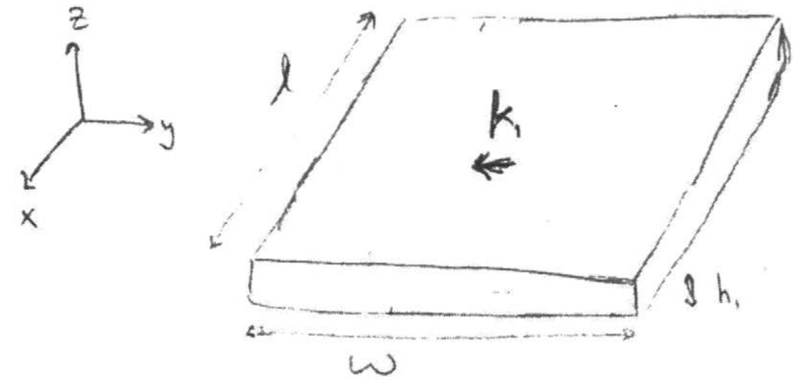
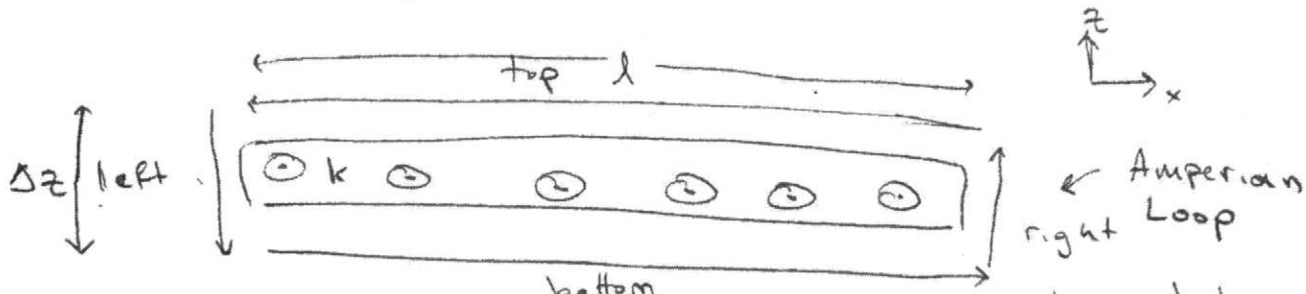
9.1.1)

Matthew Jackson
PHYS 513
HW# 9
November





$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$ (assume w is very large compared to Δz to ignore fringe fields)

$\oint \vec{B} \cdot d\vec{l} = \mu_0 k l$ ← total current enclosed ✓

→ should indicate direction that results from dot product, e.g., B^z_{right}

$B_{right} \cdot \Delta z + B_{left} \cdot \Delta z + B_{top} \cdot l + B_{bottom} \cdot l = \mu_0 k l$

assume $\Delta z \ll l$ such that $(B_{left} + B_{right})\Delta z \ll (B_{top} + B_{bottom}) \cdot l$ ✓

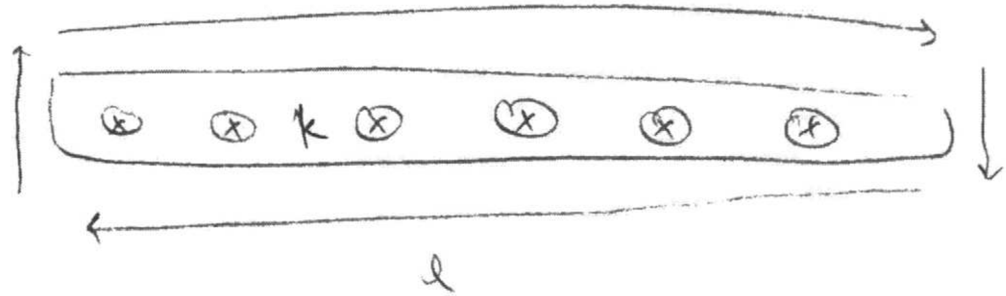
$(B_{top} + B_{bottom})\cancel{l} = \mu_0 k \cancel{l}$

