Improved Bayes Risk Can Yield Reduced Social Welfare Under Competition

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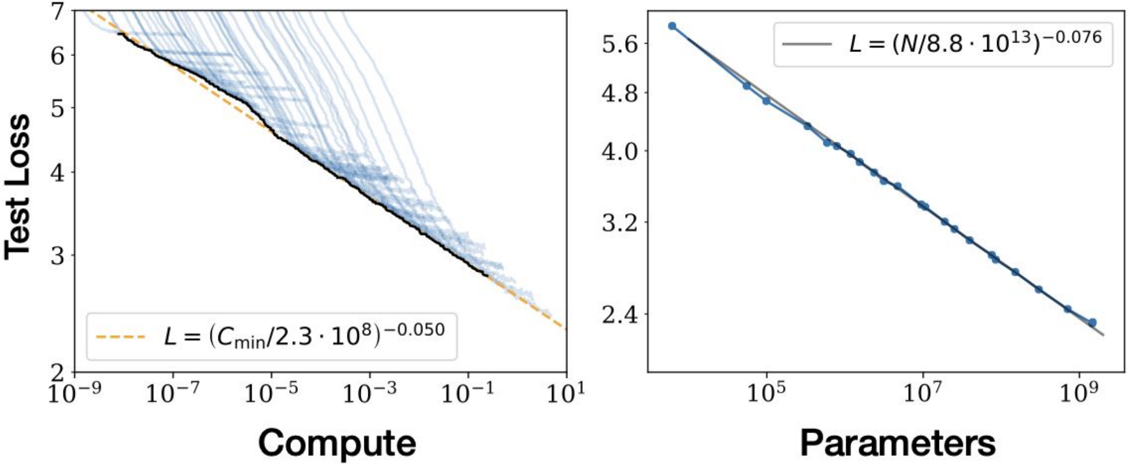
Scale improves accuracy for an isolated system

T5 11B parameters

175B parameters

540B parameters

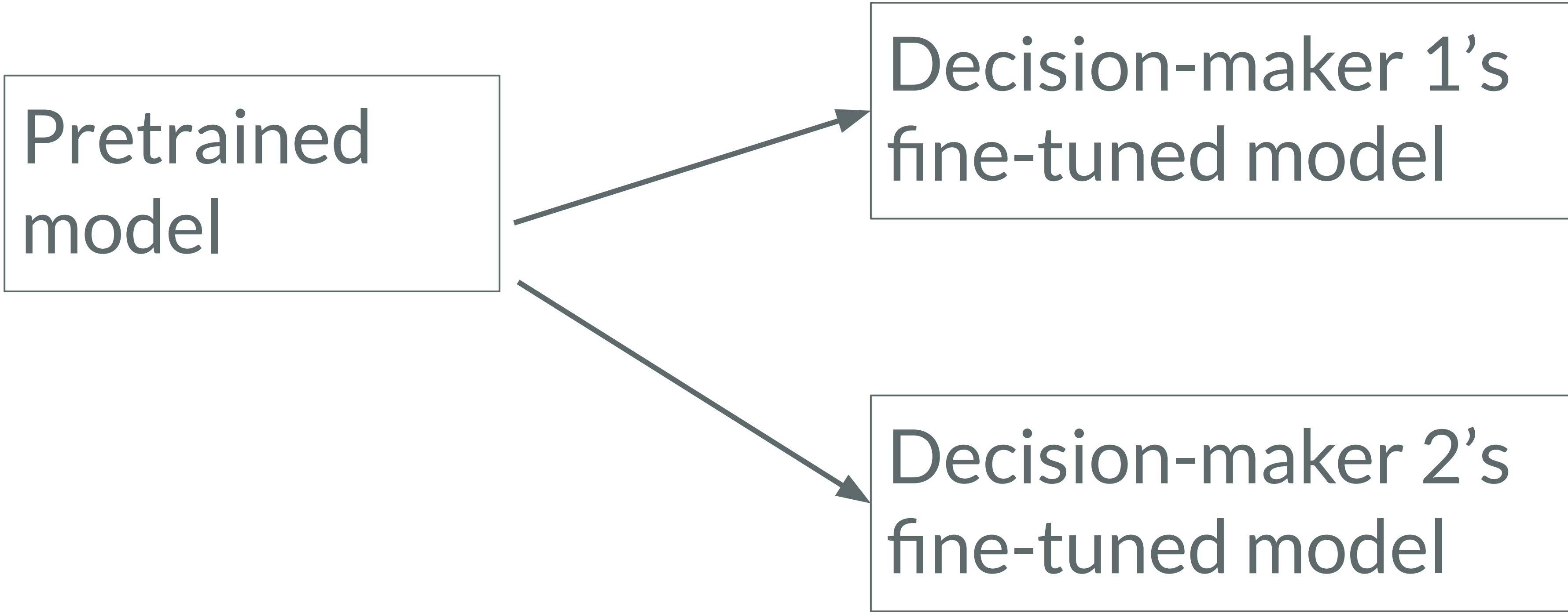
GPT-4 ? 1.8T parameters



(Kaplan et al., 2020)

