Safety vs. Performance: How Multi-Objective Learning Reduces Barriers to Market Entry

Meena Jagadeesan (UC Berkeley / Stanford)

Joint work with Michael I. Jordan and Jacob Steinhardt (UC Berkeley)

https://arxiv.org/abs/2409.03734

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We study the emerging market where companies train large language models (LLMs).

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This work: a technical framework to quantify how much data a new company needs to enter the market

Outline for the talk

1. **Background**

2. Our model

3. Our results

4. Technical ideas

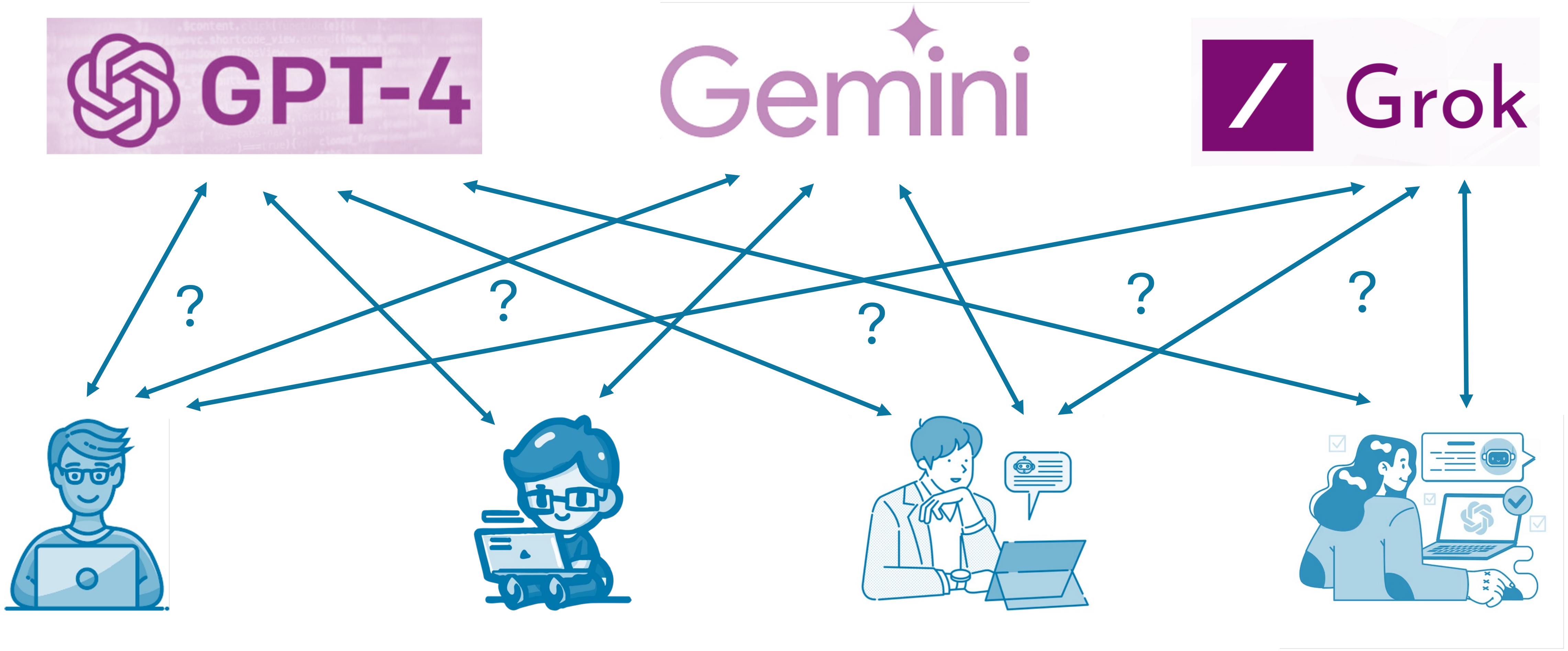
An emerging market of companies that train LLMs

GPT-4

Gemini

Grok

An emerging market of companies that train LLMs



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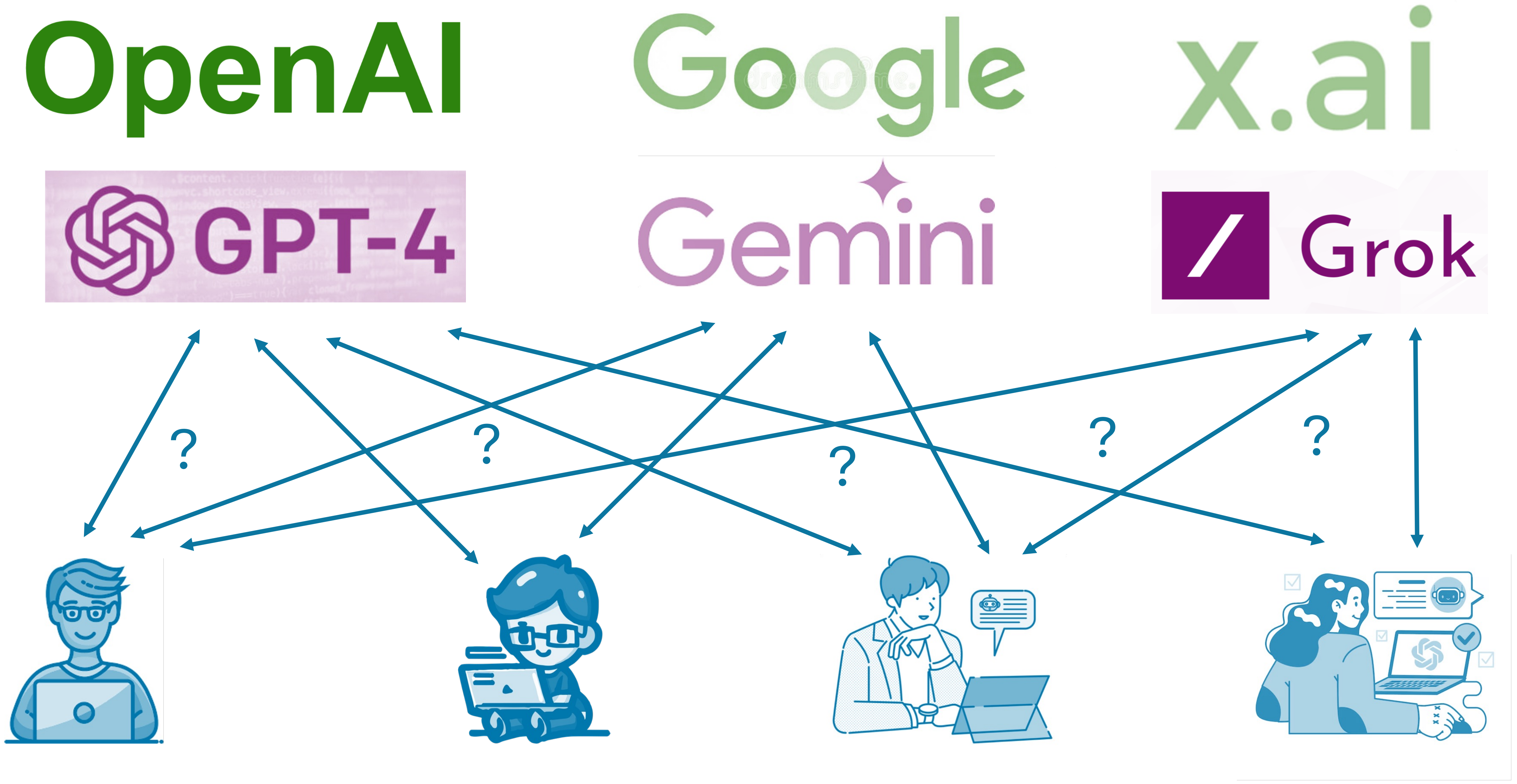

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Assumption: **Model performance** determines whether a company attracts users.

Reality: companies face pressure to consider objectives beyond performance.

Beyond performance: scrutiny of safety violations

Regulators & society scrutinize **safety violations** of deployed LLMs:

- *E.g.*, LLMs releasing dangerous information (e.g., how to create a weapon)
- *E.g.*, LLMs producing offensive content

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Scrutiny from regulators:


OCTOBER 30, 2023

Executive Order on the Safe, Secure, and Trustworthy Development and Use of Artificial Intelligence

BRIEFING ROOM › PRESIDENTIAL ACTIONS

EU Artificial Intelligence Act

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TECH • ARTIFICIAL INTELLIGENCE

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Key property: Large high-resource companies face greater scrutiny than small companies.

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- *En route: new technical tools for multi-objective, high-dim regression*

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This work: We characterize how scrutiny of safety violations shapes data-driven barriers to entry for new companies.

Key finding: Scrutiny of safety often---but not always---enables new LLM companies to enter with less data than incumbents

enter the market.

- *En route: new technical tools for multi-objective, high-dim regression*

Related Work

Competition between model-providers:

e.g., Ben-Porat, Tennenholtz ('17, '19), Feng, Gradwohl, Hartline, Johnsen, Nekipelov ('19), Dong, Elzayn, Jabbari, Kearns, Schutzman ('19), Aridor, Mansour, Slivkins, Wu ('20), Iyer and Ke ('22), Kwon, Ginart, Zou ('22), Gradwohl, Tennenholtz ('23), **J.**, Jordan, Haghtalab ('23), **J.**, Jordan, Steinhardt, Haghtalab ('23)

Broader perspectives on algorithmic competition, policy, and dynamics:

e.g., Immorlica, Kalai, Lucier, Moitra, Postlewaite, Tenneholtz ('11), Hashimoto, Srivastava, Namkoong, Liang ('18), Kleinberg, Raghavan ('21) Dean, Curmei, Ratliff, Morgenstern, Fazel ('22), Cen, Hopkins, Ilyas, Madry, Struckman, Caso ('23), Fallah, Jordan ('23), Laufer, Kleinberg, Heidari ('24), Handina, Mazumdar ('24)

Scaling laws and high-dimensional linear regression:

e.g., Hastie et al. ('19), Bordelon et al. ('20), Kaplan et al., ('20), Bahri et al. ('21), Cui et al. ('21), Hashimoto ('21) Hernandez et al. ('21), Hoffmann et al. ('22), Wei et al., ('22), Bach ('23), Jain et al. ('24), Song et al. ('24), Goyal et al. ('24), Covert et al. ('24), Shen et al. ('24), Dohmatob et al. ('24), Mallinar et al. ('24)

