10.1)

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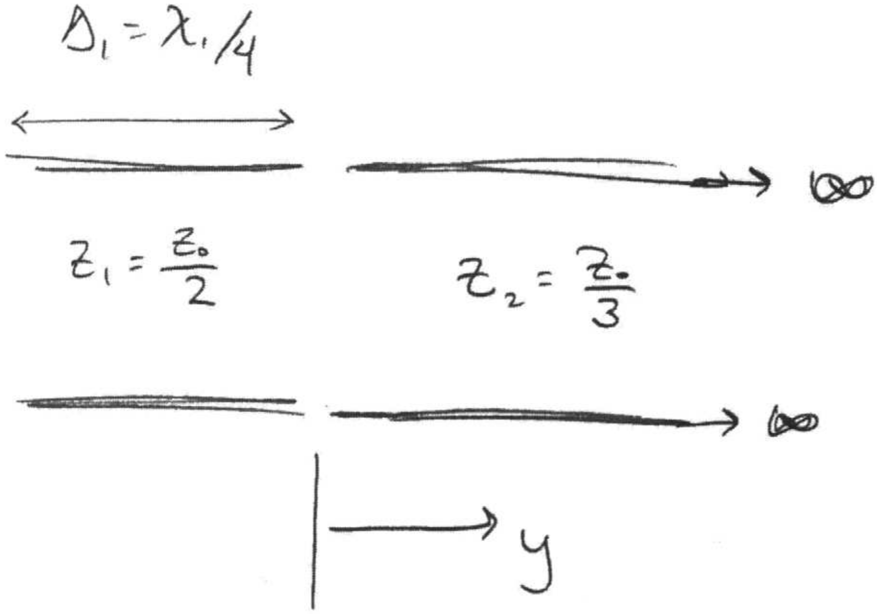
PHYS 513
HW # 10
November 30



10.1.1.1) What is $\tilde{\rho}_0(0)$?

Remember $$Z_n(-y) = Z_n \frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)}$$

at boundary

$$Z_1\left[\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right] = Z_2\left[\frac{1+\tilde{\rho}_2(0)}{1-\tilde{\rho}_2(0)}\right]$$

$\tilde{\rho}_2(0) = \tilde{\rho}_2 = 0$ because there is no reflected wave

$$z_1\left[\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right]=z_2$$

$$z_1(1+\tilde{\rho}_1(0))=z_2(1-\tilde{\rho}_1(0))$$

$$(z_1+z_2)\tilde{\rho}_1(0)=z_2-z_1$$

$$\tilde{\rho}_1(0)=\frac{z_2-z_1}{z_1+z_2}=\frac{\cancel{z_1}}{\cancel{z_1}}\frac{\left(\frac{z_2}{z_1}-1\right)}{\left(\frac{z_2}{z_1}+1\right)}$$

$$\tilde{\rho}_1(0)=\frac{\frac{\cancel{z_0}/3}{\cancel{z_0}/2}-1\rightarrow\frac{2}{3}-1}{\frac{\cancel{z_0}/3}{\cancel{z_0}/2}+1\rightarrow\frac{2}{3}+1}$$

$$\tilde{\rho}_1(0)=\frac{-1/3}{5/3}$$

$$\boxed{\tilde{\rho}_1(0)=-1/5}$$

10.1.1.2) find $Z_1(-\lambda_1/4)$

$Z_n(y) = Z_n\left[\frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)}\right]$ and $\tilde{\rho}_n(y) = \tilde{\rho}_0 e^{2j\beta y}$

$\tilde{\rho}_1(0) = \tilde{\rho}_1$

$\tilde{\rho}_1(-\lambda_1/4) = \tilde{\rho}_1(0)\, e^{2j\,\beta(-\lambda_1/4)}$ $\quad \beta = 2\pi/\lambda_1$

$= -\frac{1}{5}\, e^{\cancel{2}j\,(\cancel{2}\pi/\cancel{\lambda_1})(-\cancel{\lambda_1}/\cancel{4})}$

$= -\frac{1}{5}\, e^{-j\pi}$ $\quad e^{-j\pi} = -1$

$\tilde{\rho}_1(-\lambda_1/4) = \frac{1}{5}$

$\tilde{Z}_1(-\lambda_1/4) = Z_1\left[\frac{1+\tilde{\rho}_1(-\lambda_1/4)}{1-\tilde{\rho}_1(-\lambda_1/4)}\right]$