$B_{top} + B_{bottom} = \mu_0 k$

since only k is present $|B_{top}|$ and $|B_{bottom}|$ are same

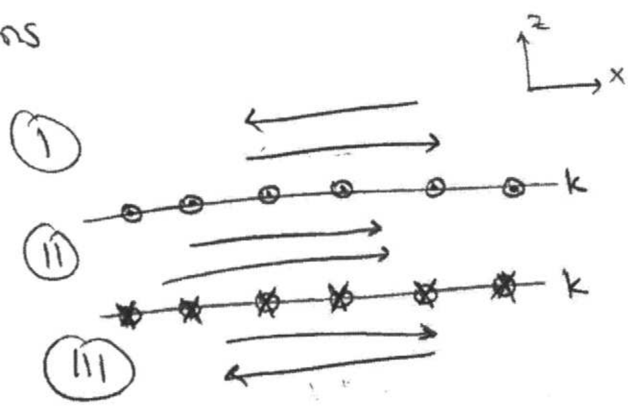
$\boxed{B = \frac{\mu k_0}{2}}$ $\quad B_{top} = \frac{\mu_0 k}{2}(-\hat{x}) \quad B_{bottom} = \frac{\mu_0 k}{2}(\hat{x})$

Same thing can be done with bottom plate



bottom and top are reversed here, but the magnitude's are the same

Since $|B| = \frac{\mu_0 k}{2}$ from each plate, this creates 3 regions



top arrow is from top plate ✓

bottom arrow is from bottom plate ✓

regions I and III cancel out

region II has magnetic field of $B = \mu_0 k\,(\hat{x})$

9.1.2) Using $\mathcal{E}_1 = -\frac{\partial \Phi_m}{\partial t}$ and $\mathcal{E}_1 = -L_1 \frac{\partial I}{\partial t}$, find Φ_m and then L_1 in terms of μ_0, l, and A_1, given as $A_1 = h_1 \omega$

$$\Phi_m = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ is only defined in the duct with $B = \mu_0 K \hat{x}$ (in l direction)

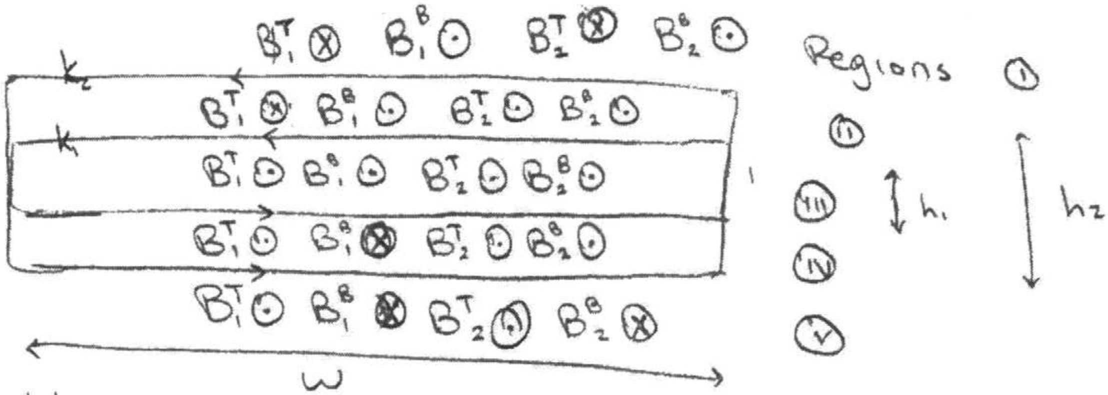
$$\Phi_m = \mu_0 K A_1$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_0 A_1 \frac{\partial K_1}{\partial t} \qquad Kl = I \qquad I = \frac{k}{l}$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t} = L \frac{\partial I}{\partial t}$$

$$\boxed{L_1 = \mu_0 \frac{A_1}{l} \qquad \mathcal{E}_1 = -\mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t}}$$

