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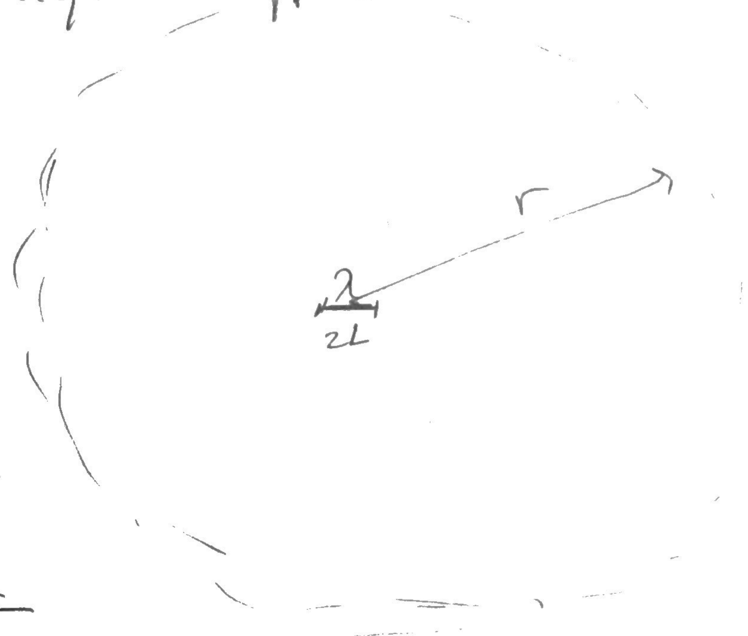
9/1/20
PHYS 513

HW 1.3

a.) Show that E from a finite line of length $2L$ with charge density λ is the same as Gauss's when taking the Taylor approx of the E field.

$r >> L$

Gauss's Law



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{2\lambda L}{\epsilon_0}$$

$$E = \boxed{\frac{2\lambda L}{4\pi\epsilon_0 r^2}\hat{r}} \rightarrow \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

Starting with E field from line charge

$$\vec{E}_z(z) = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2+L^2}}\hat{z}$$ ← from Griffith's example 2.2

$$= \frac{2\lambda}{4\pi\epsilon_0} \frac{L}{z \cdot z\sqrt{1+\left(\frac{L^2}{z^2}\right)}}\hat{z}$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \frac{L/z^2}{\sqrt{1+(L^2/z^2)}} \cdot \frac{L}{L}\hat{z}$$

$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 L} \frac{L^2/z^2}{\sqrt{1+(L^2/z^2)}} \cdot \hat{z}$$

define $f(x) = \frac{x}{\sqrt{1+x}}$ so $x = \frac{L^2}{z^2}$ & $\frac{L^2}{z^2} \ll 1$

$$f(x) = f(0) + x \cdot f'(0) + O(x^2)$$

$$f(0) = \frac{0}{\sqrt{1+0}} = 0$$

$$f'(x) = \frac{(x+2)}{2(1+x)^{3/2}} \longrightarrow f'(0) = \frac{\cancel{0}+2}{2(1+\cancel{0})^{3/2}} = 1$$

$$f(x) = x + O(x^2)$$

$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 L} f(x)\hat{z} \rightarrow \frac{2\lambda}{4\pi\epsilon_0 L} \cdot x\,\hat{z}$$

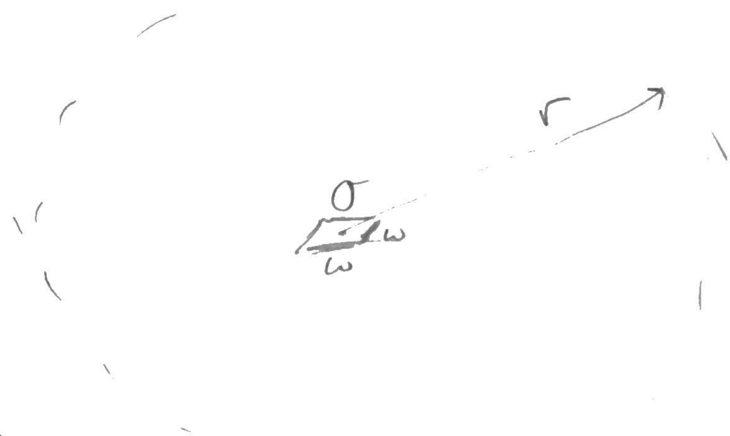
(2)

$$\vec{E}_z(z) = \frac{2\lambda}{4\pi\epsilon_0 \cancel{L}} \frac{L^{\cancel{2}}}{z^2} \rightarrow \boxed{\vec{E}_z(z) = \frac{2\lambda L}{4\pi\epsilon_0 z}\hat{z}}$$

b.) Show that E from sheet-charge of sides w and w with charge density σ is the same as Gauss's law when taking the Taylor approx of the E field.

Gauss's Law

$r \gg w$



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\sigma w^2}{\epsilon_0}$$

$$\vec{E} = \boxed{\frac{\sigma w^2}{4\pi \epsilon_0 r^2} \hat{r}} \rightarrow \frac{q}{4\pi \epsilon_0 r^2}$$

Start $\vec{E}_z(z) = \frac{\sigma_0}{\pi\epsilon_0} \tan^{-1}\left[\frac{W^2}{4z} \frac{1}{\sqrt{z^2 + \frac{W^2}{2}}}\right]\hat{z}$

$$= \frac{\sigma}{\pi\epsilon_0} \tan^{-1}\left[\frac{w^2}{4z} \frac{1}{z\sqrt{1+\frac{w^2}{2z^2}}}\right]\hat{z}$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} \tan^{-1}\left[\frac{1}{2} \frac{w^2}{2z^2} \frac{1}{\sqrt{1+\frac{w^2}{2z^2}}}\right]\hat{z}$$

define $f(x) = \tan^{-1}\left[\frac{1}{2} \frac{x}{\sqrt{1+x}}\right]$

so $x = \frac{w^2}{2z^2}$ & $\frac{w^2}{2z^2} << 1$

$$f(x) = f(0) + x \cdot f'(0) + O(x^2)$$

$$f(0) = \tan^{-1}\left[\frac{1}{2} \frac{0}{0}\right] = 0$$

$f'(x) = \frac{1}{(x+2)\sqrt{1+x}}$ $\quad f'(0) = \frac{1}{(0+2)\sqrt{1+0}} = \frac{1}{2}$

$$f(x) = 0 + x\frac{1}{2} + O(x^2)$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} f(x)\hat{z} \rightarrow \frac{\sigma}{\pi\epsilon_0} \cdot \frac{1}{2} x \hat{z}$$

$$\vec{E}_z(z) = \frac{\sigma}{\pi\epsilon_0} \frac{1}{2} \frac{w^2}{2z^2} \hat{z}$$

$$\boxed{\vec{E}_z(z) = \frac{w^2\sigma}{4\pi\epsilon_0 z^2} \hat{z}}$$