$= \frac{Z_0}{2}\left[\frac{1+1/5}{1-1/5}\right] \begin{array}{l}\rightarrow 6/5 \\ \rightarrow 4/5\end{array}$

$= \frac{Z_0}{2}\,\frac{6}{4}$

$$\boxed{\tilde{Z}_1(-\lambda/4) = \frac{3Z_0}{4}}$$

10.1.2.1) Find r & x from $z_2(0)/z_1(0)$

$$\frac{z_2(0)}{z_1(0)} = \frac{z_0/3}{z_0/2} = \frac{2}{3}$$

$$\frac{z_2(0)}{z_1(0)} = \frac{2}{3} + 0j \qquad \boxed{\begin{matrix} r = 2/3 \\ x = 0 \end{matrix}}$$

10.1.2.2) Reference slide 2 in

HW10_Smith Charts.pptx

$\boxed{\rho_1(0) = +1/5}$

10.1.2.3) Reference slide 3 in

HW10_ SmithCharts .pptx

$$\frac{z_2}{z_1} = 1.5$$

10.1.2.4) $\boxed{r = 1.5 \ , \ x = 0}$

10.1.2.5) $\boxed{z_1(-\lambda_1/4) = 1.5 \cdot z_0/2 = \frac{3}{4} z_0}$

10.2)

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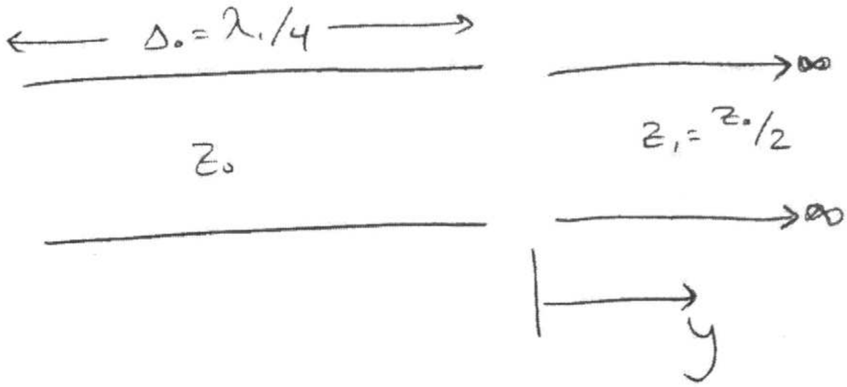

10.2.1.1) What is $\tilde{\rho}_0(0)$?

$$Z_n(0) = \tilde{Z}_0 \frac{(1+\tilde{\rho}_0(0))}{(1-\tilde{\rho}(0))} = \tilde{Z}_1 \frac{(1+\tilde{\rho}_1(0))}{(1-\tilde{\rho}_1(0))}$$

$\tilde{\rho}_1 = 0$ because no reflected wave

$$\tilde{\rho}_0(0) = \frac{\frac{Z_1}{Z_0} - 1}{\frac{Z_1}{Z_0} + 1} \qquad Z_1 = \frac{Z_0}{2}$$

$$\tilde{\rho}_0(0) = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} \rightarrow \frac{-\frac{1}{2}}{\frac{3}{2}}$$

$$\boxed{\tilde{\rho}_0(0) = \frac{-1}{3}}$$

10.2.1.2) What is $Z_0(-\lambda/4)$

$\tilde{\rho}_0(y) = \tilde{\rho}_0(0)\, e^{2j\beta y}$ $\qquad \beta = 2\pi/\lambda$

$\tilde{\rho}_0(-\lambda/4) = \frac{-1}{3}\, e^{2j(2\pi/\lambda)(-\lambda/4)}$

$\tilde{\rho}_0(-\lambda/4) = \frac{-1}{3}\, e^{-j\pi}$ $\qquad e^{-j\pi} = -1$

$\tilde{\rho}_0(-\lambda/4) = \frac{1}{3}$

$$Z_n(y) = Z_0\left[\frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)}\right]$$

$$Z_0\left(-\frac{\lambda}{4}\right) = Z_0\left[\frac{1+\tilde{\rho}_0(-\lambda/4)}{1-\tilde{\rho}_0(-\lambda/4)}\right]$$