Our focus: data-driven barriers to market entry under multi-objective learning

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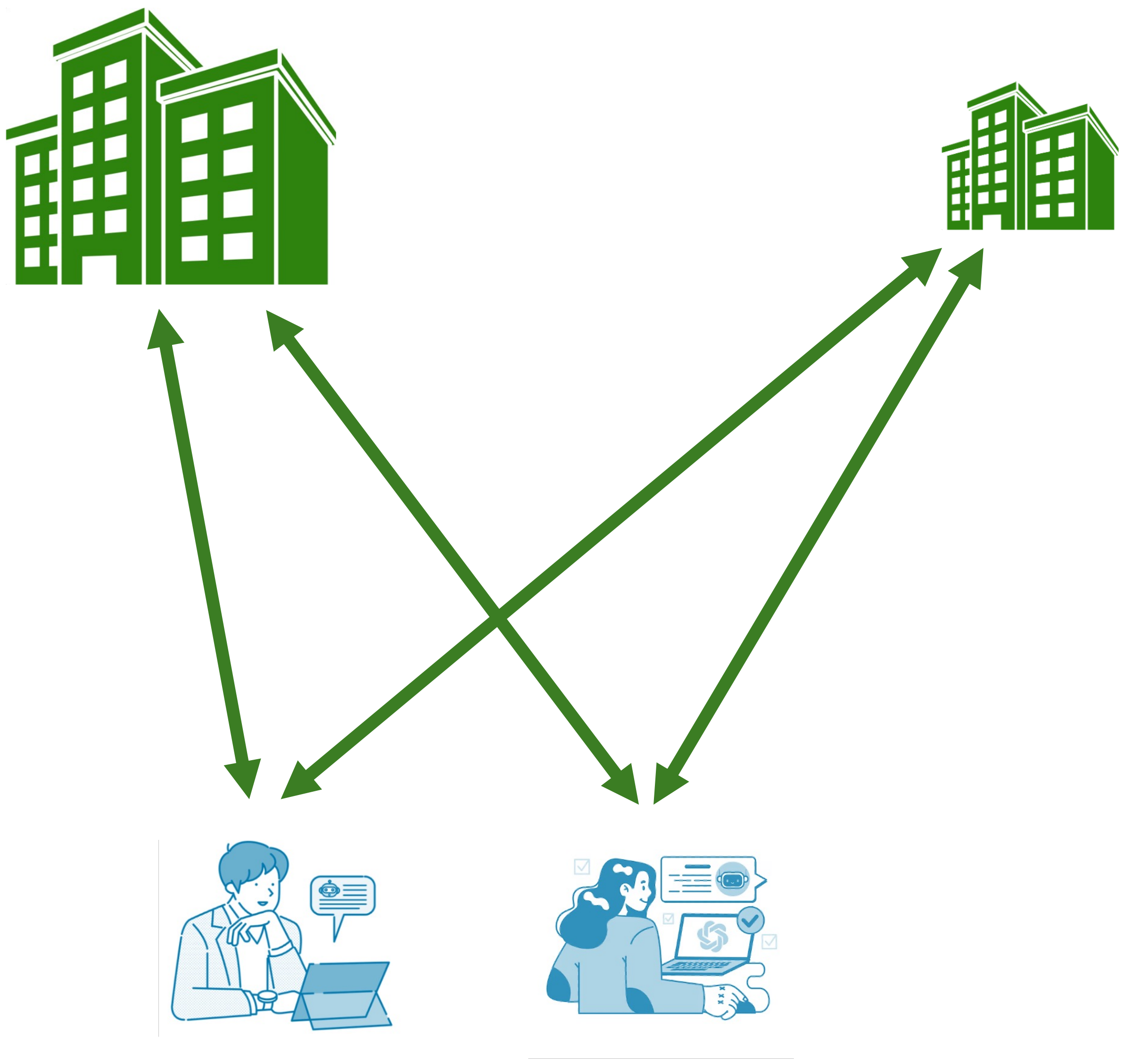

Each company strategically trains its LLM to attract consumers.

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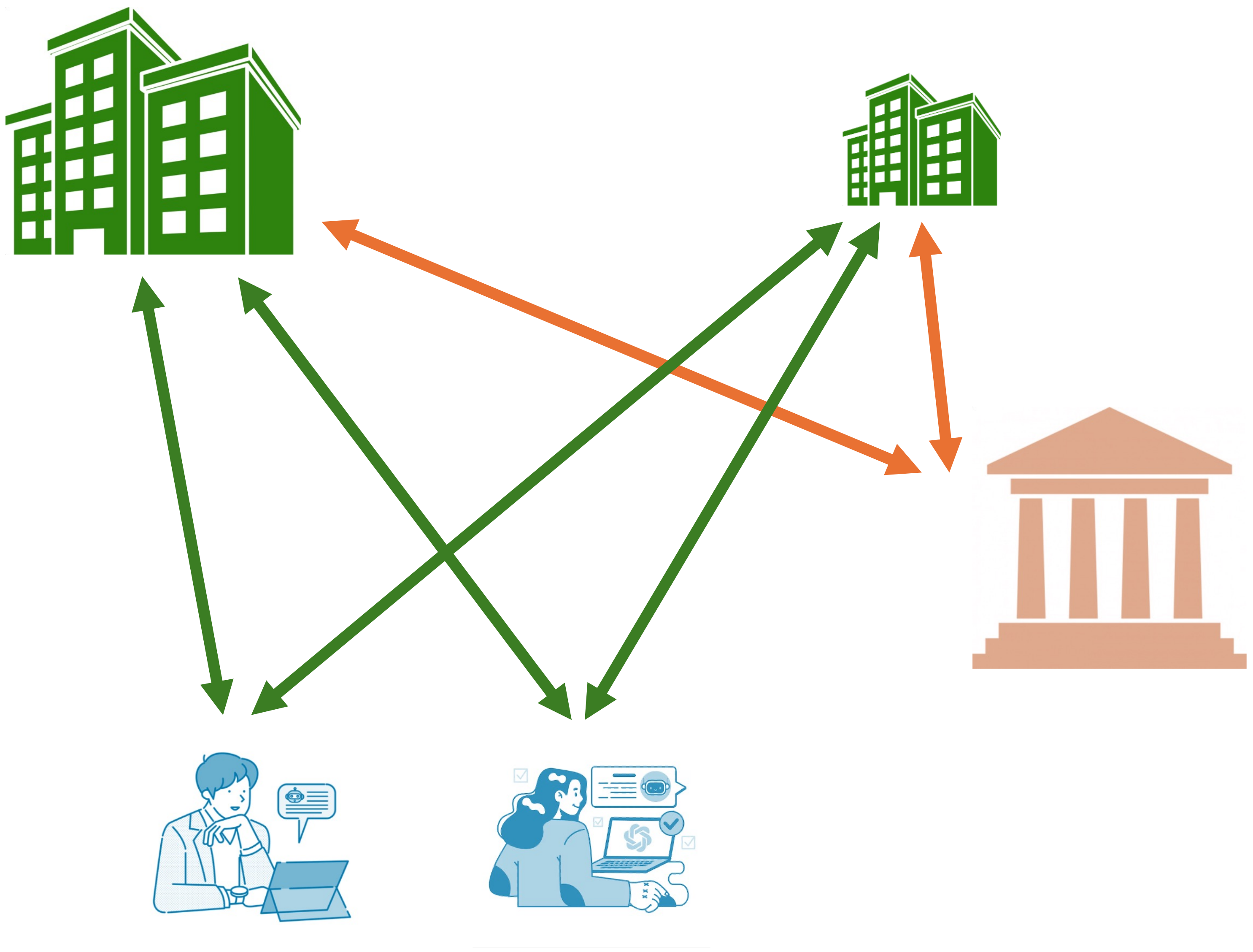

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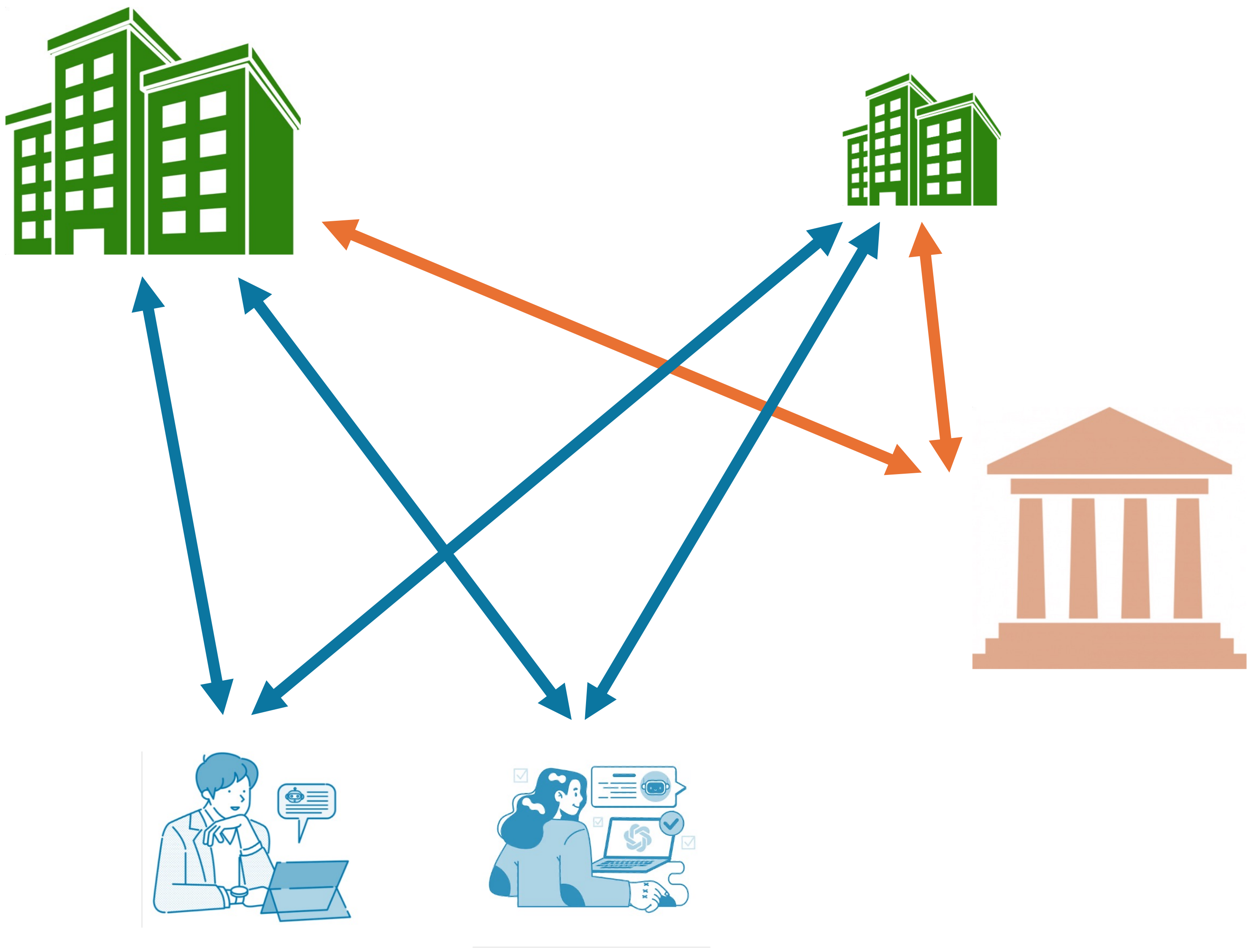

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*Consumers choose the safety-compliant model with best **performance**.*

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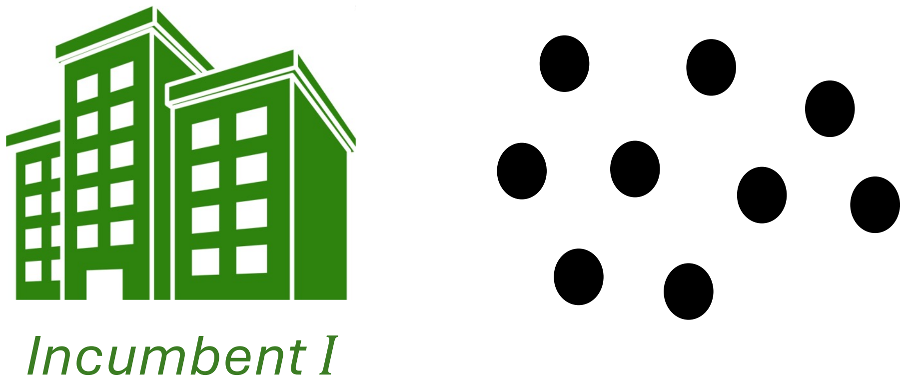

