11.2.1) Using Plots labeled Final_Part_II_1_*

Matthew Jackson
PHYS 513
Final Part 2 (HW 11)
December 2020

From the plots with the above name, one can see that the approximation holds fairly well with the inputs $V_0 = 1V$, $L = 1H$; $C = 1F$, $w = 0.005 s^{-1}$ and $N = 1000$. The $\lambda$ of the transmission line is given by the relation $\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$ (page 230 (7)) which yeilds a wave length of ~ 1257 m (or nodes). With this extremely long wave length, the differences in the solution vary by a maximum 0.52% for Voltage, 0.46% for current, and 0.25% for impedance. It should be noted that the impedance of the analytical solution is $Z = \sqrt{L/C}$ as opposed to $Z = \sqrt{L/C - \frac{(\omega L)^2}{4}}$ as discussed in the paper

The Analytical solution displayed comes from the equations shown in HW #9, specifically

$$\tilde{V}_n(x) = \tilde{V}_n^+\left(e^{-j\beta_n x}\left[1+\tilde{\rho}(x)\right]\right)$$

$$\tilde{I}_n(x) = \frac{\tilde{V}_n^+}{Z_n}\left(e^{-j\beta_n x}\left[1-\tilde{\rho}(x)\right]\right)$$

$$\tilde{Z}_n(x) = \frac{\tilde{V}_n(x)}{\tilde{I}_n(x)}$$

and $\tilde{\rho}(x) = \frac{Z_2-Z_1}{Z_2+Z_1} e^{-2j\beta_n x}$

Given how similar the answers are, I assert that this is a good approximation when Z_L = Z_O

(These plots can be generated with ANIMATE variable and the Z_L variable)

II.2.2) Using Plots labeled Final_Part_II_2_*

From the above plots, much like the other section, the Analytical and Ladder circuit align nicely. When the Load Impedance is set to $3\sqrt{L/C}$, the solutions have a mismatch of 0.0069 in voltage, 0.015 in current, and 0.0017 in impedance. The Error is rather comparable when looking at a load impedance of $10\sqrt{L/C}$ (Final_Part_II_2_2.pdf). Lastly, when supplying a more interesting impedance of 2+2j as the load, the accuracy of values is 0.04 for voltage, 0.043 for current, and 0.005 for impedance.

Since the impedance, current, and voltage align so well, I feel confident stating the Ladder circuit works well as an approximation of a transmission line.