This work: impact of increases to scale under **competing decision-makers**

Marketplace of competing decision-makers



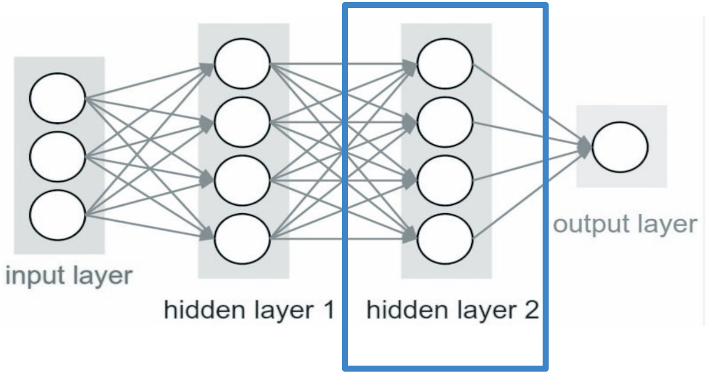
Marketplace of competing decision-makers

Pretrained model

Decision-maker 1's fine-tuned model

Decision-maker 2's fine-tuned model

Learns representations that improve with scale



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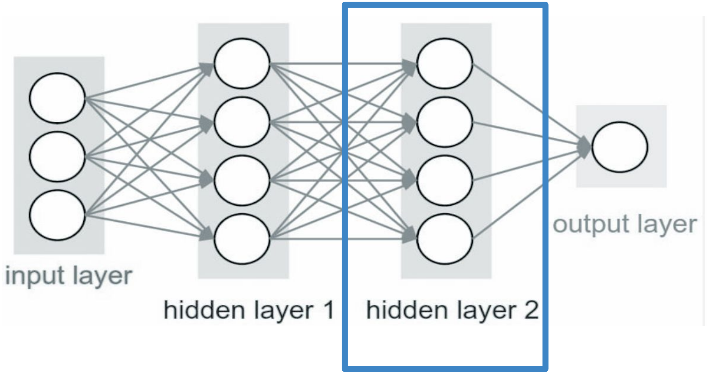

Decision-maker 1's fine-tuned model

Decision-maker 2's fine-tuned model

Leverages representations for downstream objective (**market share**)

Main question

Does improving data representation quality (Bayes risk) improve user social welfare (overall predictive accuracy) under competition?

Our main result

Result (Informal): The social welfare (overall predictive accuracy) for users can be *non-monotonic* in data representation quality (Bayes risk).

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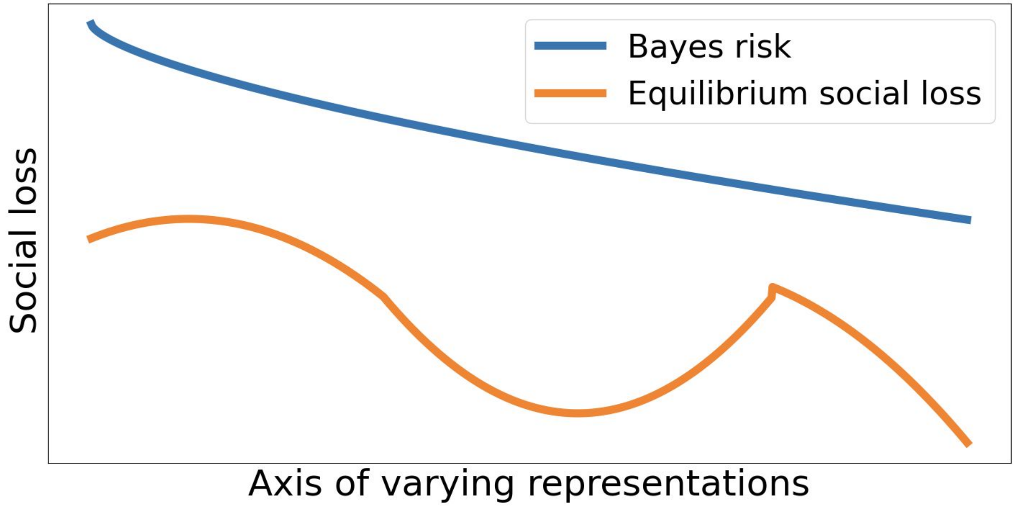