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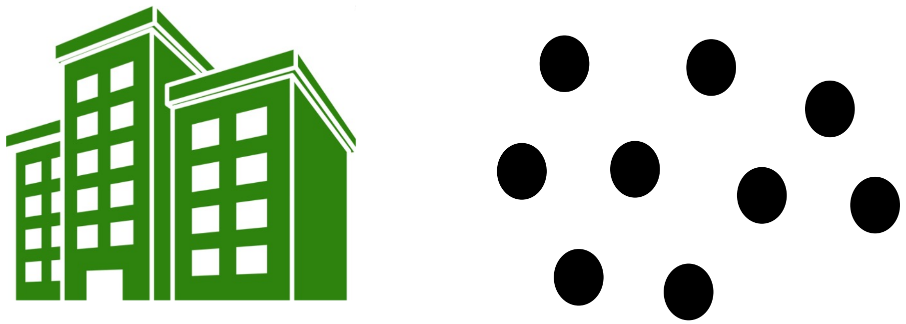

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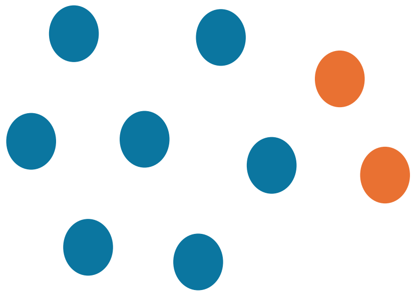

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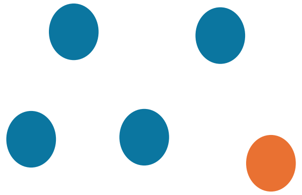

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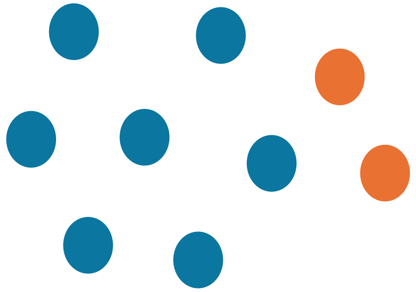

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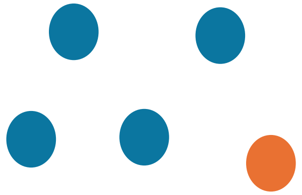

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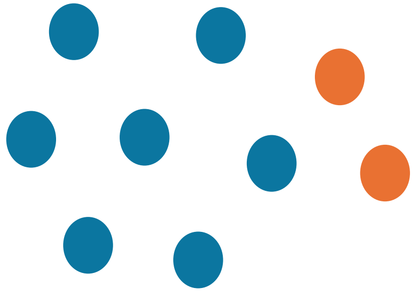

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Safety requirement:
safety loss < threshold

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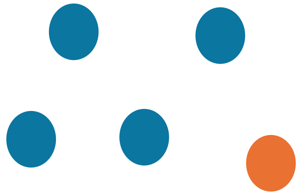

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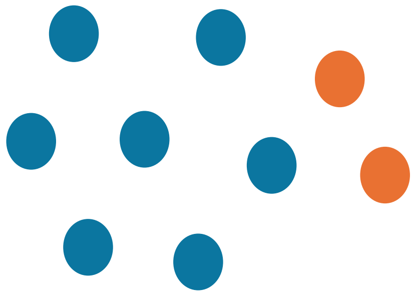

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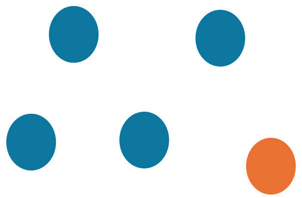

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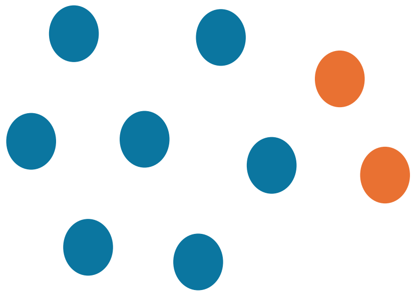

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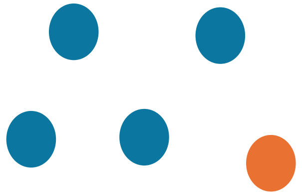

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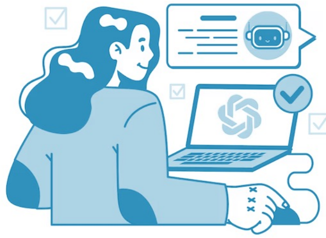

Choose safety-compliant model with best performance

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- $\boldsymbol{C}$ obtains a predictor $\hat{\beta}_C \in \mathbf{R}^d$

Model Details: High-dimensional regression assumptions

The covariates x are **high-dimensional**, i.e. $d \to \infty$ and $d \gg N$

We assume **power law decay** as a function of dimension:

- Eigenvalues of covariance matrix satisfy $\lambda_i \sim i^{-1-\gamma}$.
- Alignment coefficients satisfy $\mathrm{E}[\langle \beta, v_i \rangle]^2 \sim i^{-\delta}$.

Assumptions borrowed from Cui et al., '21, Wei et al., '22, Bach '23

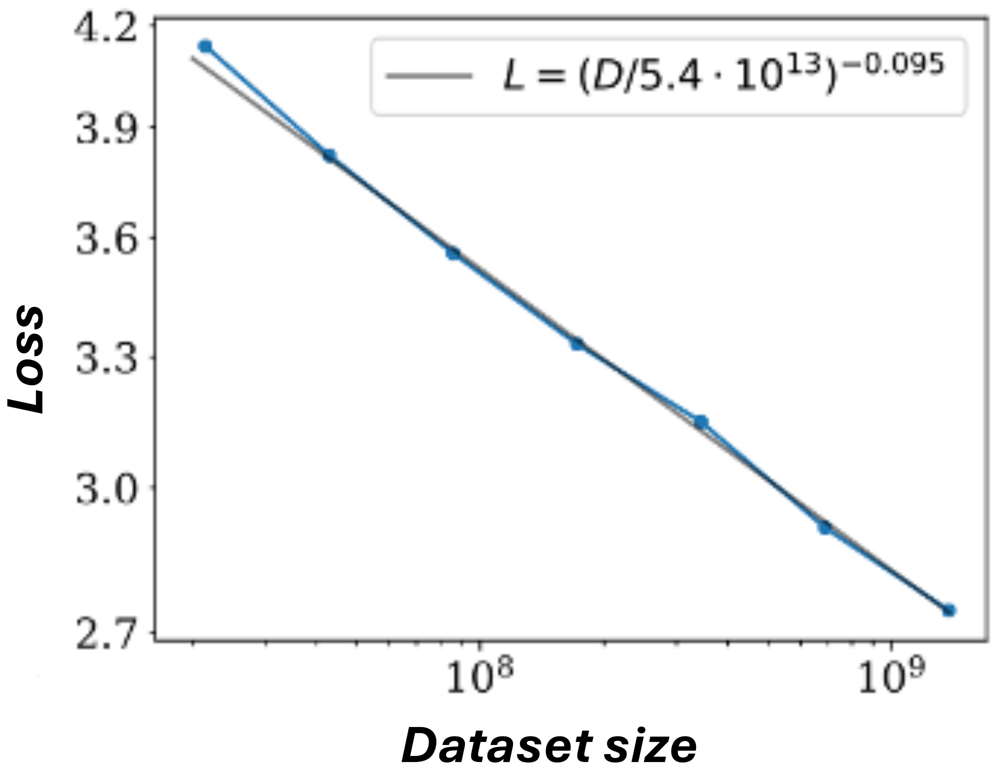
We specify the **correlation between safety and performance** as follow:

- β_1 and β_2 are drawn from a joint distribution with correlation $\rho \in [0,1]$ within each eigendimension, i.e. such that: $\mathrm{E}[\langle \beta_1, v_i \rangle \langle \beta_2, v_i \rangle] \sim \rho \cdot i^{-\delta}$.

Digression: Why high-dimensional regression?

For single-objective data scaling: high-dim regression captures LLM behavior.

LLMs:



e.g., Kaplan et al., 2020

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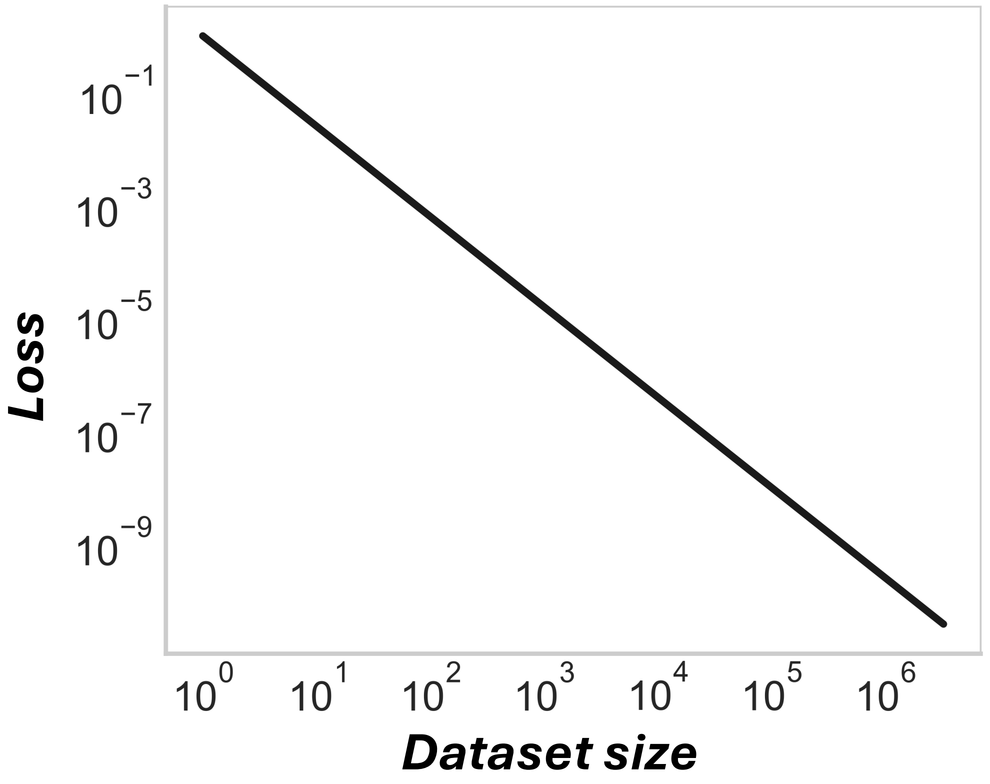

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Model Details: Evaluation of Safety and Performance

A company $C \in \{I, E\}$ is* **safety compliant** if:

$$\mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}_C\,, x\rangle - \langle\beta_2\,, x\rangle\right)^2\right] \leq \tau_C$$

Safety loss

Safety compliance threshold

**Caveat: we approximate the safety / performance loss by a deterministic equivalent*

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Consumers choose* the safety-compliant company that **maximize performance**, i.e. that minimize:

$$\mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}_C\,, x\rangle - \langle\beta_1\,, x\rangle\right)^2\right].$$

Performance loss

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Model Details: Company choices and Market entry threshold

Each C chooses* α_C and λ_C to ***maximize performance subject to safety compliance.***

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Regularization level

$$\min_{\alpha_C, \lambda_C} \mathbf{E}_{x\sim D}\left[\left(\langle \hat{\boldsymbol{\beta}}_C , \boldsymbol{x}\rangle - \langle \boldsymbol{\beta}_1 , \boldsymbol{x}\rangle\right)^2\right]$$

$$\text{s.t. } \mathbf{E}_{x\sim D}\left[\left(\langle \hat{\boldsymbol{\beta}}_C , \boldsymbol{x}\rangle - \langle \boldsymbol{\beta}_2 , \boldsymbol{x}\rangle\right)^2\right] \leq \boldsymbol{\tau}_C$$

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Market entry threshold := minimum dataset size N_E^* such that the new company E:

- Satisfies* safety compliance* $\mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}_E, x\rangle - \langle\beta_2, x\rangle\right)^2\right] \leq \tau_E$, and
- Achieves* performance $\mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}_E, x\rangle - \langle\beta_1, x\rangle\right)^2\right] \leq \mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}_I, x\rangle - \langle\beta_1, x\rangle\right)^2\right]$.

