PHYS 510 **Homework 1**

(1) (a) Write a program to compute an approximate value for the derivative of a function using the finite difference formula

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

Test your program using the function $\tan(x)$ for $x = 1$. Determine the error by comparing with the square of the built-in function $\sec(x)$. Plot the magnitude of the error as a function of h, for $h = 10^{-k}, k = 0, \dots, 16$. You should use log scale for h and for the magnitude of the error. Is there a minimum value for the magnitude of the error?

(b) Repeat the exercise using the centered difference approximation

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

(2) The polynomial $(x-1)^6$ has the value zero at $x = 1$ and is positive elsewhere. The expended form of polynomial, $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$, is mathematically equivalent but may not give the same results numerically. Compute and plot the value of this polynomial, using each of the two forms, for 101 equally spaced points in the interval [0.995, 1.005], i.e., with a spacing 0.0001. Your plot should be scaled so that the values for *x* and for the polynomial use the full ranges of their respective axes.

(3) The standard quadratic equation,

$$ax^2 + bx + c = 0 \qquad (3.1)$$

has an analytical solution that can be written as either

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (3.2)$$

or

$$x'_{1,2} = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}} \qquad (3.3)$$

Inspection of Eq. (3.2) and Eq. (3.3) indicates that subtractive cancellation (and consequently an increase in error) arises when $b^2 \gg 4ac$ because then the square root and its preceding term nearly cancel for one of the roots.

Please do the following for this problem:

(a) Write a program that calculate the four solutions $x_{1,2}$ and $x'_{1,2}$ for the quadratic equation with a set of arbitrary values of a, b, and c.

(b) Take $a = b = 1$, and $c = 10^{-n}, n = 1, 2, 3, \ldots$ and compute the four solutions for increasing *n*. Please make sure that your output data have enough significant figures.

(c) What is the largest *n* when machine precision causes Eq. (3.3) to fail?

(d) Since $b > 0$, subtractive cancellation will NOT occur in x'_1 and x_2. Treat these two values as the accurate solutions to the quadratic equation and calculate the errors in the other two solutions x_1 and x'_2. Plot the relative errors of solutions x_1 and x'_2 as a function of 4*ac* in a log-log graph.

(4) Consider the finite sum

$$S_N^{(1)} = \sum_{n=1}^{2N} (-1)^n \frac{n}{n+1} \qquad (4.1)$$

If you sum the even and odd values of *n* separately, you get two sums:

$$S_N^{(2)} = -\sum_{n=1}^{N} \frac{2n-1}{2n} + \sum_{n=1}^{N} \frac{2n}{2n+1} \qquad (4.2)$$

If you combine the series analytically, you obtain:

$$S_N^{(3)} = \sum_{n=1}^{N} \frac{1}{2n(2n+1)} \qquad (4.3)$$

Even though all three summations $S^{(1)}$, $S^{(2)}$ and $S^{(3)}$ are mathematically equal, they may give different numerical results.

Please do the following for this problem:

(a) Write a single-precision program that calculates $S^{(1)}$, $S^{(2)}$ and $S^{(3)}$, analyze the results for $1 \leq N \leq 1{,}000{,}000$.

(b) Assume $S^{(3)}$ to be the exact answer. Make a log-log plot of the relative error versus the number of terms, that is, of $log_{10}|(S_N^{(1)} - S_N^{(3)})/S_N^{(3)}|$ verses $log_{10}(N)$ for $1 \leq N \leq 1{,}000{,}000$.

(c) See whether straight-line behavior for the error occurs in some region of your plot. This indicates that the error is proportional to a power of N.