9.1.3)



Start with B^x_y where x is B from top (T) or bottom plate with current sheet from k_1 (1) or k_2 (2)

assuming $k_1 = k_2$

regions (I) and (V) have total B of

$B - B + B - B = 0$

<u>Region (II)</u>

$-B + B + B + B = 2B = \underline{\mu_0 k}$ ✓

<u>Region (III)</u>

$B + B + B + B = 4B = \underline{2\mu_0 k}$ ✓

<u>Region (IV)</u>

$B + B - B + B = 2B = \underline{\mu_0 k}$ ✓

Look at the areas encased by the inner current area is encased by w and h_1

$$\mathcal{E}_1 = -\frac{d\Phi_m}{dt}$$

$$\Phi_m = \int \vec{B}_{(III)} \cdot d\vec{A}$$

$$\Phi_m = 2\mu_0 k A_1$$

$$\Phi_m = 2\mu_0 \frac{A_1}{l} I$$

$$\mathcal{E}_1 = -2\mu_0 \frac{A_1}{l} \frac{\partial I}{dt}$$

$$\mathcal{E}_2 = -\frac{\partial \Phi_m}{dt}$$

$$\Phi_m = \underbrace{\int \vec{B}_{(II)} \cdot d\vec{A}_{(II)}}_{\text{Region (II)}} + \underbrace{\int B_{(III)} \cdot d\vec{A}_{(III)}}_{\text{Region (III) (same as } \mathcal{E}_1)} + \underbrace{\int \vec{B}_{(IV)} \cdot d\vec{A}_{(IV)}}_{\text{Region (IV)}}$$

Region (II) and Region (IV) are symmetric so combine $B_{(II)}$ and $B_{(IV)}$ ✓

$$\Phi_m = B_{(11)} \left(A_{(11)} + A_{(IV)} \right) + \mathcal{E}_1$$

$A_{(11)} + A_{(IV)}$ is the same as $A_{(11)} + A_{(III)} + A_{(IV)} -$ $A_{(14)}$ which equals $wh_2 - wh_1 \Rightarrow w(h_2 - h_1)$

$$\Phi_m = \mu_0 k \, w(h_2 - h_1) + 2\mu_0 k \, wh_1$$

$$\Phi_m = (\mu_0 w h_2 + \mu_0 w h_1) k$$

$$\mathcal{E}_2 = -\frac{\partial \Phi_m}{\partial t}$$

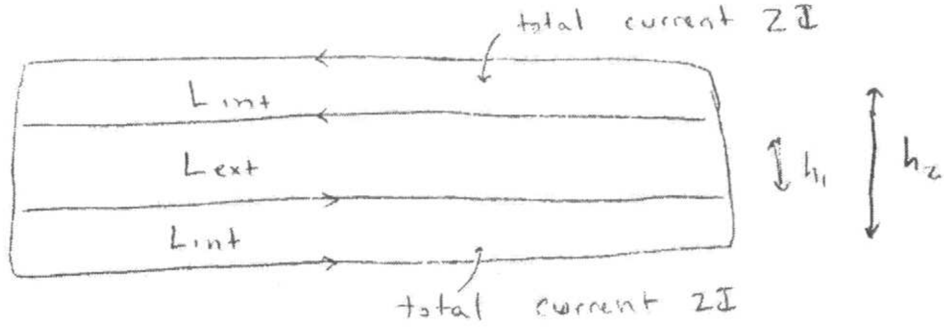
$$\mathcal{E}_2 = -\mu_0 \frac{(A_2 + A_1)}{l} \frac{\partial I}{\partial t}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -2\mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t} - \mu_0 \frac{(A_2 + A_1)}{l} \frac{\partial I}{\partial t}$$

$$\boxed{\begin{aligned} \mathcal{E} &= -\left(\mu_0 \frac{(A_2 + 3A_1)}{l} \right) \frac{\partial I}{\partial t} \\ L &= \mu_0 \frac{(A_2 + 3A_1)}{l} \end{aligned}}$$