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Our goal: characterize the market entry threshold $\boldsymbol{N_E^*}$

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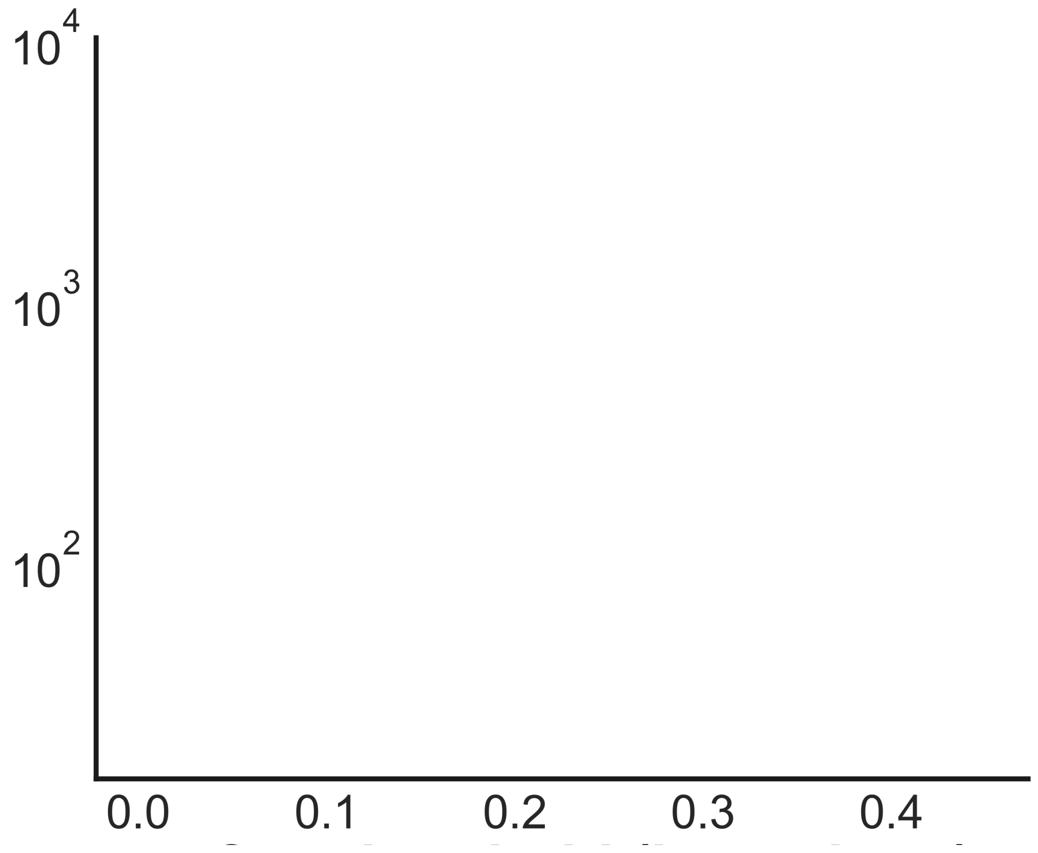
Warmup result

Setup: Incumbent has infinite data $N_I = \infty$*; new company faces no safety constraint* $\tau_E = \infty$.



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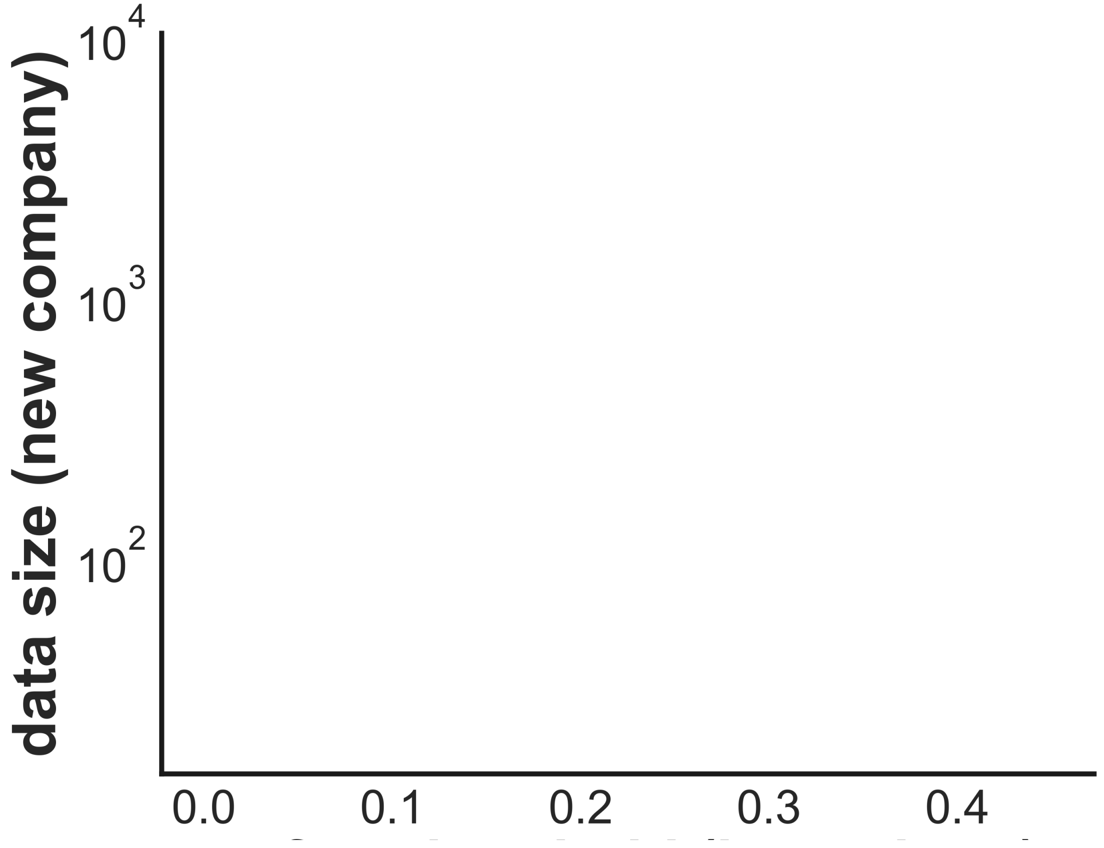

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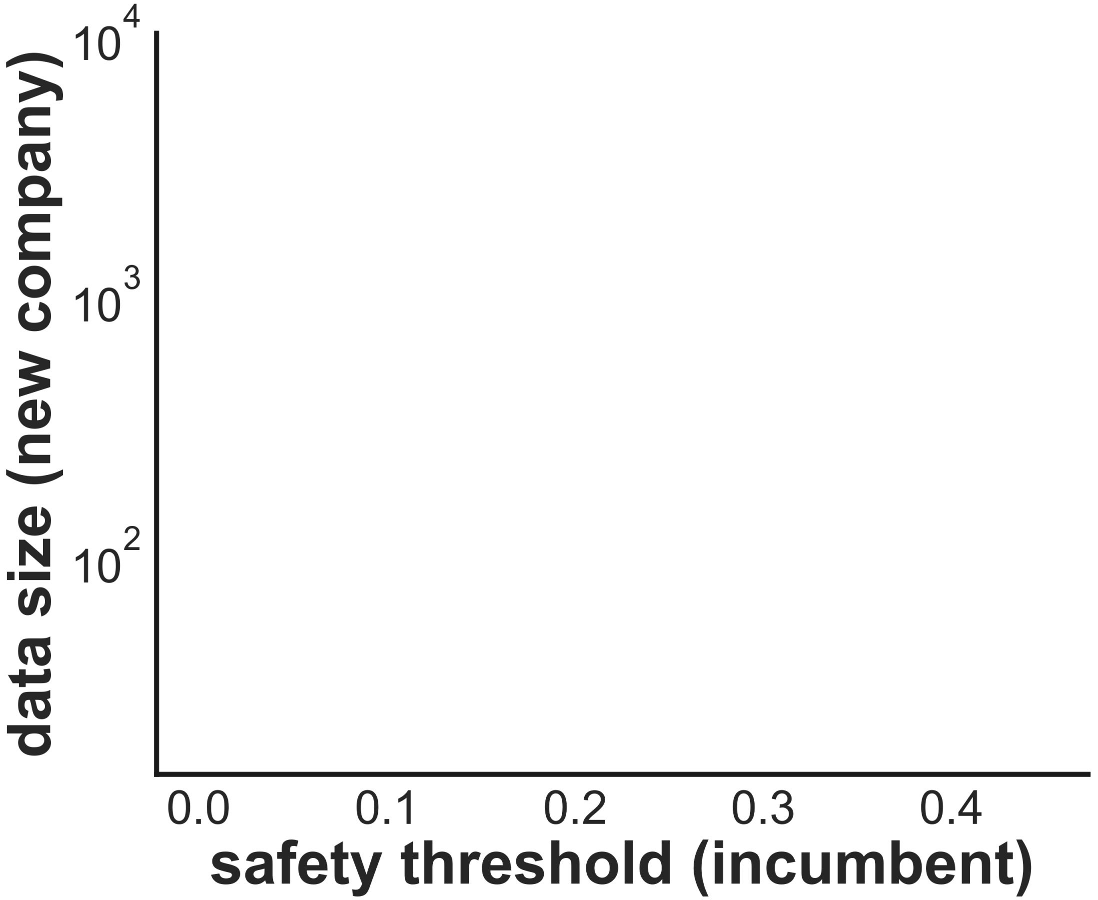

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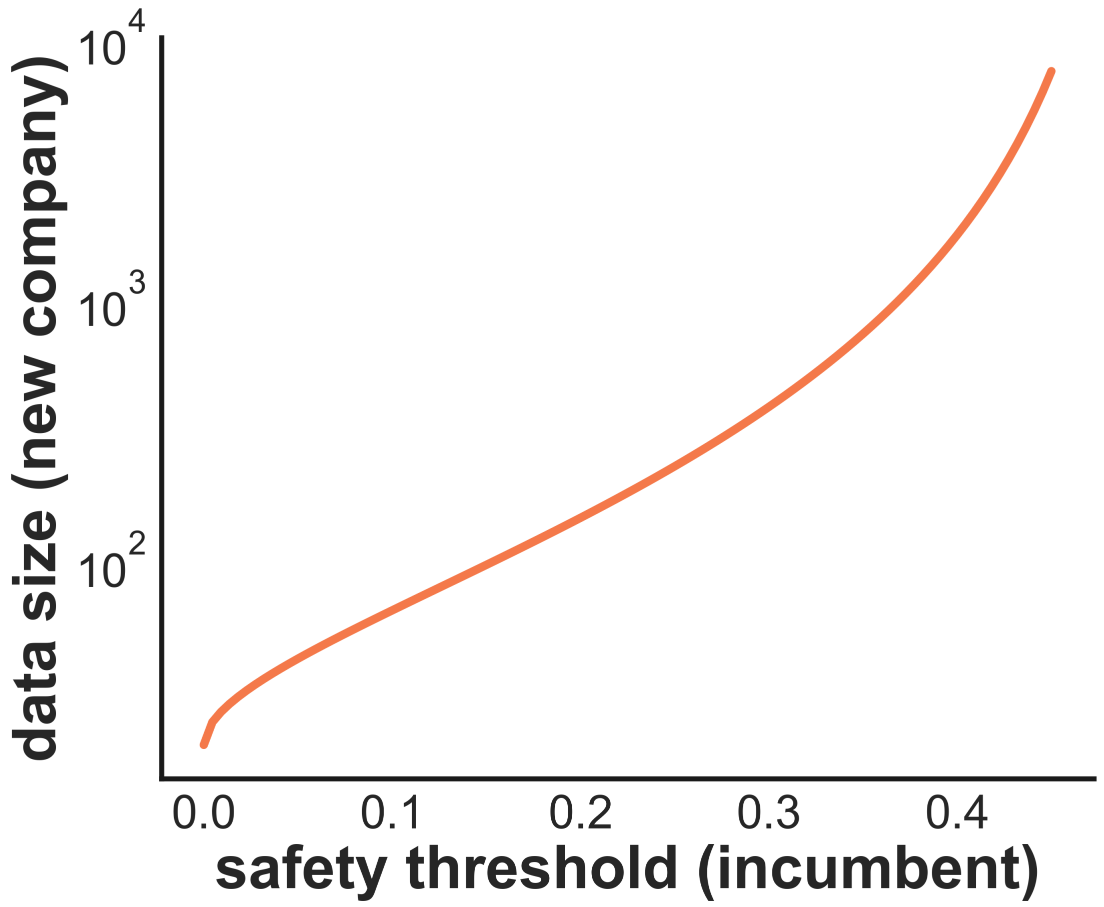

Takeaway: New company can enter with finite data, even with an infinite-data incumbent.

