Matthew Jackson
PHYS 513
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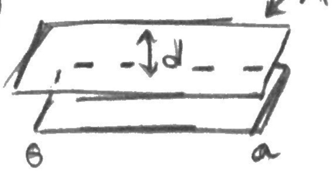
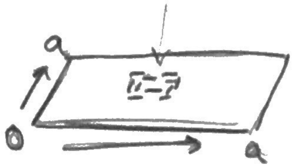
HW 3

3.1.1)



Gauss's Law
(Do the pill box method)

What are the bounds of the pill box? Either way, one end cap will have a zero electric field. I'll discuss this more in class.



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

The problem statement did not specify "a" or that the plates were square. So technically you should be using A instead of a^2.

the only direction I care about is away from plate

$$\underset{A \to a^2}{\iint} \int \vec{E} \cdot d\vec{a} = \frac{\underset{V}{\iiint} \rho \, dV}{\epsilon_0} \leftarrow \frac{\sigma a^2}{2}$$

Why 1/2 here?

$$E a^2 = \frac{\sigma a^2}{2\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{x}$$ ← 2 plates so multiply by 2

This is actually the wrong field between plates.

$$\psi = -\int_0^d \vec{E} \cdot d\vec{l}$$

Why zero?

$$\Delta\Psi = \frac{\sigma d}{\epsilon_0} - \cancel{\frac{\sigma (0)}{\epsilon_0}} \to 0$$

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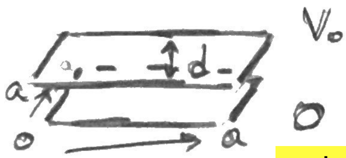
$$C = \frac{\cancel{\sigma} a^2}{\frac{(\cancel{\sigma} d)}{\epsilon_0}} \to \boxed{C = \frac{a^2 \epsilon_0}{d}}$$

3 1.1) Boundary Value



(i) $\Psi(d) = V_o$

(ii) $\Psi(0) = 0$

(iii) $\left.\frac{\partial \Psi}{\partial x}\right|_d = \frac{-\sigma}{\epsilon_o}$

+x is upwards based on work you did later on the page, but you should provide this in the diagram.

$\nabla^2 \Psi = 0$ ← 1-D solution

$$\frac{\partial^2 \Psi}{\partial x^2} = 0$$

$$\frac{\partial \Psi}{\partial x} = C_1$$

$$\Psi(x) = C_1 x + C_2$$

(ii) $\Psi(0) = \cancel{C_1 0} + C_2 = 0$

$C_2 = 0$

(i) $\Psi(d) = V_o = C_1 d$

$$C_1 = \frac{V_o}{d}$$

$$\Psi(x) = \frac{V_o}{d} x$$

(iii) $\frac{\partial \Psi}{\partial x} = -\frac{\sigma}{\epsilon_o}$

$$\left.\frac{\partial \Psi}{\partial x}\right|_d = \frac{\sigma}{\epsilon_o} = \frac{V_o}{d}$$

$$V_o = \frac{\sigma d}{\epsilon_o}$$

$$\Psi(x) = \frac{\sigma}{\epsilon_o} x$$

You've provided three boundary conditions, but only two are needed. This is OK, but note that you could also have given (iv) being dpsi/dx at x=0 = -sigma/epsilon_o.

Also, in (iii), the negative sign should not be there. E_n = sigma/epsilon_0 and the normal direction to this surface is in the -x direction, so E_n = -E_x. With E_x = -dpsi/dx, you get dpsi/dx at x = d = sigma/epsilon_o.

3.1.1) $\psi(x) = \frac{\sigma}{\epsilon_0} d$

$$\Delta V = V_0 - 0 = V_0$$

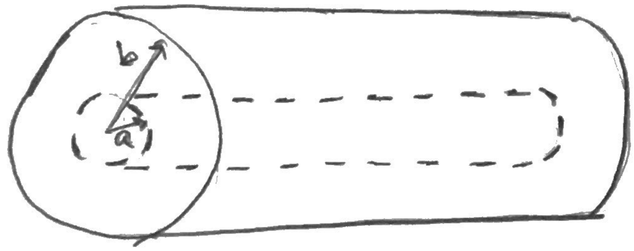
$$V_0 = \frac{\sigma d}{\epsilon_0}$$

$$C = \frac{Q}{|\Delta V|} \longrightarrow Q = \sigma a^2$$

$$C = \frac{\cancel{\sigma} a^2}{\left(\frac{\cancel{\sigma} d}{\epsilon_0}\right)}$$

$$\boxed{C = \frac{a^2 \epsilon_0}{d}}$$

3.1.2)



$$\oint_R \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \rightarrow \iiint_V \rho\, dV$$

$$\iint_A \vec{E}\cdot d\vec{a} = \frac{\sigma\, 2\pi a L}{\epsilon_0} \leftarrow Q$$

$$E\cdot \cancel{2\pi} x \cancel{L} = \frac{\cancel{2\pi} a \cancel{L}\, \sigma}{\epsilon_0}$$

$$E = \frac{a\sigma}{\epsilon_0}\frac{1}{x}\,\hat{x}$$

$$\Delta V = -\int_b^a E\cdot dl$$

$$\Delta V = \int_a^b \frac{a\sigma}{\epsilon_0}\frac{1}{x}\,dx$$

$$\Delta V = \frac{a\sigma}{\epsilon_0}\ln\Big|_a^b$$

$$\Delta V = \frac{a\sigma}{\epsilon_0}\ln(b) - \ln(a)$$

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$$\Delta V = \frac{a\sigma}{\epsilon_0}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{\cancel{\sigma}\, 2\pi \cancel{a} L}{\frac{\cancel{a\sigma}}{\epsilon_0}\ln\left(\frac{b}{a}\right)} \rightarrow \boxed{C = \frac{2\pi L\epsilon_0}{\ln(b/a)}\ F}$$