9.1.4)



L_{ext} is given by the 2 currents which results in a magnetic field of $2\mu_0 k$

$$L_{ext} = \frac{1}{I_{tot}} \int \vec{B} \cdot d\vec{A}$$

$$L_{ext} = \frac{1}{2I} \int 2\mu_0 \frac{k}{l}\, dA$$

$$L_{ext} = \frac{1}{\cancel{2}(Kl)} \cancel{2}\mu_0 \cancel{K}\, w h_1$$

$$L_{ext} = \frac{\mu_0 w h_1}{l} = \frac{\mu_0 A_1}{l}$$

$$L_{int} = \frac{1}{I_{tot}} \int \vec{B} \cdot d\vec{A}$$

$$L_{int} = \frac{1}{I_{tot}} \left(\mu_0 k\, w (h_2 - h_1) \right)$$ I_{tot} is one kl

$$L_{int} = \frac{1}{\cancel{k}l} \left(\mu_0 \cancel{k} w (h_2 - h_1) \right)$$

9.1.5) Flux linkages seem to be a difference between accounting for the way current will have a mutual inductance with another current in the same region. If you don't account for this, mutual inductance terms get left out.

9.2.1) Given

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$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta z} + \tilde{V}_n^- e^{j\beta z}$$

where

$$V_n(z,t) = \mathrm{Re}\left[\tilde{V}_n(z) e^{j\omega t}\right]$$

similarly

$$\tilde{E}_n(z) = \tilde{E}_n^+ e^{-jk_n z} + \tilde{E}^- e^{jk_n z}$$

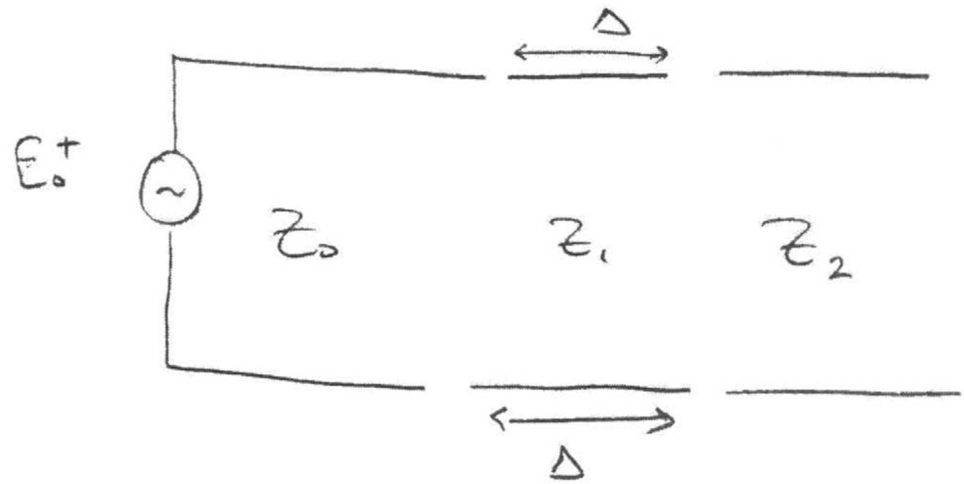
where

$$E_n(z,t) = \mathrm{Re}\left[\tilde{E}_n(z) e^{j\omega t}\right]$$

Assume $\tilde{V}_0^+$ is known. At $z=0$ & $z=\Delta$, V & I are continous. Assume $V_2^- = 0$

find $\tilde{V}_0^-(z)$, $\tilde{V}_1^-(z)$, and $\tilde{V}_2^+(z)$. find $\tilde{V}_0^-(z) / \tilde{V}_0^+(z)$ and $\tilde{V}_1^-(z) / \tilde{V}_1^+(z)$