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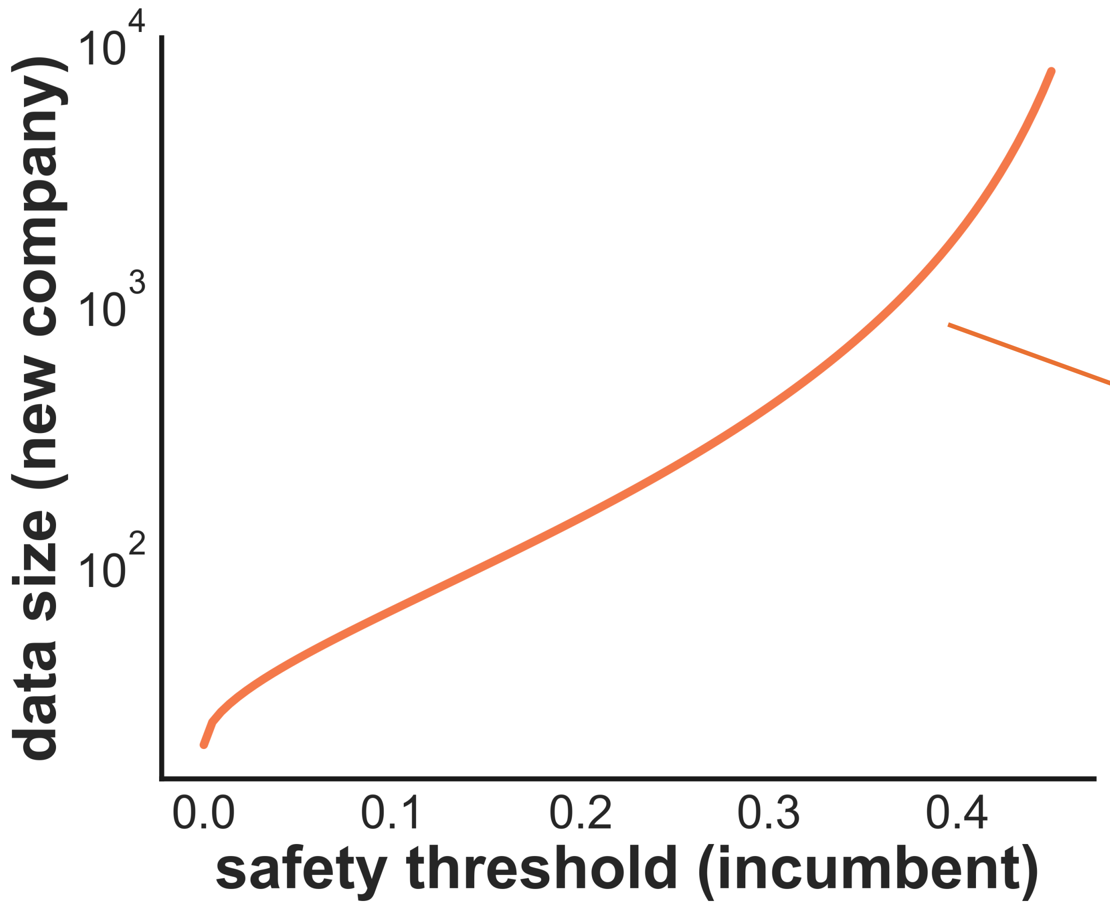


<u>Takeaway</u>: New company can enter with finite data, even with an infinite-data incumbent.

$$N_E^* = \Theta\left(\left(\sqrt{\boldsymbol{L}} - \sqrt{\min(\boldsymbol{L}, \boldsymbol{\tau_I})}\right)^{-\frac{2}{\nu}}\right).$$

L = Optimal infinite-data loss w/o safety

Data efficiency $\nu = min(2(1+\gamma), \gamma + \delta)$

Intuition for warmup

Key driver: The new company can train more unsafe models.

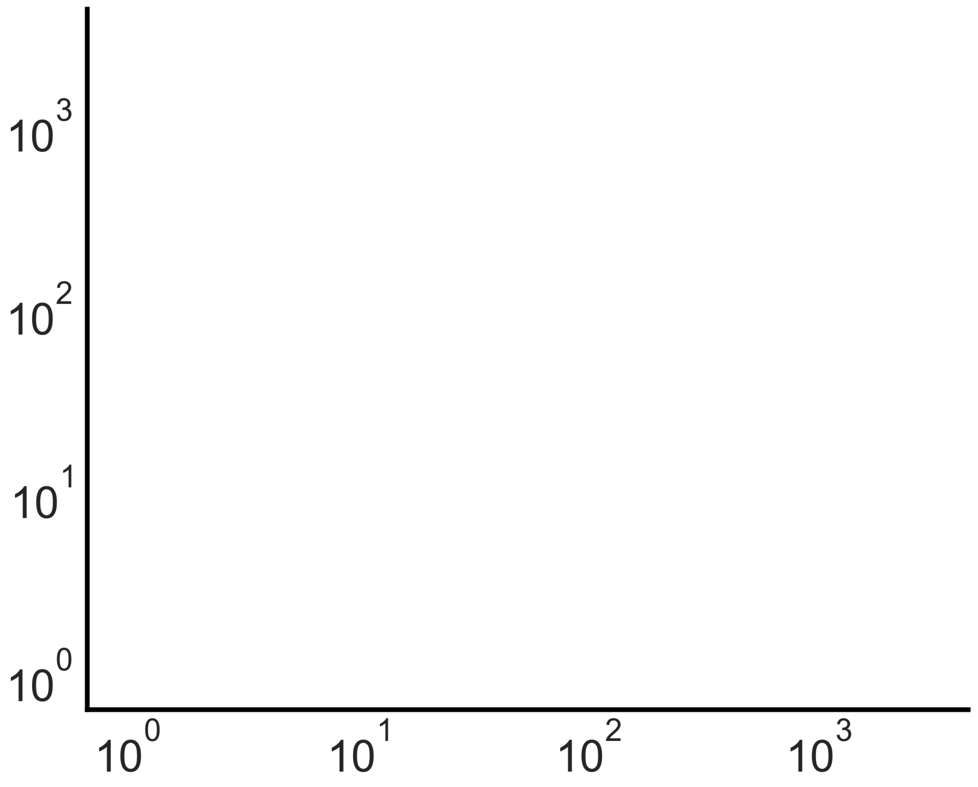
The incumbent must conservatively balance safety and performance, but the new company can focus more on performance.

$\Rightarrow$ The new company **curates its training data** to prioritize performance.

$\Rightarrow$ **The new company can enter the market with less data than the incumbent.**

Role of the incumbent's dataset size N_I

Setup: New company faces no safety constraint (i.e., $\tau_E = \infty$)



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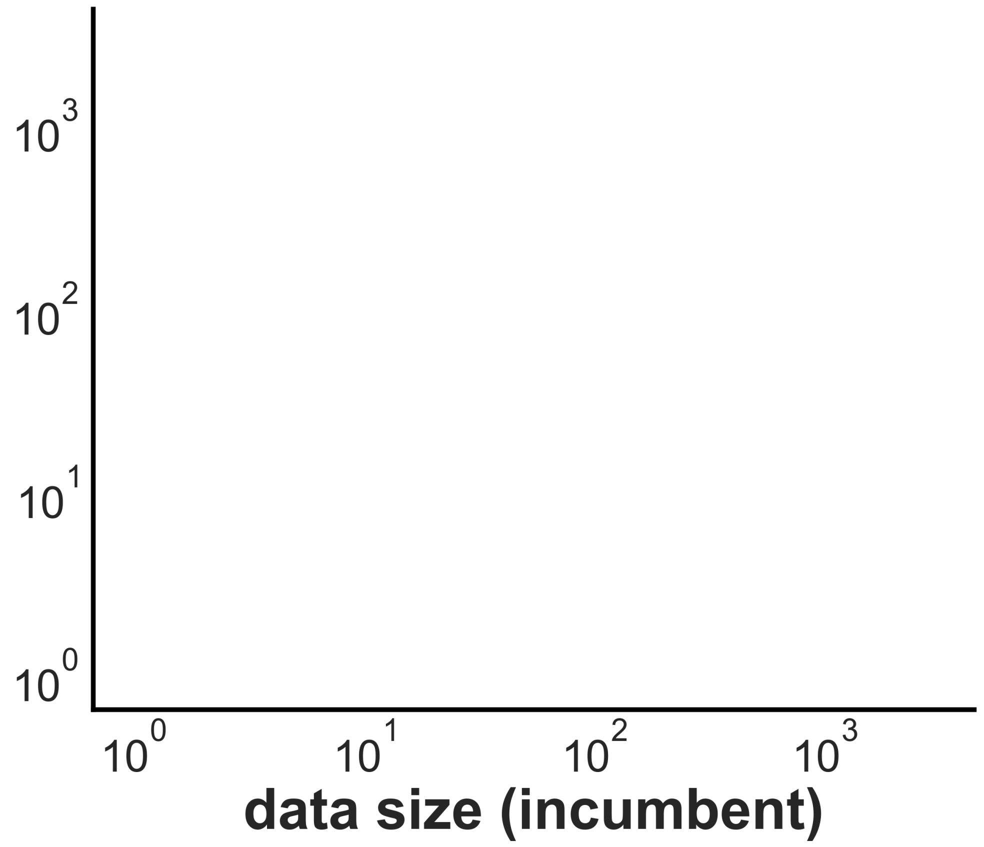