$$= Z_0\left[\frac{1+1/3}{1-1/3}\right] \to \frac{4/3}{2/3}$$

$$\boxed{Z_0\left(-\frac{\lambda}{4}\right) = 2Z_0}$$

10.2.2.1) Compute $z_1(0)/z_0(0)$ and r & x

$$\frac{z_1(0)}{z_0(0)} = \frac{1}{2}$$ $\boxed{r = 1/2 \quad x = 0}$

10.2.2.2) Reference slide 5 from

HW10_Smith Charts.pptx

$\rho_0(0) = 0.5$

10.2.2.3) Reference slide 6 from

HW10_Smith Charts.pptx

$$\frac{z_*}{z_0} = 2$$

10.2.2.4) $\boxed{r = 2 \quad x = 0}$ ✓

10.2.2.5) $\boxed{z_0(-\lambda/4) = 2 z_0}$ ✓

10.3)

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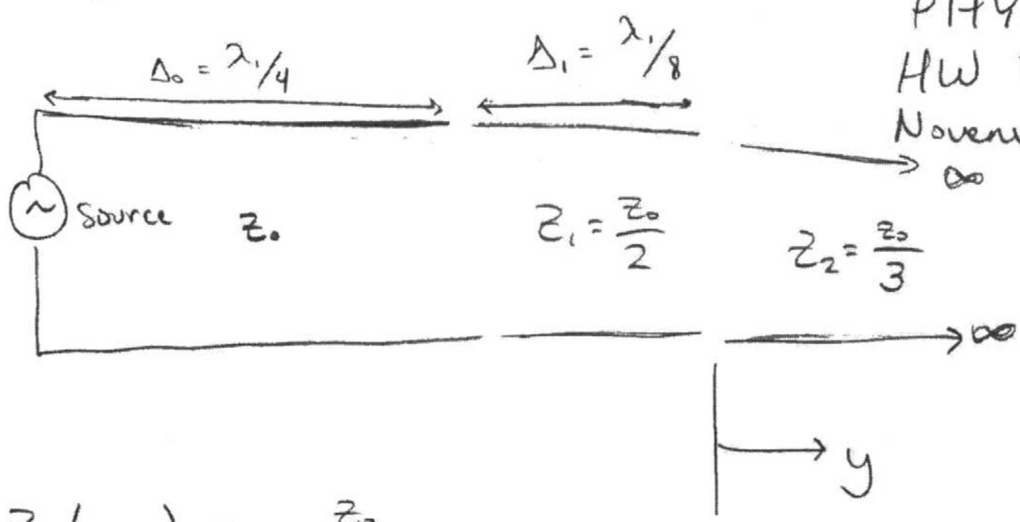

$$Z_2(y>0) = Z_2 = \frac{Z_0}{3}$$

$$Z_n(0) = Z_2 = Z_1 \frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}$$

$$Z_2(1-\tilde{\rho}_1(0)) = Z_1(1+\tilde{\rho}_1(0))$$

$$Z_2 - Z_1 = (Z_2+Z_1)\tilde{\rho}_1(0)$$

$$\tilde{\rho}_1(0) = \frac{Z_2 - Z_1}{(Z_2+Z_1)} = \frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}$$

$$\tilde{\rho}_1(0) = \frac{\frac{Z_0/3}{Z_0/2} - 1}{\frac{Z_0/3}{Z_0/2} + 1} \rightarrow \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1}$$

$$\tilde{\rho}_1(0) = \frac{-1}{5}$$ ✓ (Also see slide 8)

$$\tilde{\rho}_1\left(-\frac{\lambda_1}{8}\right) = \tilde{\rho}_1(0)\, e^{2j\beta\left(-\frac{\lambda}{8}\right)} \qquad \beta = \frac{2\pi}{\lambda_1}$$

$$\tilde{\rho}_1\left(-\frac{\lambda}{8}\right) = \frac{-1}{5}\, e^{\cancel{4}\pi j \frac{1}{\cancel{\lambda_1}}\left(\frac{-\cancel{\lambda}}{8\,2}\right)}$$

$$\tilde{\rho}_1\left(\frac{-\lambda}{8}\right) = \frac{-1}{5}\, e^{-\frac{\pi}{2}j} \qquad e^{-\frac{\pi}{2}j} = -j$$