step 1 break the problem down



$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^L}{Z_L} \quad (\text{I}) \qquad Z_L = Z_1 + Z_2 \quad (\text{III})$$

$$V_0^+ + V_0^- = V_0^L \quad (\text{II})$$

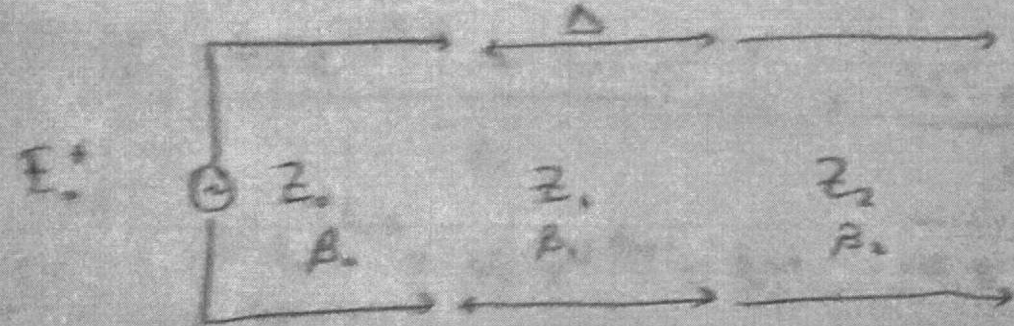
$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$\frac{V_0^+ - V_0}{Z_0} = \frac{V_0^+ + V_0^-}{Z_L}$$

$$Z_L(V_0^+ - V_0^-) = Z_0(V_0^+ + V_0^-)$$

$$V_0^+(Z_L - Z_0) = V_0^-(Z_0 + Z_L)$$

$$V_0^- = \frac{(Z_L - Z_0)}{(Z_0 + Z_L)} V_0^+$$



$$\left(\frac{V_0^+ e^{-j\beta_0 z}}{Z_0} - \frac{V_0^-}{Z_0} e^{j\beta_0 z}\right) = \frac{V_1^+ e^{-j\beta_1 z}}{Z_1}$$

and $V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z} = V_1^+ e^{-j\beta_1 z}$

Start with these 2 equations

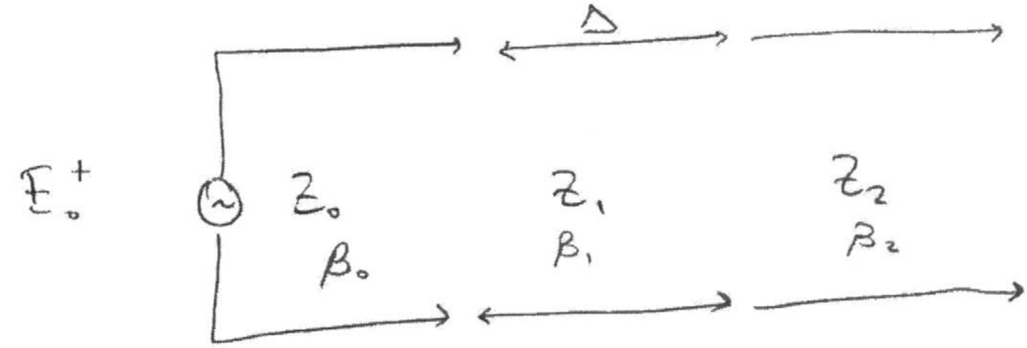
$$\frac{V_0^+ e^{-j\beta_0 z} - V_0^- e^{j\beta_0 z}}{Z_0} = \frac{V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z}}{Z_1}$$

$Z_1 (V_0^+ e^{-j\beta_0 z} - V_0^- e^{j}$ [illegible] $Z_0 (V_0^+ e^{-j}$ [illegible] $e^{j\beta_0 z})$