(i) $\Psi(a) = V_0$
(ii) $\Psi(b) = 0$
(iii) $\frac{\partial \Psi}{dx}\Big|_a = \frac{-\sigma}{\epsilon_0}$

3.1.2) Boundary Value

$\nabla^2 \Psi = 0$ ← cylindrical

$$\cancel{\frac{1}{x}} \frac{d}{dx}\left(x \frac{d\Psi}{dx} \right) = 0$$

$$x \frac{d\Psi}{dx} = C_1$$

$$\frac{d\Psi}{dx} = \frac{C_1}{x}$$

$$\Psi = C_1 \ln(x) + C_2$$

(ii) $\Psi(b) = 0 = C_1 \ln(b) + C_2$

$$C_2 = -C_1 \ln(b)$$

(i) $\Psi(a) = C_1 \ln(a/b) = V_0$

$$C_1 = \frac{V_0}{\ln(a/b)}$$

(iii) $\frac{\partial \Psi}{dx}\Big|_a = \frac{-\sigma}{\epsilon_0} = \frac{V_0}{\ln(a/b)} \frac{1}{a}$ absorb negative into ln

$$V_0 = \frac{\sigma a \ln(b/a)}{\epsilon_0} = \underline{\underline{\Delta V}}$$

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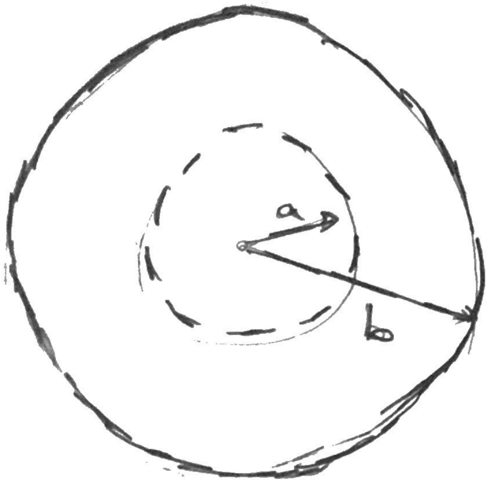
$$C = \frac{Q_{enc}}{\left(\frac{\sigma a \ln(b/a)}{\epsilon_0}\right)} \rightarrow \frac{\cancel{\sigma} \cancel{a}\, 2\pi L}{\frac{\cancel{\sigma} \cancel{a} \ln(b/a)}{\epsilon_0}} \qquad \boxed{C = \frac{2\pi L \epsilon_0}{\ln(b/a)}}$$

3.1.3)



Gauss's Law

$$\oint_A \vec{E} \cdot d\vec{a} = \frac{\iiint_V \rho \, dV}{\epsilon_0}$$

$$E \cdot \cancel{4\pi} x^2 = \frac{\sigma \cancel{4\pi} a^2}{\epsilon_0}$$

$$E = \frac{\sigma a^2}{\epsilon_0 x^2}$$

$$\Delta V = -\int_b^a \frac{\sigma a^2}{\epsilon_0 x^2} \, dx$$

$$\Delta V = \frac{\sigma a^2}{\epsilon_0} \left(\frac{1}{x} \right) \Big|_b^a$$

$$\Delta V = \frac{\sigma a^2}{\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$C = \frac{\cancel{\sigma} 4\pi \cancel{a^2}}{\frac{\cancel{\sigma} \cancel{a^2}}{\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)}$$

$$C = \frac{4\pi\epsilon_0}{\left(\frac{b-a}{ab} \right)}$$

$$\boxed{C = \frac{4\pi\epsilon_0 ab}{b-a}}$$

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(i) $\Psi(a) = V_0$
(ii) $\Psi(b) = 0$
(iii) $\left.\frac{d\Psi}{dx}\right|_a = -\frac{\sigma}{\epsilon_0}$

3.1.3) Boundary Value

$\nabla^2 \Psi = 0$ ← spherical

$$\frac{1}{\cancel{x^2}} \frac{d}{dx}\left(x^2 \frac{d\Psi}{dx}\right) = 0$$

$$x^2 \frac{d\Psi}{dx} = C_1$$

$$\frac{d\Psi}{dx} = \frac{C_1}{x^2}$$

$$\Psi(x) = \frac{-C_1}{x} + C_2$$

(ii) $\Psi(b) = 0 = \frac{-C_1}{b} + C_2$

$$C_2 = \frac{C_1}{b}$$

(i) $\Psi(a) = V_0 = \frac{-C_1}{a} + \frac{C_1}{b}$

$$V_0 = C_1\left(\frac{-1}{a} + \frac{1}{b}\right) \to \frac{1}{b} - \frac{1}{a}$$

$$C_1 = \left(\frac{1}{b} - \frac{1}{a}\right)^{-1} V_0$$

(iii) $\left.\frac{d\Psi}{dx}\right|_a = \frac{-\sigma}{\epsilon_0} = \frac{V_0\left(\frac{1}{b} - \frac{1}{a}\right)}{a^2}$

$$V_0 = \frac{\sigma a^2}{\epsilon_0\left(\frac{1}{a} - \frac{1}{b}\right)} = \Delta V$$

$$C = \frac{4\pi \cancel{a^2 \sigma}}{\frac{\cancel{\sigma a^2}}{\epsilon_0\left(\frac{1}{a} - \frac{1}{b}\right)}} \to \boxed{\frac{4\pi a b \epsilon_0}{(b-a)}}$$

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