11.2.1) Using Plots labeled Final_Part_II_1_*

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From the plots with the above name, one can see that the approximation holds fairly well with the inputs $V_0 = 1\,V$, $L = 1\,H$; $C = 1\,F$, $\omega = 0.005\,s^{-1}$ and $N = 1000$. The λ of the transmission line is given by the relation $\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$ (page 230 (7)) which yeilds a wave length of ~ 1257 m (or nodes). With this extremely long wave length, the differences in the solution vary by a maximum 0.52% for Voltage, 0.46% for current, and 0.25% for impedance. It should be noted that the impedance of the analytical solution is $Z = \sqrt{L/C}$ as opposed to $Z = \sqrt{L/C - \frac{(\omega L)^2}{4}}$ as discussed in the paper

The Analytical solution displayed comes from the equations shown in HW #9, specifically

$$\tilde{V}_n(x) = \tilde{V}_n^+\left(e^{-j\beta_n x}\left[1+\tilde{\rho}(x)\right]\right)$$

$$\tilde{I}_n(x) = \frac{\tilde{V}_n^+}{Z_n}\left(e^{-j\beta_n x}\left[1-\tilde{\rho}(x)\right]\right)$$

$$\tilde{Z}_n(x) = \frac{\tilde{V}_n(x)}{\tilde{I}_n(x)}$$

and $\tilde{\rho}(x) = \frac{Z_2 - Z_1}{Z_2 + Z_1} e^{-2j\beta_n x}$

Given how similar the answers are, I assert that this is a good approximation when Z_L = Z_0

(These plots can be generated with ANIMATE variable and the Z_L variable)

11.2.2) Using Plots labled

Final_Part_II_2_*

From the above plots, much like the other section, the Analytical and Ladder circuit align nicely. When the Load Impedance is set to $3\sqrt{L/C}$, the solutions have a mismatch of 0.0069 in voltage, 0.015 in current, and 0.0017 in impedance. The Error is rather comparable when looking at a load impedance of $10\sqrt{L/C}$ (Final_Part_II_2_2.pdf). Lastly, when supplying a more interesting impedance of $2+2j$ as the load, the accuracy of values is 0.04 for voltage, 0.043 for current, and 0.005 for impedance.

Since the impedance, current, and voltage align so well, I feel confident stating the Ladder circuit works well as an approximation of a transmission line.

11.2.3) Using plots titled final_Part_II_3_*

As can be seen with the 5 plots I have submitted, increasing ω causes the approximation to fall apart. This can best be see at the characteristic imdedance at $\omega = 2$, where the ladder circuit has increasing voltages and currents at each node. Going above $\omega = 2$ actually caused the ladder circuit to crash meaning I couldn't show plots above $\omega = 2$.

The characteristic impedance is given by $z_0 = \sqrt{(L/C) - (\frac{\omega^2 L^2}{4}} = 0$. Since the equation presented is the equivalent impedance

For the system, increasing ω lowers the impedance, which further diverges from the $\sqrt{(L/C)}$ that was being used by my code. Lastly, the ω values plotted were [0.005, 0.02, 0.1, 0.5, 2] with an accidental double plot of 0.005. The approximation holds for 0.02, but $\omega = 0.1$ shows a rather wave like impedance, and $\omega = 0.5$ shows very large errors. One thing I found rather interesting was that the Voltage and current kept increasing at $\omega = 2$, and I believe this is due to there being no impedance, so everything harmonically adds as the circuit goes down the nodes.