$(Z_1 - Z_0) V_0^+ e^{-j\beta_0 z} = ($ [illegible] $V_0^- e^{j\beta}$ [illegible]

$$\boxed{V_0^- = \frac{(Z_1 - Z_0)}{(Z_1 + Z} \text{[illegible]}}$$

$$\boxed{\frac{V_0^-}{V_0^+} \text{[illegible]}}$$



$$\left(\frac{V_0^+ e^{-j\beta_0 z}}{Z_0} - \frac{V_0^- e^{j\beta_0 z}}{Z_0}\right) = \frac{V_1^+ e^{-j\beta_1 z}}{Z_1}$$

and $$V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z} = V_1^+ e^{-j\beta_1 z}$$

Start with these 2 equations

$$\frac{V_0^+ e^{-j\beta_0 z} - V_0^- e^{j\beta_0 z}}{Z_0} = \frac{V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z}}{Z_1}$$

$$Z_1\left(V_0^+ e^{-j\beta_0 z} - V_0^- e^{j\beta_0 z}\right) = Z_0\left(V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z}\right)$$

$$(Z_1 - Z_0) V_0^+ e^{-j\beta_0 z} = (Z_1 + Z_0) V_0^- e^{j\beta_0 z}$$ ✓

$$\boxed{V_0^- = \frac{(Z_1 - Z_0)}{(Z_1 + Z_0)} V_0^+ e^{-2j\beta_0 z}}$$ ✓

$$\boxed{\frac{V_0^-}{V_0^+} = \frac{(Z_1 - Z_0)}{(Z_1 + Z_0)} e^{-2j\beta_0 z}}$$ ✓

Using $$\frac{V_0^+ e^{-j\beta_0 z} - V_0^- e^{j\beta_0 z}}{Z_0} = \frac{V_1^+ e^{-j\beta_1 z}}{Z_1}$$

and $$V_0^+ e^{-j\beta_0 z} + V_0^- e^{j\beta_0 z} = V_1^+ e^{-j\beta_1 z}$$

$$V_0^- e^{j\beta_0 z} = V_1^+ e^{-j\beta_1 z} - V_0^+ e^{-j\beta_0 z}$$

$$\frac{V_0^+ e^{-j\beta_0 z} - \left(V_1^+ e^{-j\beta_1 z} - V_0^+ e^{-j\beta_0 z}\right)}{Z_0} = \frac{V_1^+ e^{-j\beta_1 z}}{Z_1}$$

$$\left(2V_0^+ e^{-j\beta_0 z} - V_1^+ e^{-j\beta_1 z}\right) Z_1 = Z_0 V_1^+ e^{-j\beta_1 z}$$

$$2 Z_1 V_0^+ e^{-j\beta_0 z} = (Z_1 + Z_0) V_1^+ e^{-j\beta_1 z}$$

$$V_1^+ = \frac{2 Z_1 V_0^+ e^{-j\beta_0 z}}{Z_1 + Z_0} e^{j\beta_1 z}$$

$$\boxed{V_1^+ = \frac{2 Z_1}{(Z_1 + Z_0)} V_0^+ e^{-j(\beta_0 - \beta_1) z}}$$

$$\boxed{\frac{V_1^+}{V_0^+} = \frac{2 Z_1}{(Z_1 + Z_0)} e^{-j(\beta_0 - \beta_1) z}}$$ ✓

could use τ here.

Do the same process for Z_2

$$\left(\frac{V_1^+ e^{-j\beta_1 z} - V_1^- e^{j\beta_1 z}}{Z_1}\right) = \frac{V_2^+ e^{-j\beta_2 z}}{Z_2}$$

and $V_1^+ e^{-j\beta_1 z} + V_1^- e^{j\beta_1 z} = V_2^+ e^{-j\beta_2 z}$

$$\left(\frac{V_1^+ e^{-j\beta_1 z} - V_1^- e^{j\beta_1 z}}{Z_1}\right) = \frac{\left(V_1^+ e^{-j\beta_1 z} + V_1^- e^{j\beta_1 z}\right)}{Z_2}$$

