5.3) Two antenna combine to form a configuration given by

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HW 5

$$E_x = A_1 \cos(kx - \omega t + \delta_1) + A_2 \cos(kx - \omega t + \delta_2)$$

Can they be combined into a singular Antenna defined by

$$E_x = A_3 \cos(kx - \omega t + \delta_3)?$$

Given that there are two unknown values, A_3 & δ_3, with only one equation, I don't think there is a general solution for A_3 & δ_3. However, there are solutions where A_3 & δ_3 can be found

IR $A_1 = A_2$

$$A_3 \cos(kx-\omega t+\delta_3) = A_1\left(\cos(kx-\omega t+\delta_1)+\cos(kx-\omega t+\delta_2)\right)$$

$$\cos(kx-\omega t+\delta_1)+\cos(kx-\omega t+\delta_2) = \left(\frac{e^{i(kx-\omega t+\delta_1)}}{2}+\frac{e^{-i(kx-\omega t+\delta_1)}}{2}\right) + \left(\frac{e^{i(kx-\omega t+\delta_2)}}{2}+\frac{e^{-i(kx-\omega t+\delta_2)}}{2}\right)$$

I call this the "partial" complex exponential approach - you rewrite trig functions in terms of complex exponentials and then end up having to use a trig identity at the end. This approach works well for deriving trig identities.

This is a bit different that the "full" complex exponential approach that I used in class, which requires fewer steps to get to an answer.

set $\phi = kx-\omega t$,

$$= \frac{e^{i(\phi+\delta_1)}}{2}+\frac{e^{-i(\phi+\delta_1)}}{2}+\frac{e^{i(\phi+\delta_2)}}{2}+\frac{e^{-i(\phi+\delta_2)}}{2}$$

Break all δ into $2\cdot\delta/2$

$$= \frac{e^{i(\phi+2\delta_1/2)}}{2}+\frac{e^{-i(\phi+2\delta_1/2)}}{2}+\frac{e^{i(\phi+2\delta_2/2)}}{2}+\frac{e^{-i(\phi+2\delta_2/2)}}{2}$$

Add the other δ via $\delta/2-\delta/2$

$$= \frac{e^{i(\phi+(\delta_1/2+\delta_2/2)+(\delta_1/2-\delta_2/2))}}{2}+\frac{e^{-i(\phi+(\delta_1/2+\delta_2/2)+(\delta_1/2-\delta_2/2))}}{2}$$
$$+\frac{e^{i(\phi+(\delta_2/2+\delta_1/2)+(\delta_2/2-\delta_1/2))}}{2}+\frac{e^{-i(\phi+(\delta_2/2+\delta_1/2)+(\delta_2/2-\delta_1/2))}}{2}$$

$$= \frac{e^{i\left(\frac{\delta_1-\delta_2}{2}\right)} e^{i\left(\phi+\left(\frac{\delta_1+\delta_2}{2}\right)\right)}}{2} + \frac{e^{-i\left(\frac{\delta_1-\delta_2}{2}\right)} e^{-i\left(\phi+\left(\frac{\delta_1+\delta_2}{2}\right)\right)}}{2}$$

$$+ \frac{e^{-i\left(\frac{\delta_1-\delta_2}{2}\right)} e^{i\left(\phi+\left(\frac{\delta_1+\delta_2}{2}\right)\right)}}{2} + \frac{e^{i\left(\frac{\delta_1-\delta_2}{2}\right)} e^{-i\left(\phi+\frac{(\delta_1+\delta_2)}{2}\right)}}{2}$$

$$a = \left(\frac{\delta_1-\delta_2}{2}\right) \qquad b \rightarrow \left(\phi+\left(\frac{\delta_1+\delta_2}{2}\right)\right)$$

$$= \frac{e^{ia} e^{ib}}{2} + \frac{e^{-ia} e^{-ib}}{2} + \frac{e^{-ia} e^{ib}}{2} + \frac{e^{ia} e^{-ib}}{2}$$

$$= \frac{1}{2}\left[e^{ia}\left(e^{ib}+e^{-ib}\right) + e^{-ia}\left(e^{ib}+e^{-ib}\right)\right].$$

$$= \frac{1}{2}\left[\left(e^{ia}+e^{-ia}\right)\left(e^{ib}+e^{-ib}\right)\right]$$

$$= 2\cos(a)\cos(b)$$

$$= 2\cos\left(\frac{\delta_1-\delta_2}{2}\right)\cos\left(kx-\omega t+\frac{\delta_1+\delta_2}{2}\right)$$

$$\therefore \quad A_3 = 2A_1\cos\left(\frac{\delta_1-\delta_2}{2}\right)$$

$$\delta_3 = \frac{\delta_1+\delta_2}{2}$$

when $A_1 = A_2$

When $\delta_1 = \delta_2$

$$A_3 \cos(kx - \omega t + \delta_3) = A_1 \cos(kx - \omega t + \delta_1) + A_2 \cos(kx - \omega t + \delta_1)$$

$\therefore$

$\delta_3 = \delta_1 \qquad A_3 = A_1 + A_2$