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PHYS 513
HW #5
September 28, 2020

5.1) Describe $\vec{\nabla}\cdot\vec{B}=0$, $\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$ and $\vec{B}=-\vec{\nabla}\Phi_M$ in your own words. Also talk about the tangential and perpendicular fields across a surface.

$\vec{\nabla}\cdot\vec{B}=0$

This equation is difficult for me to describe without the integral form. I am going to cheat a little and look at the integral form to help explain my understanding of this equation

$$\int_V \vec{\nabla}\cdot\vec{B} = \int_V 0\,dV \xrightarrow{\text{div}} \oint_S \vec{B}\cdot d\vec{s} = \int_V 0\,dV$$

Using this integral form, it is easy to see that there is no volume V that will have a magnetic field that doesn't have equal parts entering and exiting that volume.

This can be contrasted with the electric field equivalent, which states $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$. This difference highlights that Maxwell's equation's don't need a magnetic monopole. The book talks about how there can be configurations where ρ_m can be used, but they approximations or simplifications.

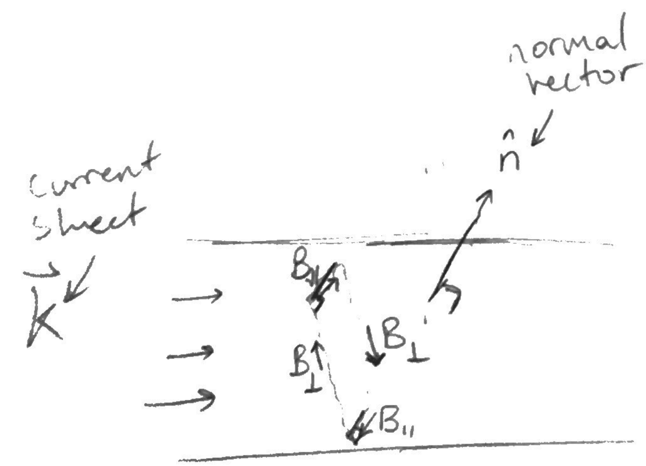
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

This equation relates a magnetic field to a current density by showing that a current will rotate around a magnetic field. In the integral form, the magnetic field is related to current by wrapping around a closed loop. This can be further expanded upon by looking at a current sheet at a surface. The magnetic field arising from that current can found from a loop around the surface such that there is a parallel and perpendicular component. The parallel component will have to be continuous across the surface the perpendicular component will be discontinous

$$\vec{B} = -\vec{\nabla}\,\Psi_m$$

This one is harder for me to understand. It would seem that configurations like this one can only arise when $\vec{J} = 0$, due to $\vec{\nabla} \times \vec{\nabla}\varphi = 0$. The book describes scenarios where this can happen when $\vec{B} = \mu(\vec{H} + \vec{M})$ such that there is no current being moved, but a configuration of $\vec{M}$ that produces a $\vec{B}$.

Current sheet graphic



The B field that is parallel with $\hat{n}$ will be continuous because it passes through the current sheet, but $B_{\perp}$ goes around the current sheet, such that it will have to be discontinous as the parallel $\Delta l_{\parallel} \to 0$