$$Z_2\left(V_1^+ e^{-j\beta_1 z} - V_1^- e^{j\beta_1 z}\right) = Z_1\left(V_1^+ e^{-j\beta_1 z} + V_1^- e^{j\beta_1 z}\right)$$

$$(Z_2 - Z_1)\, V_1^+ e^{-j\beta_1 z} = (Z_1 + Z_2)\, V_1^- e^{j\beta_1 z}$$

$$V_1^- = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} e^{-2j\beta_1 z}\, V_1^+$$

$$V_1^- = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} e^{-2j\beta_1 z} \left(\frac{2Z_1}{Z_1 + Z_2} e^{-j(\beta_0 - \beta_1)z}\right) V_0^+$$

$$\boxed{V_1^- = \frac{2Z_1(Z_2 - Z_1)}{(Z_1 + Z_2)} V_0^+ e^{-j(\beta_1 + \beta_0)z}}$$

$$\boxed{\frac{V_1^-}{V_1^+} = \frac{(Z_2 - Z_1)}{(Z_1 + Z_2)} e^{-2j\beta_1 z}}$$

✓

$$\frac{\left(V_1^+ e^{-j\beta_1 z} - V_1^- e^{j\beta_1 z}\right)}{Z_1} = \frac{V_2^+ e^{-j\beta_2 z}}{Z_2}$$

$$V_1^+ e^{-j\beta_1 z} + V_1^- e^{j\beta_1 z} = V_2^+ e^{-j\beta_2 z}$$

$$V_1^- e^{j\beta_1 z} = V_2^+ e^{-j\beta_2 z} - V_1^+ e^{-j\beta_1 z}$$

$$\frac{\left(V_1^+ e^{-j\beta_1 z} - \left(V_2^+ e^{-j\beta_2 z} - V_1^+ e^{-j\beta_1 z}\right)\right)}{Z_1} = \frac{V_2^+ e^{-j\beta_2 z}}{Z_2}$$

$$Z_2\left(2V_1^+ e^{-j\beta_1 z} - V_2^+ e^{-j\beta_2 z}\right) = Z_1 V_2^+ e^{-j\beta_2 z}$$

$$2V_1^+ Z_2 e^{-j\beta_1 z} = (Z_1 + Z_2) V_2^+ e^{-j\beta_2 z}$$

$$V_2^+ = \frac{2Z_2}{(Z_1 + Z_2)} e^{-j(\beta_1 - \beta_2)z} V_1^+$$

$$V_2^+ = \frac{2Z_2}{(Z_1 + Z_2)} e^{-j(\beta_1 - \beta_2)z} \frac{2Z_1}{(Z_1 + Z_0)} e^{-j(\beta_0 - \beta_1)z}$$

$$\boxed{V_2^+ = \frac{4Z_1 Z_2}{(Z_1 + Z_2)(Z_1 + Z_0)} e^{-j(\beta_0 - \beta_2)z} V_0^+}$$

could use τ_1, τ_2

$$\frac{\left(V_1^+ e^{-j\beta_1 z} - V_1^- e^{j\beta_1 z}\right)}{Z_1} = \frac{V_2^+ e^{-j\beta_2 z}}{Z_2}$$

$$V_1^+ e^{-j\beta_1 z} + V_1^- e^{j\beta_1 z} = V_2^+ e^{-j\beta_2 z}$$

$$V_1^- e^{j\beta_1 z} = V_2^+ e^{-j\beta_2 z} - V_1^+ e^{-j\beta_1 z}$$

$$\frac{\left(V_1^+ e^{-j\beta_1 z} - \left(V_2^+ e^{-j\beta_2 z} - V_1^+ e^{-j\beta_1 z}\right)\right)}{Z_1} = \frac{V_2^+ e^{-j\beta_2 z}}{Z_2}$$