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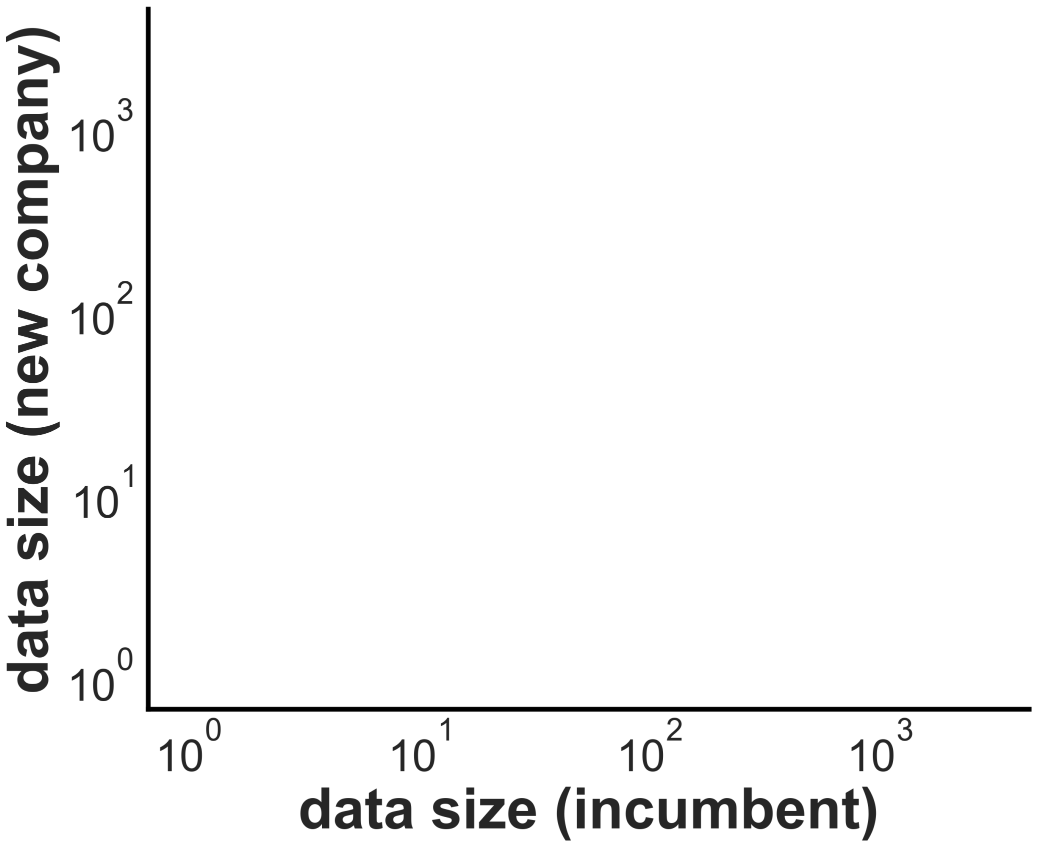

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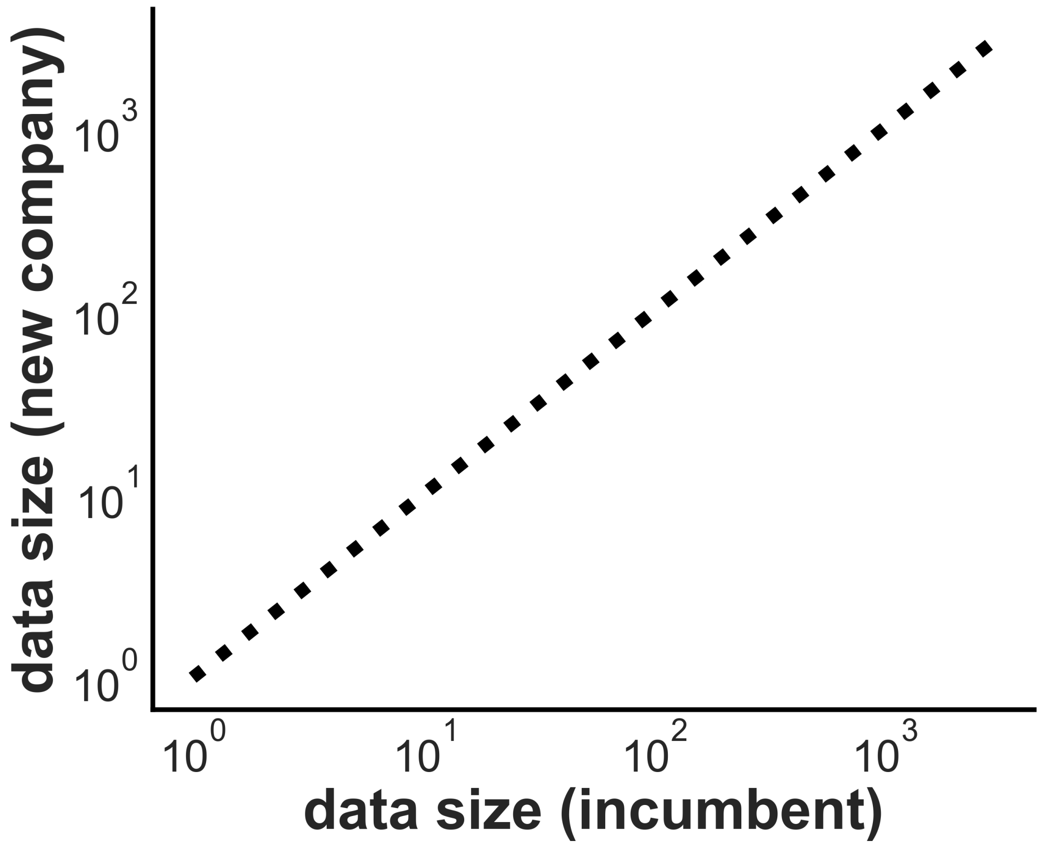

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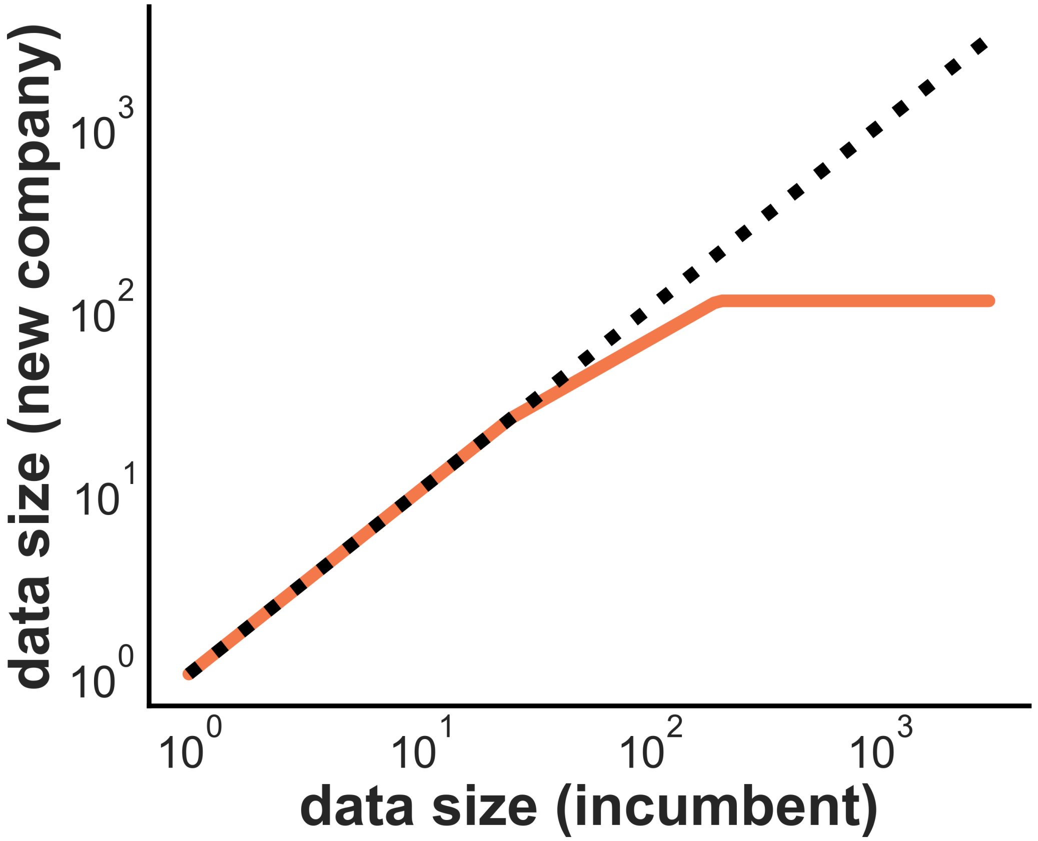

Three regimes of behavior

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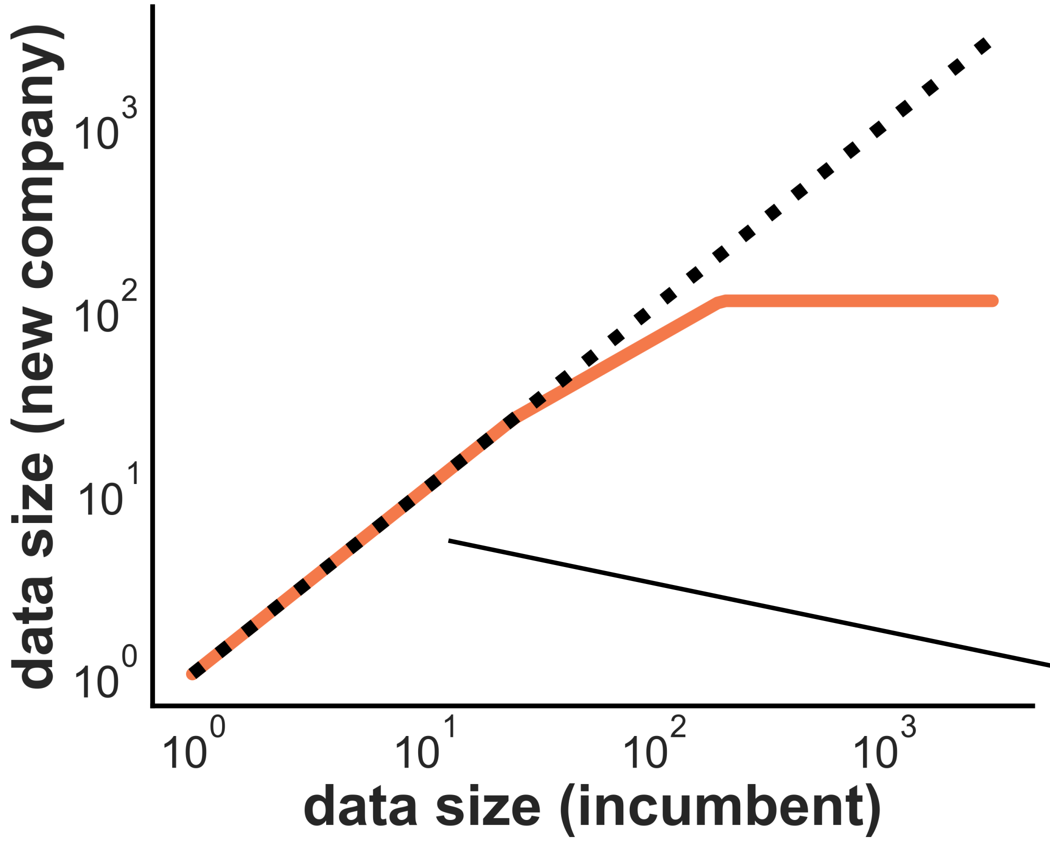

