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PHYS 513
HW # 8
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8.3.1) 5.8f

Show that

$$V = V_+ e^{-j\beta z} + V_+ |\rho| e^{j(\theta_\rho + \beta z)}$$

can be written as a standing and travelling wave

Start with magnitude

$$|V|^2 = \left(\underline{V}_+ e^{-j\beta z} + \underline{V}_+ |\rho| e^{j(\theta_\rho + \beta z)} \right) *$$

$$\left(\underline{V}_+^* e^{j\beta z} + \underline{V}_+^* |\rho| e^{-j(\theta_\rho + \beta z)} \right)$$

$$= |V_+|^2 \left(e^{-j\beta z} e^{j\beta z} + |\rho|^2 e^{j(\theta_\rho + \beta z)} e^{-j(\theta_\rho + \beta z)} + e^{j\beta z} |\rho| e^{j(\theta_\rho + \beta z)} + |\rho| e^{-j(\theta_\rho + \beta z)} e^{-j\beta z} \right)$$

$$= |V_+|^2 \left(1 + |\rho|^2 + |\rho| \left(e^{j(\theta_\rho + 2\beta z)} + e^{-j(\theta_\rho + 2\beta z)} \right) \right)$$

$a + bi + a - bi = 2a$

$$|V|^2 = |V_+|^2 \left(1 + |\rho|^2 + |\rho| \, 2 \operatorname{Re}\left(e^{j(\theta_\rho + 2\beta z)} \right) \right)$$

$$\boxed{|V| = \sqrt{|V_+|^2 \left(1 + |\rho|^2 + |\rho| \, 2 \operatorname{Re}\left(e^{j(\theta_\rho + 2\beta z)} \right) \right)}}$$

$$|V| = \sqrt{|V_+|^2 (1 + |\rho|^2 + |\rho| 2 \cos(2(\omega t + \beta z) + \Theta_\rho)}$$

$$\omega t = -\frac{\Theta_\rho}{2} + \pi/4$$

$$= \sqrt{|V_+|^2 (1 + |\rho|^2 + |\rho| 2 \cos(2(\frac{-\Theta_\rho}{2} + \frac{\pi}{4} + \beta z) + \Theta_\rho))}$$

$$= \sqrt{|V_+|^2 (1 + |\rho|^2 + |\rho| 2 \cos(\cancel{-\Theta_\rho} + \frac{\pi}{2} + 2\beta z + \cancel{\Theta_\rho}))}$$

$$\boxed{|V| = \sqrt{|V_+|^2 (1 + |\rho|^2 + 2|\rho| \cos(2\beta z + \frac{\pi}{2})}}$$