$$\underline{\tilde{\rho}_1\left(-\frac{\lambda}{8}\right) = \frac{1}{5}j} \quad \text{(Also see slide 9)}$$

$$Z_n\left(\frac{-\lambda}{8}\right) = Z_0\,\frac{1+\tilde{\rho}_0\left(\frac{-\lambda_1}{8}\right)}{1-\tilde{\rho}_0\left(\frac{-\lambda_1}{8}\right)} = Z_1 \underbrace{\left[\frac{1+\tilde{\rho}_1\left(-\lambda_1/8\right)}{1-\tilde{\rho}_1\left(-\lambda_1/8\right)}\right]}_{\gamma}$$

$$Z_0\left[\frac{1+\tilde{\rho}_0\left(-\lambda_1/8\right)}{1-\tilde{\rho}_0\left(-\lambda/8\right)}\right] = Z_1\gamma$$

$$Z_0\left(1+\tilde{\rho}_0\left(-\lambda/8\right)\right) = Z_1\gamma\left(1-\tilde{\rho}_0\left(-\lambda/8\right)\right)$$

$$\left(Z_0+\gamma Z_1\right)\tilde{\rho}_0\left(-\lambda/8\right) = Z_1\gamma - Z_0$$

$$\tilde{\rho}_0\left(-\lambda/8\right) = \frac{Z_1\gamma - Z_0}{Z_1\gamma + Z_0} \rightarrow \underline{\frac{\frac{Z_1\gamma}{Z_0} - 1}{\frac{Z_1\gamma}{Z_0} + 1}}$$

$$\frac{Z_1 \gamma}{Z_0} \rightarrow r + xj \qquad \gamma = \left[\frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)}\right]$$

$$\frac{Z_1}{Z_0}\left[\frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)}\right] = r + xj$$

$$\frac{\frac{Z_0}{2}}{Z_0}\left[\frac{1+\frac{1}{5}j}{1-\frac{1}{5}j}\right] = r + xj$$

$$\frac{1}{2}\left[\frac{(1+1/5j)(1+1/5j)}{(1-1/5j)(1+1/5j)}\right] \begin{array}{l} \rightarrow 1^2 + 1/5j + 1/5j + (1/5j)^2 \\ \rightarrow 1^2 + \cancel{1/5j} - \cancel{1/5j} - (1/5j)^2 \end{array}$$

$$\frac{1}{2}\left[\frac{\frac{25}{25} - \frac{1}{25} + 2/5j}{1^2 + 1/25}\right] \rightarrow \frac{\frac{24}{25} + \frac{10}{25}j}{26/25}$$

$$\frac{1}{2}\ \frac{24+10j}{26}$$

$$\frac{12+5j}{26} = r + xj$$

$r = \frac{6}{13}$ $\quad x = \frac{5}{26}$ $\quad$ see smith Chart slide 10

$r = 0.462 \quad x = 0.192$

$\tilde{\rho}_0(-\lambda_1/8) = .39 \angle 153$ (from Smith chart Slide 10)

$\tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4) = 0.39\, e^{j\frac{153\pi}{180}}\, e^{2j\frac{2\pi}{\lambda}\left(-\frac{\lambda_0}{4}\right)}$

$\tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4) = 0.39\, e^{j\left(\frac{153\pi}{180} - \pi\right)}$

(see smith chart Slide 11)

$$Z_0(-\lambda_1/8 - \lambda_0/4) = Z_0\left[\frac{1 + \tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4)}{1 - \tilde{\rho}_0(-\lambda_1/8 - \lambda_0/4)}\right]$$

$$\boxed{Z_0(-\lambda_1/8 - \lambda_0/4) = \left[1.85 - 0.77j\right] Z_0}$$

(from smith chart slide 11)

→ logic for this step here

$\tilde{\rho}_0(y) = \rho_0(0)\, e^{2j\beta y}$ → $\tilde{\rho}_0(y_1 + y_2) = \rho_0(0)\, e^{j\beta(y_1 + y_2)}$

$\tilde{\rho}_0(y_1 + y_2) = \boxed{\rho_0(0)\, e^{j\beta(y_1)}}\, e^{j\beta(y_2)}$

$\tilde{\rho}_0(y_1 + y_2) = \rho_0(y_1)\, e^{j\beta(y_2)}$