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PHYS 513
HW # 9
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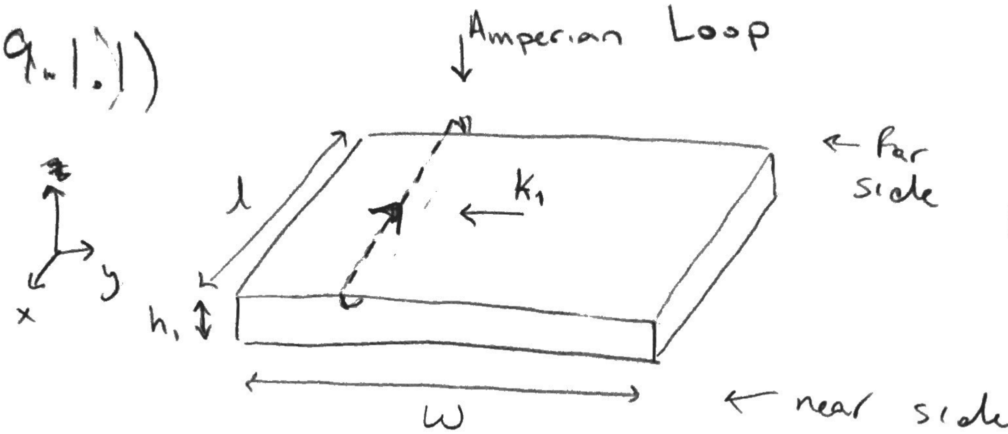
9.1.1)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \rightarrow \int_0^{l} -k_1 \cdot dl$$ for the top plate

Amperian loop is rectangle

$h_1 << w$ and $h_1 << l$

$$\oint \vec{B}\cdot d\vec{l} = \cancel{B_{near}\cdot h_1} + B_{up}\cdot l - \cancel{B_{far}\cdot h_1} + B_{bottom}\cdot l$$

assume $h_1 \rightarrow 0$ compared to l

$$\oint \vec{B}\cdot d\vec{l} = B_{bottom}\cdot l - B_{top}\cdot l$$

$$B_{bottom}\cdot \cancel{l} - B_{top}\cdot \cancel{l} = -\mu_0 k \cdot \cancel{l}$$

$$B_{bottom} - B_{top} = -\mu_0 k$$

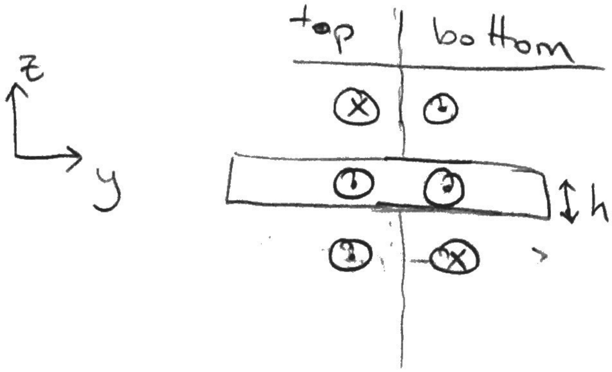
$$B_{top} - B_{bottom} = \mu_0 k$$ top plate

the $-\mu_0 k$ becomes $\mu_0 k$ for the bottom plate

$$B_{bottom} - B_{top} = \mu_0 k$$ bottom plate

look at the fields along

zy plane



Region 1 $\quad B_{top} - B_{top} = 0$

Region 2 $\quad B_{bottom} + B_{top} \neq 0$

Region 3 $\quad B_{bottom} - B_{bottom} = 0$

Given that $B_{bottom} - B_{top} = \mu_0 K$ (this is based on direction)

$B_{bottom}\,\hat{x} + B_{top}(+\hat{x}) = \mu_0 k$

$B_{bottom} + B_{top} = \mu_0 k$

assume B_{bottom} and B_{top} are same given they come from same source

$B = \frac{\mu_0 k}{2}$

$\boxed{\text{so Region 2 } B \text{ is } \mu_0 k}$

9.1.2) Using $\mathcal{E}_1 = -\frac{\partial \Phi_m}{\partial t}$ and $\mathcal{E}_1 = -L_1 \frac{\partial I}{\partial t}$, find Φ_m and then L_1 in terms of μ_0, l, and A_1, given as $A_1 = h_1 w$

$$\Phi_m = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ is only defined in the duct with $B = \mu_0 k \hat{x}$ (in l direction)

$$\Phi_m = \mu_0 K A_1$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_0 A_1 \frac{\partial k_1}{\partial t} \qquad k l = I \qquad I = \frac{k}{l}$$

$$\frac{\partial \Phi_m}{\partial t} = \mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t} = L \frac{\partial I}{\partial t}$$

$$\boxed{L_1 = \mu_0 \frac{A_1}{l} \qquad \mathcal{E}_1 = -\mu_0 \frac{A_1}{l} \frac{\partial I}{\partial t}}$$

9.1.3) $\mathcal{E}_1 = -\mu_0 \frac{A_1}{l} \frac{dI_1}{dt}$

since the difference for $\mathcal{E}_2$ is $K_2 (I_2/l)$ and A_2, change the values

$$\mathcal{E}_2 = -\mu_0 \frac{A_2}{l} \frac{dI_2}{dt}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$= -\mu_0 \frac{A_1}{l} \frac{dI_1}{dt} - \mu_0 \frac{A_2}{l} \frac{dI_2}{dt}$$

$$\boxed{\mathcal{E} = -\frac{\mu_0}{l}\left(A_1 \frac{dI_1}{dt} + A_2 \frac{dI_2}{dt} \right)}$$