11.2.1) Using Plots labeled Final_Part_II_1_*

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From the plots with the above name, one can see that the approximation holds fairly well with the inputs $V_0 = 1V$, $L = 1H$; $C = 1F$, $\omega = 0.005 s^{-1}$ and $N = 1000$. The λ of the transmission line is given by the relation

In this equation, L and C are per unit length.

$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$ (page 230 (7)) which yeilds a wave length of ~ 1257 m (or nodes). With this extremely long wave length, the differences in the solution vary by a maximum 0.52% for Voltage, 0.46% for current, and 0.25% for impedance. It should be noted that the impedance of the analytical solution is $Z = \sqrt{L/C}$ as opposed to $Z = \sqrt{L/C - \frac{(\omega L)^2}{4}}$ as discussed in the paper

This equation is the impedance of a discrete ladder network with an infinite number of nodes. So the comparison is not so direct.

The Analytical solution displayed comes from the equations shown in HW #9, specifically

$$\tilde{V}_n(x) = \tilde{V}_n^+\left(e^{-j\beta_n x}\left[1+\tilde{\rho}(x)\right]\right)$$

$$\tilde{I}_n(x) = \frac{\tilde{V}_n^+}{Z_n}\left(e^{-j\beta_n x}\left[1-\tilde{\rho}(x)\right]\right)$$

$$\tilde{Z}_n(x) = \frac{\tilde{V}_n(x)}{\tilde{I}_n(x)}$$

I was hoping students would look back at the previous HW and make a comparison like this!

and $\tilde{\rho}(x) = \dfrac{Z_2 - Z_1}{Z_2 + Z_1} e^{-2j\beta_n x}$

Given how similar the answers are, I assert that this is a good approximation when Z_L = Z_0

(These plots can be generated with ANIMATE variable and the Z_L variable)

11.2.2) Using Plots labeled
Final_Part_II_2_*

From the above plots, much like the other section, the Analytical and Ladder circuit align nicely. When the Load Impedance is set to $3\sqrt{L/C}$, the solutions have a mismatch of 0.0069 in voltage, 0.015 in current, and 0.0017 in impedance. The Error is rather comparable when looking at a load impedance of $10\sqrt{L/C}$ (Final_Part_II_2_2.pdf). Lastly, when supplying a more interesting impedance of $2+2j$ as the load, the accuracy of values is 0.04 for voltage, 0.043 for current, and 0.005 for impedance.

Since the impedance, current, and voltage align so well, I feel confident stating the Ladder circuit works well as an approximation of a transmission line.

11.2.3) Using plots titled final_Part_II_3_*

As can be seen with the 5 plots I have submitted, increasing ω causes the approximation to fall apart. This can best be see at the characteristic imdedance at $\omega = 2$, where the ladder circuit has increasing voltages and currents at each node. Going above $\omega=2$ actually caused the ladder circuit to crash meaning I couldn't show plots above $\omega=2$.

The characteristic impedance is given by $z_0 = \sqrt{(L/C) - (\frac{\omega^2 L^2}{4}} = 0$. Since the equation presented is the equivalent impedance for the system, increasing ω lowers the impedance, which further diverges from the $\sqrt{(L/C)}$ that was being used by my code. Lastly, the ω values plotted were [0.005, 0.02, 0.1, 0.5, 2] with an accidental double plot of 0.005. The approximation holds for 0.02, but $\omega = 0.1$ shows a rather wave like impedance, and $\omega = 0.5$ shows very large errors. One thing I found rather interesting was that the Voltage and current kept increasing at $\omega = 2$, and I believe this is due to there being no impedance, so everything harmonically adds as the circuit goes down the nodes.

The approximation breaks down for short wavelengths. In this case, major differences between ladder and continuous case are not surprising.