = \mu_0 K$ (this is based on direction)

$B_{bottom}\,\hat{x} + B_{top}(+\hat{x}) = \mu_0 k$

$B_{bottom} + B_{top} = \mu_0 k$

assume B_{bottom} and B_{top} are same given they come from same source

$B_{top} = \dfrac{\mu_0 k}{2} = B_{bottom} \Rightarrow B = B_{top} + B_{bottom} = \mu_0 K$

$\boxed{\text{so Region 2 } B \text{ is } \mu_0 k}$ ✓

9.1.2) Using $\mathcal{E}_1 = -\frac{\partial \Phi_m}{\partial t}$ and $\mathcal{E}_1 = -L_1 \frac{\partial I}{\partial t}$, find Φ_m and then L_1 in terms of μ_o, l, and A_1, given as $A_1 = h_1 \omega$

$$\Phi_m = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ is only ~~defined~~ nonzero in the duct with $B = \mu_o K \hat{x}$ (in l direction)

$$\Phi_m = \mu_o K A_1$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_o A_1 \frac{\partial K_1}{\partial t} \qquad \begin{aligned} K l &= I \\ I &= \frac{K}{l} \end{aligned}$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_o \frac{A_1}{l} \frac{\partial I}{\partial t} = L \frac{\partial I}{\partial t}$$

$$\boxed{\begin{aligned} L_1 &= \mu_o \frac{A_1}{l} \\ \mathcal{E}_1 &= -\mu_o \frac{A_1}{l} \frac{\partial I}{\partial t} \end{aligned}}$$

9.1.3) $\mathcal{E}_1 = -\mu_0 \frac{A_1}{l} \frac{dI_1}{dt}$ $\qquad \mathcal{E}_1 = -\frac{\partial \Phi_1}{\partial t}$

Φ_1 has contributions from currents on both ducts.

since the difference for $\mathcal{E}_2$ is K_2 (I_2/l) and A_2, change the values

$B_1A_1 + B_2A_1 = \Phi_1$

$\mathcal{E}_2 = -\mu_0 \frac{A_2}{l} \frac{dI_2}{dt}$ $\qquad \Phi_2$ has contributions from currents on both ducts.

$\Phi_2 = B_1A_1 + B_2(A_2 - A_1)$

$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$

$= -\mu_0 \frac{A_1}{l} \frac{dI_1}{dt} - \mu_0 \frac{A_2}{l} \frac{dI_2}{dt}$

$$\boxed{\mathcal{E} = -\frac{\mu_0}{l}\left(A_1 \frac{dI_1}{dt} + A_2 \frac{dI_2}{dt}\right)}$$

9.2.1) Given

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$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta z} + \tilde{V}_n^- e^{j\beta z}$$

where

$$V_n(z,t) = Re\left[\tilde{V}_n(z) e^{j\omega t}\right]$$

similarly

$$\tilde{E}_n(z) = \tilde{E}_n^+ e^{-jk_n z} + \tilde{E}^- e^{jk_n z}$$

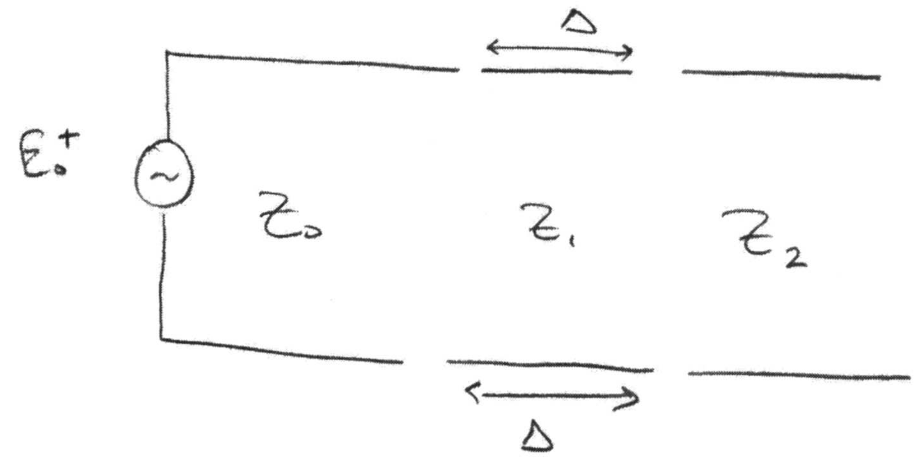
where

$$E_n(z,t) = Re\left[\tilde{E}_n(z) e^{j\omega t}\right]$$

Assume $\tilde{V}_0^+$ is known. At $z=0$ & $z=\Delta$, V & I are continous. Assume $V_2^- = 0$