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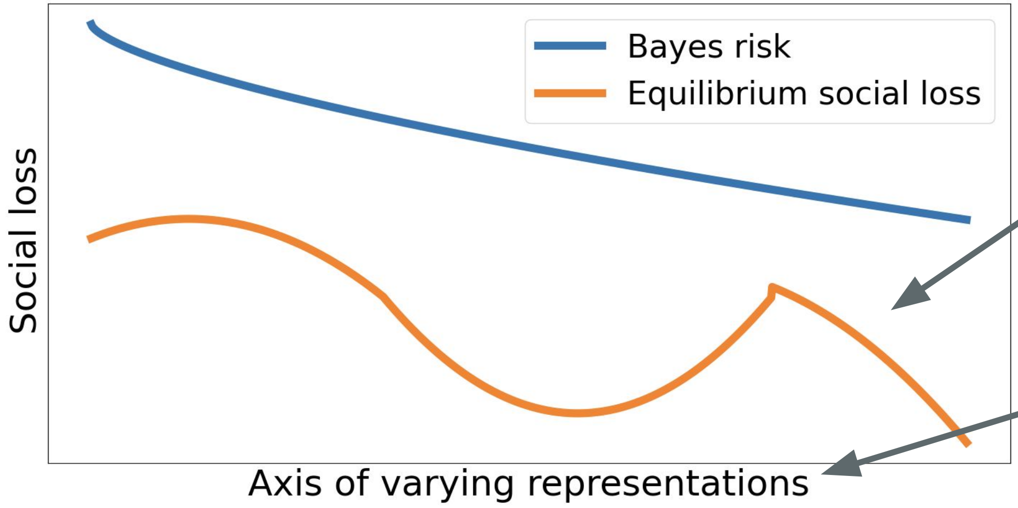

Shape of this curve varies for different axes of varying representations

e.g., noise, dimension, quality of pretrained model

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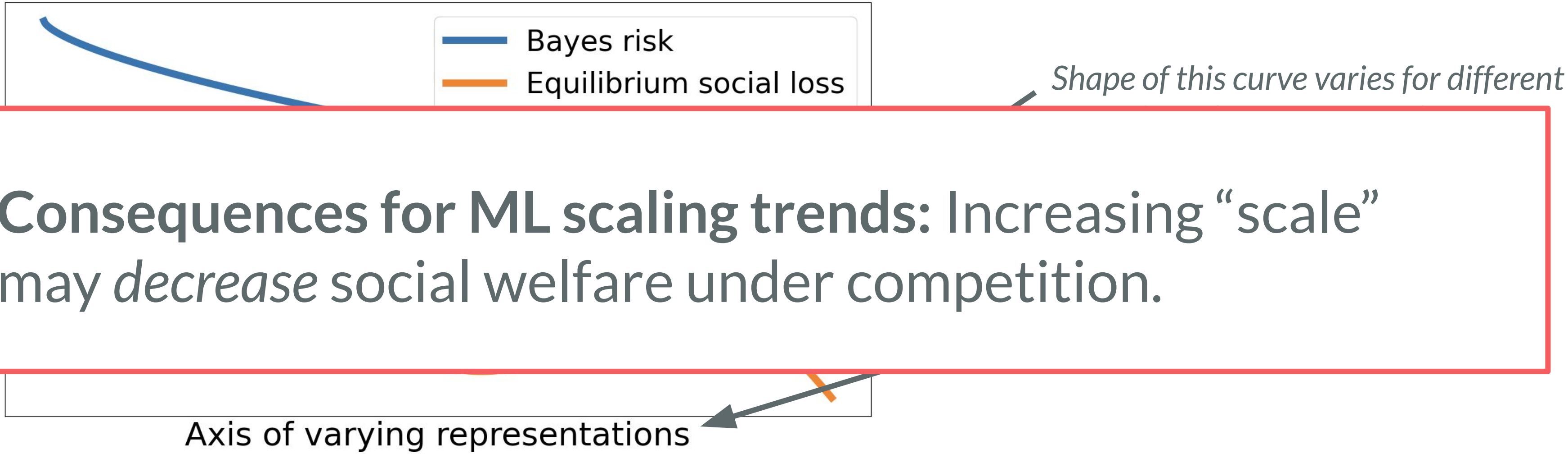


Consequences for ML scaling trends: Increasing "scale" may *decrease* social welfare under competition.