$$Z_2\left(2V_1^+ e^{-j\beta_1 z} - V_2^+ e^{-j\beta_2 z}\right) = Z_1 V_2^+ e^{-j\beta_2 z}$$

$$2V_1^+ Z_2 e^{-j\beta_1 z} = (Z_1 + Z_2) V_2^+ e^{-j\beta_2 z}$$

$$V_2^+ = \frac{2Z_2}{(Z_1+Z_2)} e^{-j(\beta_1-\beta_2)z} V_1^+$$

$$V_2^+ = \frac{2Z_2}{(Z_1+Z_2)} e^{-j(\beta_1-\beta_2)z} \frac{2Z_1}{(Z_1+Z_0)} e^{-j(\beta_0-\beta_1)z}$$

$$\boxed{V_2^+ = \frac{4Z_1 Z_2}{(Z_1+Z_2)(Z_1+Z_0)} e^{-j(\beta_0-\beta_2)z} V_0^+}$$

9.2.~~2~~1) Show

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HW # 9
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$$V(z,t) = V^{+}[\cos(\omega t - \beta z) + \rho\cos(\omega t + \beta z)]$$

can be written as two standing waves

$$V(z,t) = A\cos(\omega t)\cos(\beta z) + B\sin(\omega t)\sin(\beta z)$$

and find A + B

$$\cos(\omega t - \beta z) \rightarrow \cos(\mu - \nu)$$
$$\cos(\omega t + \beta z) \rightarrow \cos(\mu + \nu)$$

$$\cos(\mu - \nu) = \cos\mu\cos\nu + \sin\mu\sin\nu \quad (I)$$
$$\cos(\mu + \nu) = \cos\mu\cos\nu - \sin\mu\sin\nu \quad (II)$$

$$(I) + (II) = 2\cos\mu\cos\nu = \underline{\cos(\mu-\nu) + \cos(\mu+\nu)}$$
$$(I) - (II) = 2\sin\mu\sin\nu = \underline{\cos(\mu-\nu) - \cos(\mu+\nu)}$$

define γ such that

$$\gamma + (\gamma - \rho) = 1$$

$$V(z,t) = V^+\Big[\big(\gamma + (\gamma - \rho)\big)\cos(\mu - \nu) + \big(\gamma + (\rho - \gamma)\big)\cos(\mu + \nu)\Big]$$

$$= V^+\Big[\gamma\{\cos(\mu - \nu) + \cos(\mu + \nu)\} + \qquad \leftarrow (I) + (II)$$

$$(\gamma - \rho)\{\cos(\mu - \nu) - \cos(\mu + \nu)\}\Big] \qquad \leftarrow (I) - (II)$$

$$= V^+\Big[2\gamma \cos\mu \cos\nu + 2(\gamma - \rho)\sin\mu \sin\nu\Big]$$

Solve for γ

$$2\gamma - \rho = 1 \Rightarrow \gamma = \frac{\rho + 1}{2}$$

$$= V^+\Big[(\rho + 1)\cos\mu\cos\nu + (1 - \rho)\sin\mu\sin\nu\Big]$$

$$\boxed{V(z,t) = V^+\Big[(\rho + 1)\cos\omega t \cos\beta z + (1 - \rho)\sin\omega t \sin\beta z\Big]}$$

9.2.2.3) The plot generated from the code creates a standing wave with V_{max} and V_{min}. These V_{max} and V_{min} relate to the two standing waves found in part 9.2.1 by the relationship of the coefficients $(1-\rho)$ and $(1+\rho)$.

If $\rho < 0$ $|(1-\rho)| \to (1+|\rho|)$ and $|(1+\rho)| \to (1-|\rho|)$. Therefore, the standing wave ratio is given by

$$S = \frac{1+|\rho|}{1-|\rho|}$$

Good that you tried to explain the abs values in S. what about $\rho > 0$ case?

$|\rho|$ can be determined from the plot of the ratios, and if the reflected wave is given, so can the sign.