find $\tilde{V}_0^-(z)$, $\tilde{V}_1^-(z)$, and $\tilde{V}_2^+(z)$. find $\tilde{V}_0^-(z) / \tilde{V}_0^+(z)$ and $\tilde{V}_1^-(z) / \tilde{V}_1^+(z)$

Step 1 break the problem down



$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^L}{Z_L} \quad (\text{I}) \qquad Z_L = Z_1 + Z_2 \quad (\text{III})$$

$$V_0^+ + V_0^- = V_0^L \quad (\text{II})$$

Impedances don't add like this for transmission lines.

$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$\frac{V_0^+ - V_0}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$Z_L(V_0^+ - V_0^-) = Z_0(V_0^+ + V_0^-)$$

$$V_0^+(Z_L - Z_0) = V_0^-(Z_0 + Z_L)$$

$$V_0^- = \frac{(Z_L - Z_0)}{(Z_0 + Z_L)} V_0^+$$

$$V_1^- = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} V_1^+$$

$$V_1^- = \frac{(Z_2 - Z_1)}{\cancel{(Z_1 + Z_2)}} \frac{2\cancel{(Z_1 + Z_2)}}{(Z_0 + Z_1 + Z_2)} V_0^+$$

$$\boxed{V_1^- = \frac{2(Z_2 - Z_1)}{(Z_0 + Z_1 + Z_2)} V_0^+}$$

Do the same as earlier with

$V_1^- = V_2^+ - V_1^+$ (sorry for changing notation it seemed easier with less sections)

$$\frac{V_1^+}{Z_1} - \frac{V_2^+ - V_1^+}{Z_1} = \frac{V_2^+}{Z_2}$$

$$\frac{V_1^+ - V_2^+ + (+V_1^+)}{Z_1} = \frac{V_2^+}{Z_2}$$

$$\frac{2V_1^+ - V_2^+}{Z_1} = \frac{V_2^+}{Z_2}$$

$$Z_2(2V_1^+ - V_2^+) = Z_1 V_2^+$$

$$2V_1^+ Z_2 = V_2^+(Z_1 + Z_2)$$

$$V_2^+ = \frac{2Z_2}{(Z_1+Z_2)} V_1^+$$

$$V_2^+ = \frac{2Z_2}{\cancel{(Z_1+Z_2)}} \frac{2\cancel{(Z_1+Z_2)}}{(Z_0+Z_1+Z_2)} V_0^+$$

$$\boxed{V_2^+ = \frac{4Z_2}{(Z_1+Z_2+Z_3)} V_0^+}$$

$$\boxed{\frac{V_0^-}{V_0^+} = \frac{(Z_1+Z_2-Z_0)}{(Z_1+Z_2+Z_0)}}$$

$$\boxed{\frac{V_1^-}{V_1^+} = \frac{(Z_2-Z_1)}{(Z_1+Z_2)}}$$

$$V_0^L = \frac{2(Z_1+Z_2)}{(Z_0+Z_1+Z_2)} V_0^+$$

$$V_0^L = \boxed{V_1^+ = \frac{2(Z_1+Z_2)}{(Z_0+Z_1+Z_2)} V_0^+}$$

Do the same for region

Start with (1) and (11)

$$\frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = \frac{V_1^+ + V_1^-}{Z_2} \qquad \begin{array}{l} Z_L = Z_2 \\ V_1^L = V_2^+ \end{array}$$

$$\frac{(V_1^+ - V_1^-)}{Z_1} = \frac{(V_1^+ + V_1^-)}{Z_2}$$

$$Z_2(V_1^+ - V_1^-) = Z_1(V_1^+ + V_1^-)$$

$$V_1^+(Z_2 - Z_1) = V_1^-(Z_1 + Z_2)$$

$$\boxed{V_0^- = \frac{(Z_1 + Z_2 - Z_0)}{(Z_1 + Z_2 + Z_0)} V_0^+}$$

$\times$

if $Z_1 = Z_2$, get unexpected result.