Our results

We study a model for competing model-providers, and we show non-monotonicity through:

1. A theoretical analysis of a stylized setup with closed-form equilibria

2. An empirical analysis on synthetic data simulations and CIFAR-10 representations from pretrained models for linear predictors

Overview of our model

Task: multi-class classification with:

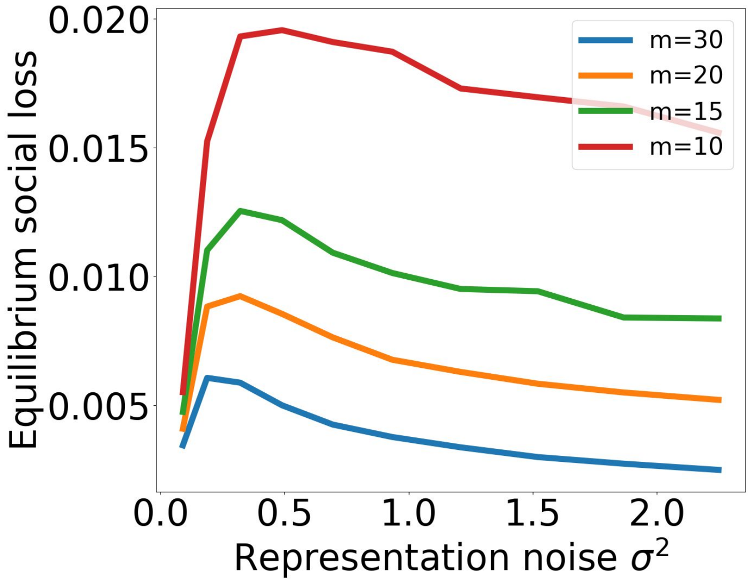
- User distribution $(x, y) \sim D$ where $x \in R^d$ and $y \in \{0,1, 2, \ldots, K\text{-}1\}$
- Model family F of predictors f mapping $R^d \rightarrow \{0,1, 2, \ldots, K\text{-}1\}$

Interaction between model-providers and users:

- Each of m model-providers chooses a predictor in F.
- Each user (x, y) noisily chooses the model-provider offering them the best prediction.
- A model-provider's utility is equal to the market share.

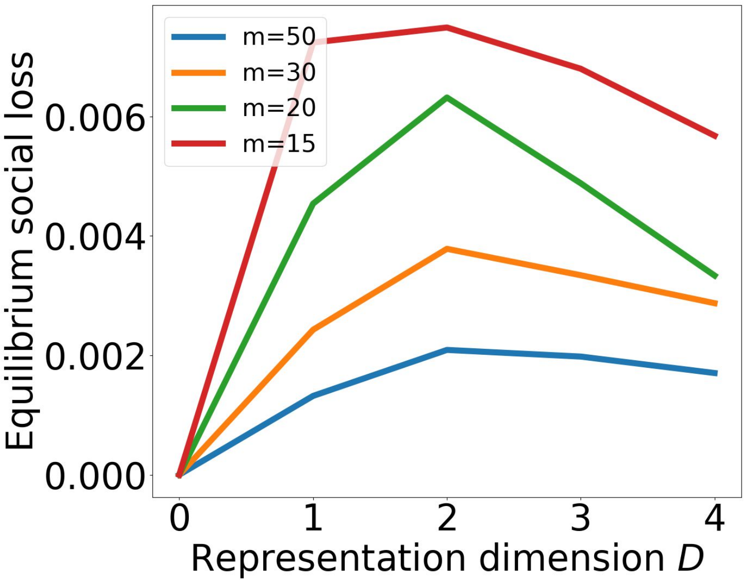
We study the **Nash equilibria** between model-providers.