$$N_E^* = \Theta(N_I)$$

New company needs **as much data** as the incumbent

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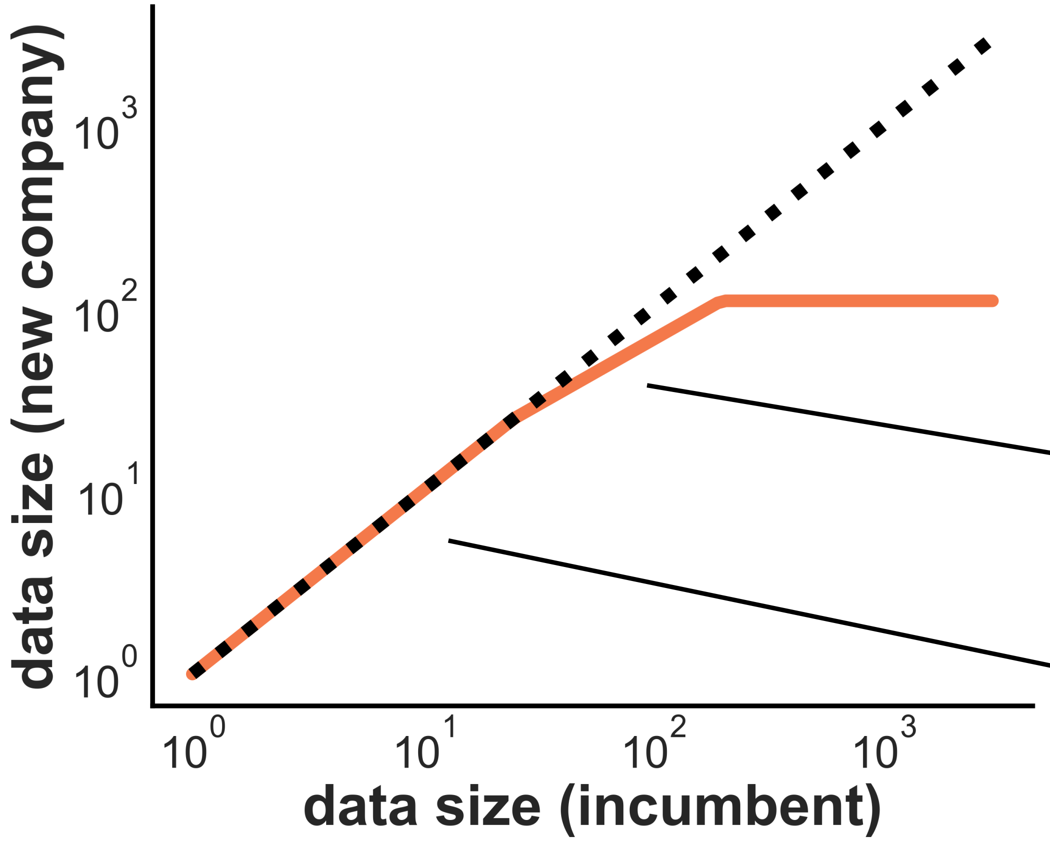

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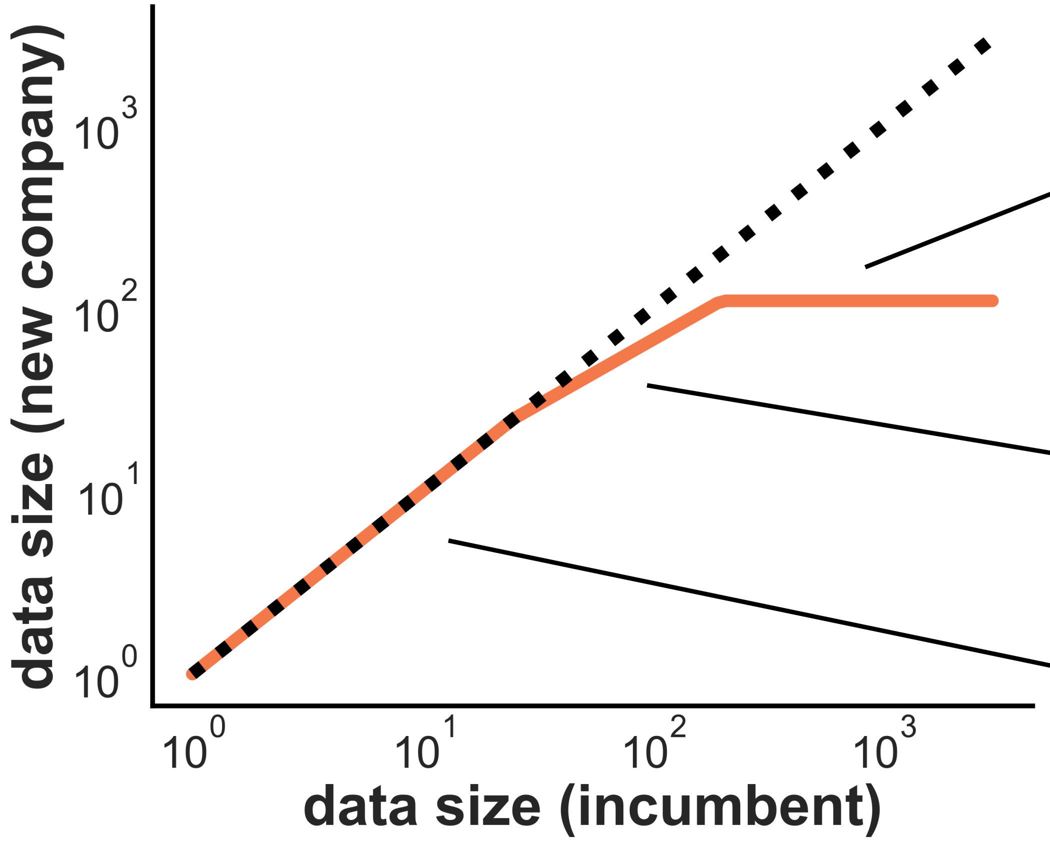

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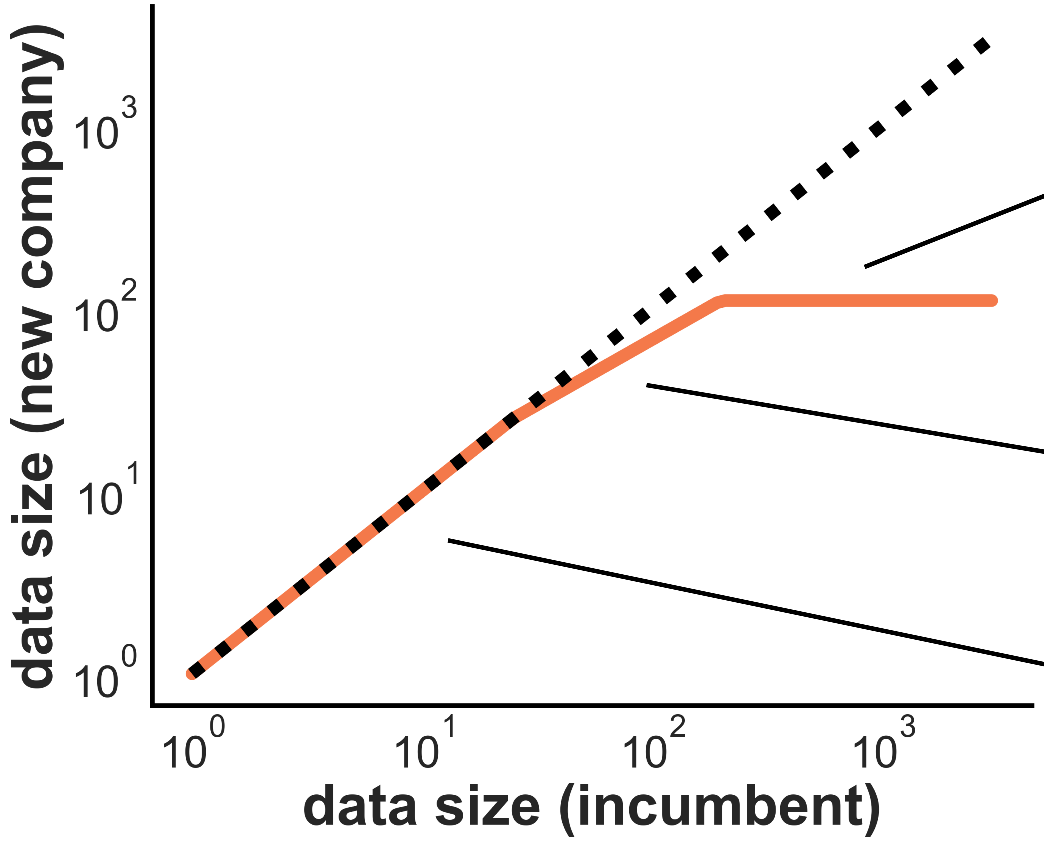

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$$N_E^* = \Theta(1)$$

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New company needs **less data** than the incumbent

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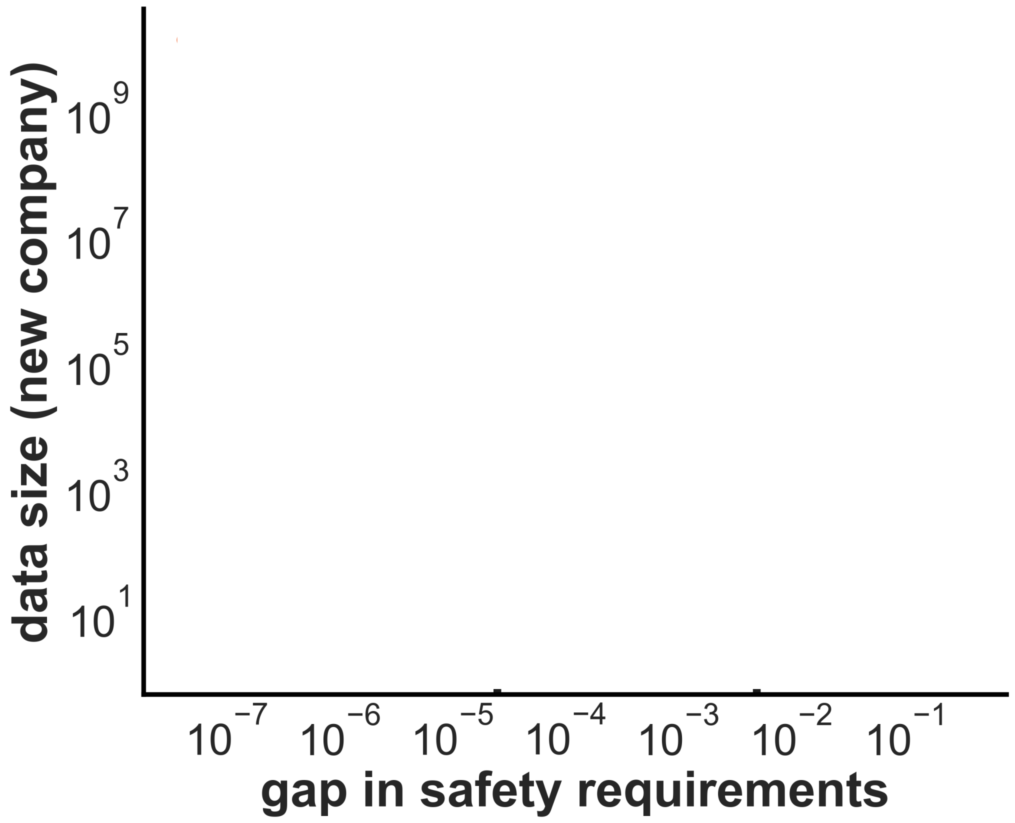
Role of the gap $\boldsymbol{D}$ in safety thresholds

Setup: Incumbent has infinite data ($N_I = \infty$), D = performance gap in infinite−data regime