Use (1) and (11) but (11) rearranged to $V_0^- = V_0^L - V_0^+$

$$\frac{V_0^+}{Z_0} - \frac{V_0^L - V_0^+}{Z_0} = \frac{V_0^L}{Z_L}$$

$$\frac{V_0^+ - V_0^L + (+V_0^+)}{Z_0} = \frac{V_0^L}{Z_L}$$

$$\frac{2V_0^+ - V_0^L}{Z_0} = \frac{V_0^L}{Z_L}$$

$$(2V_0^+ - V_0^L) Z_L = V_0^L Z_0$$

$$2V_0^+ Z_L = V_0^L (Z_0 + Z_L)$$

$$V_0^L = \frac{2 Z_L}{(Z_0 + Z_L)} V_0^+$$

9.2.21) Show

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$$V(z,t) = V^+[\cos(\omega t - \beta z) + \rho\cos(\omega t + \beta z)]$$

can be written as two standing waves

$$V(z,t) = A\cos(\omega t)\cos(\beta z) + B\sin(\omega t)\sin(\beta z)$$

and find A + B

$$\cos(\omega t - \beta z) \rightarrow \cos(\mu - \nu)$$
$$\cos(\omega t + \beta z) \rightarrow \cos(\mu + \nu)$$

$$\cos(\mu - \nu) = \cos\mu\cos\nu + \sin\mu\sin\nu \quad (1)$$
$$\cos(\mu + \nu) = \cos\mu\cos\nu - \sin\mu\sin\nu \quad (11)$$

$$(1) + (11) = 2\cos\mu\cos\nu = \underline{\cos(\mu-\nu) + \cos(\mu+\nu)}$$
$$(1) - (11) = 2\sin\mu\sin\nu = \underline{\cos(\mu-\nu) - \cos(\mu+\nu)}$$

define γ such that

$$\gamma + (\gamma - \rho) = 1$$

$$V(z,t) = V^+\Big[\big(\gamma + (\gamma - \rho)\big)\cos(\mu - \nu) + \big(\gamma + (\rho - \gamma)\big)\cos(\mu + \nu)\Big]$$

$$= V^+\Big[\gamma\{\cos(\mu - \nu) + \cos(\mu + \nu)\} + \quad \leftarrow (\text{I}) + (\text{II})$$

$$(\gamma - \rho)\{\cos(\mu - \nu) - \cos(\mu + \nu)\}\Big] \quad \leftarrow (\text{I}) - (\text{II})$$

$$= V^+\Big[2\gamma \cos\mu \cos\nu + 2(\gamma - \rho)\sin\mu \sin\nu\Big]$$

Solve for γ

$$2\gamma - \rho = 1 \Rightarrow \gamma = \frac{\rho + 1}{2}$$

(this is the origin of "standing wave ratio" $\frac{1 + |\rho|}{1 - |\rho|}$)

$$= V^+\Big[(\rho + 1)\cos\mu \cos\nu + (1 - \rho)\sin\mu \sin\nu\Big]$$

$$\boxed{V(z,t) = V^+\Big[(\rho + 1)\cos\omega t \cos\beta z + (1 - \rho)\sin\omega t \sin\beta z\Big]}$$

9.2.23) The plot generated from the code creates a standing wave with V_{max} and V_{min}. These V_{max} and V_{min} relate to the two standing waves found in part 9.2.1 by the relationship of the coefficients $(1-\rho)$ and $(1+\rho)$.

If $\rho < 0$ $|(1-\rho)| \to (1+|\rho|)$ and $|(1+\rho)| \to (1-|\rho|)$. Therefore, the standing wave ratio is given by

$$S = \frac{1+|\rho|}{1-|\rho|}$$

V. good that you explained origin of abs. value.

$|\rho|$ can be determined from the plot of the ratios, and if the reflected wave is given, so can the sign.