Theoretical analysis of equilibria in stylized setups



Mixture of 1d Gaussians with means 0 and 1

σ := std dev of Gaussians



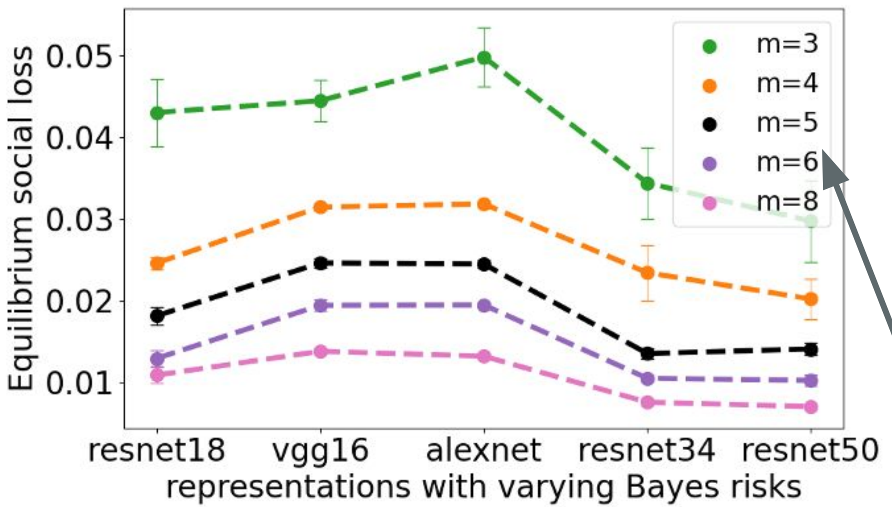
4 subpopulations that need increasing #s of dimensions to detect and classify

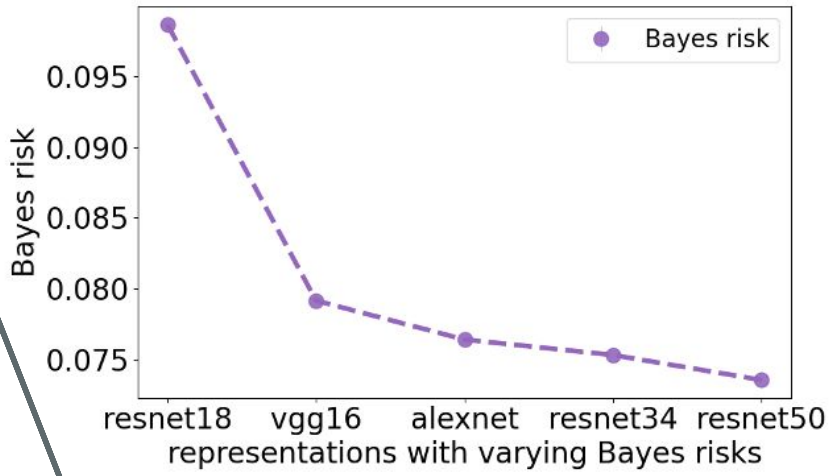
D := representation dimension

Overall predictive loss at equilibrium is non-monotonic in Bayes risk.

Simulations for linear predictors on CIFAR-10

Classification on CIFAR-10 with representations from pretrained networks





m=# of model-providers

Overall predictive loss at equilibrium is non-monotonic in Bayes risk.

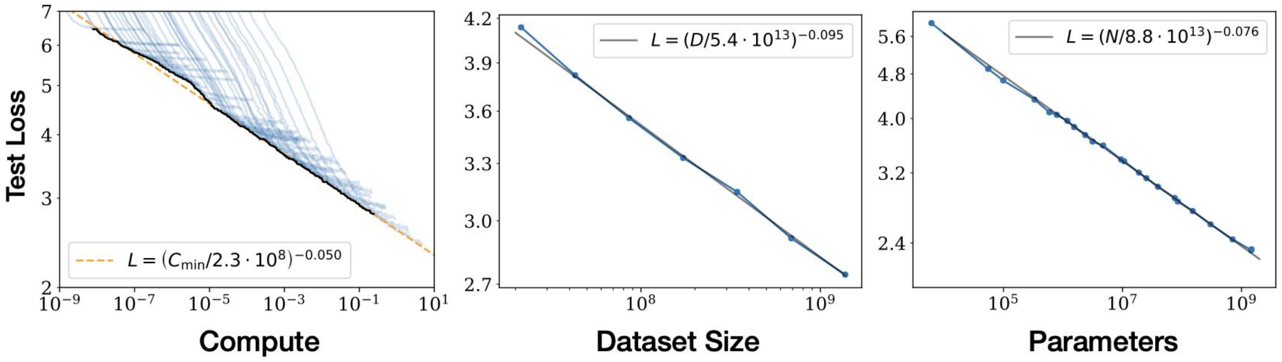
Takeaways

We showed that under competition, **the equilibrium social welfare can be non-monotonic in representation quality** (as measured by Bayes risk).

Consequence for ML scaling laws: **Increases to "scale" may reduce overall predictive accuracy for users** in real-world marketplaces with competing model-providers.

Future work: scaling laws under competition?

Model-provider
in isolation



Competing
model-providers

???