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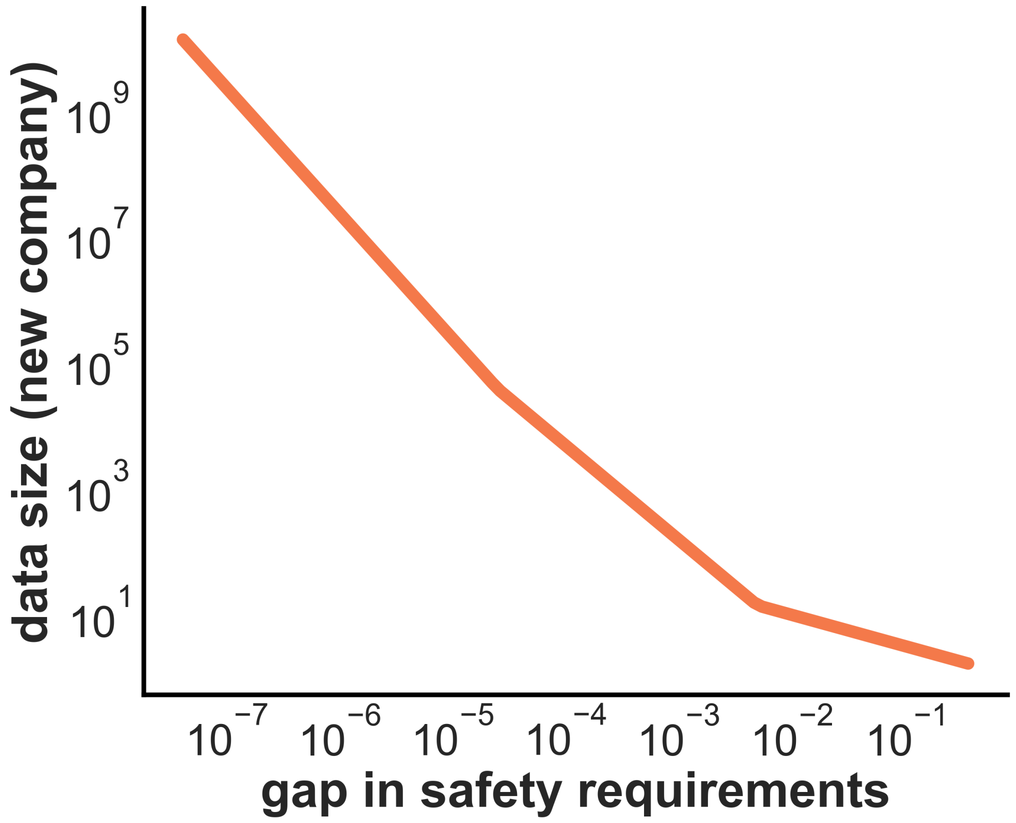

Takeaways:

- New company only needs **finite data**

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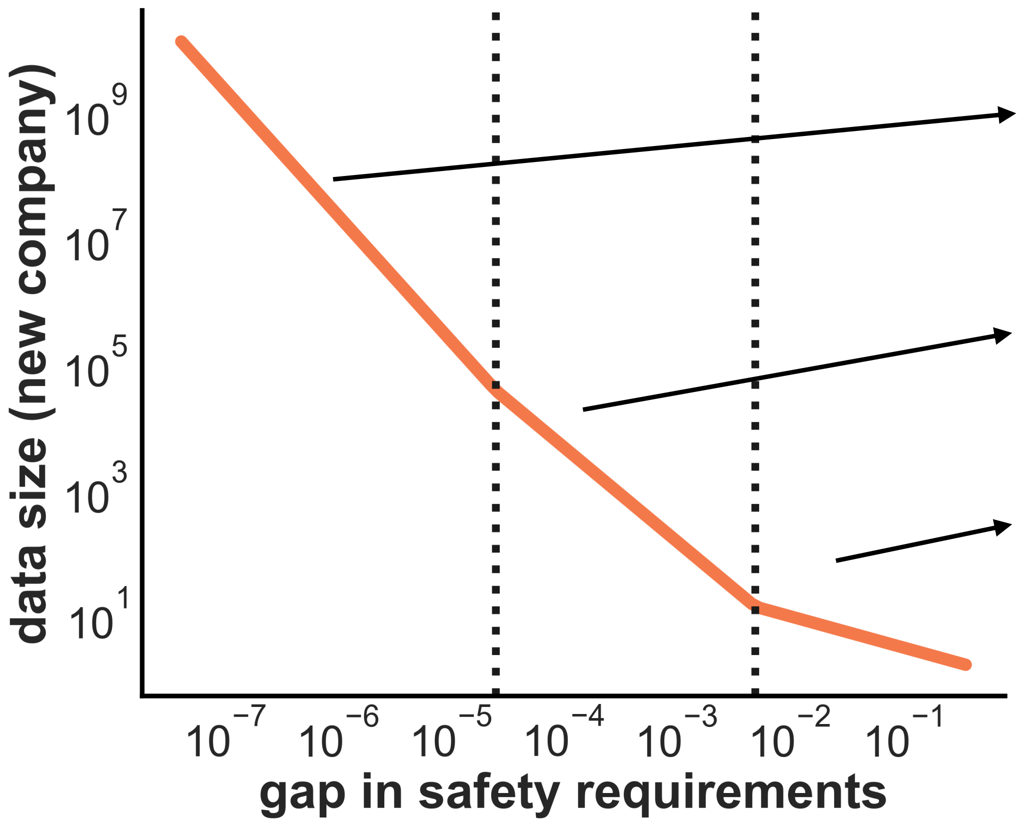

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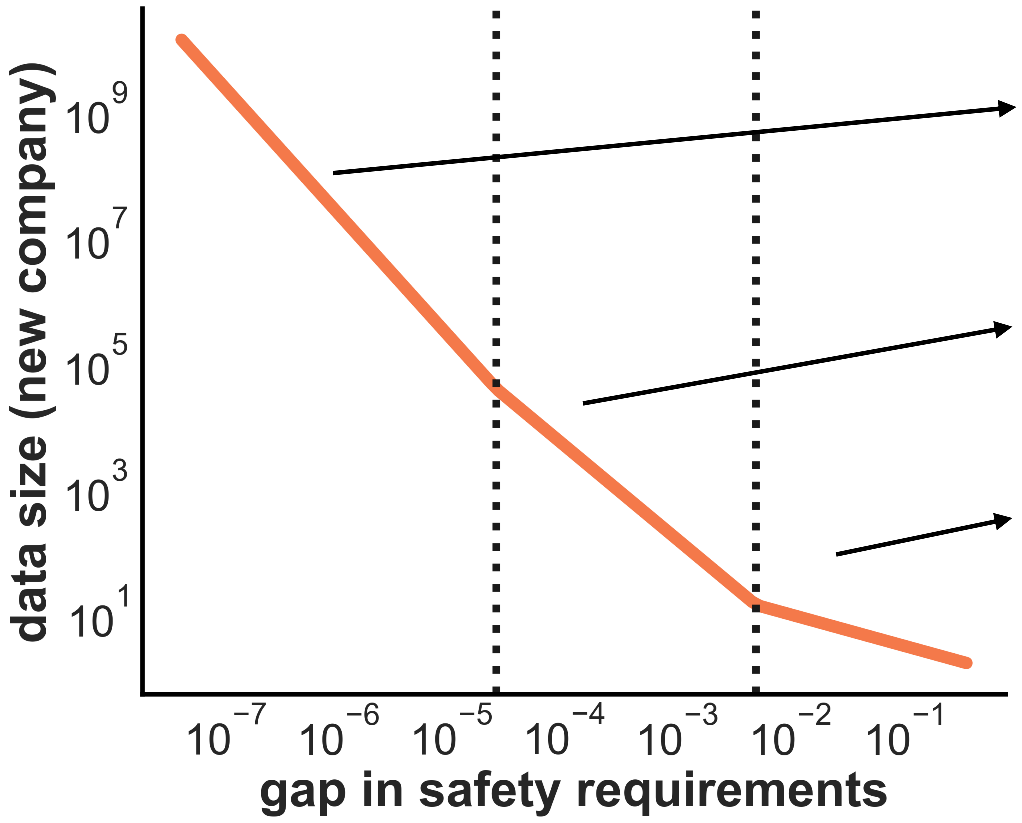

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Takeaways:

- New company only needs **finite data**
- New company must **scale up data** faster when safety thresholds are more even.

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Implication: when does scrutiny of safety reduce data-driven barriers to entry?

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Our findings:

- Uneven scrutiny of safety reduces data-driven barriers to entry ***only when the incumbent’s dataset size is sufficiently large***.
- If the scrutiny is more even, then the data-driven barriers to entry not only increase but also ***scale up at a faster rate***.

Outline for the talk

1. Background
2. Our model
3. Our results
4. **Technical ideas**

Summary

Technical tool: derive multi-objective data scaling laws

<u>**Result**</u>: We characterize how the **loss** of **optimally regularized** ridge regression in terms of the **training data size** N and **data mixture level** α.

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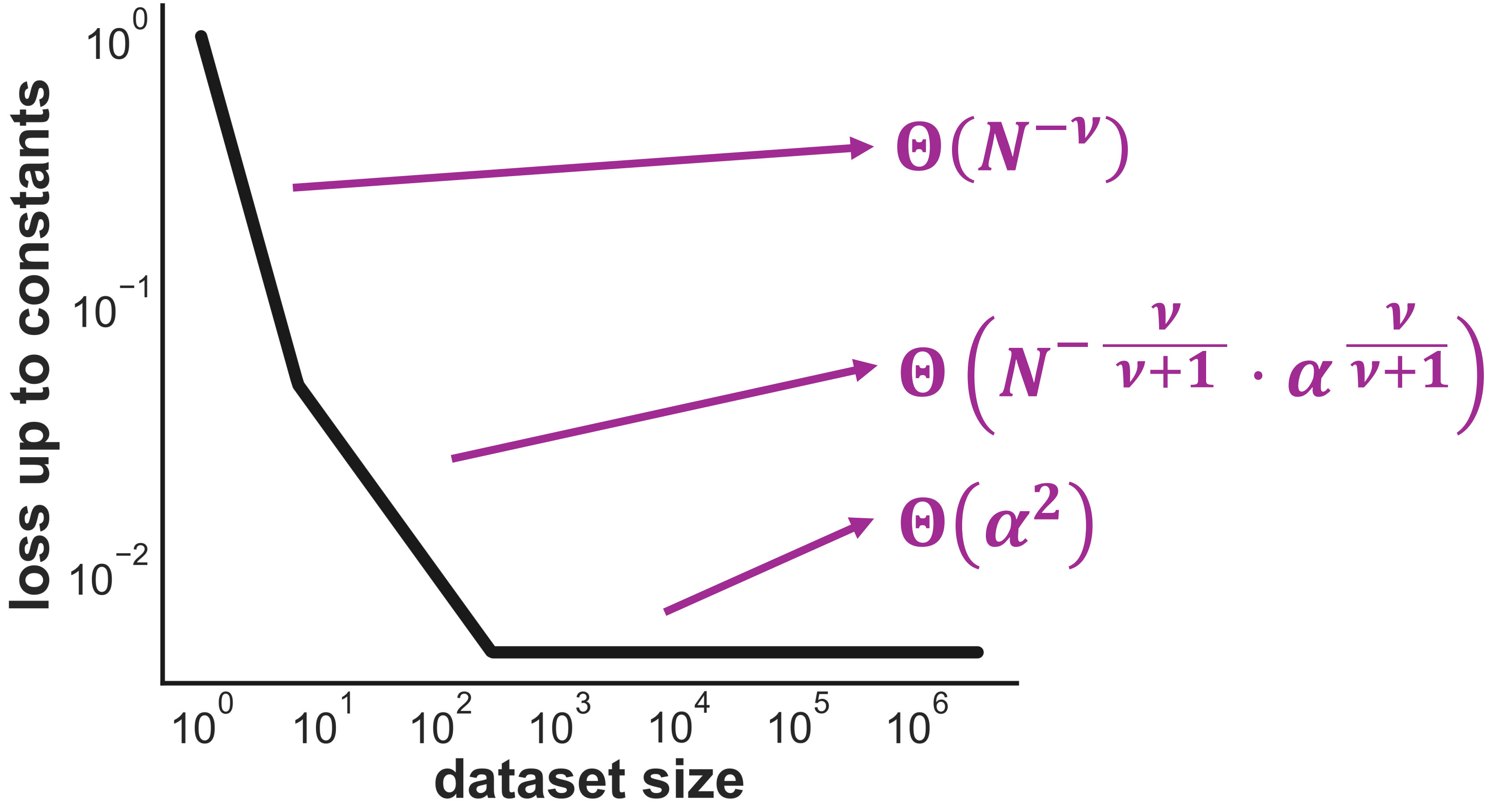

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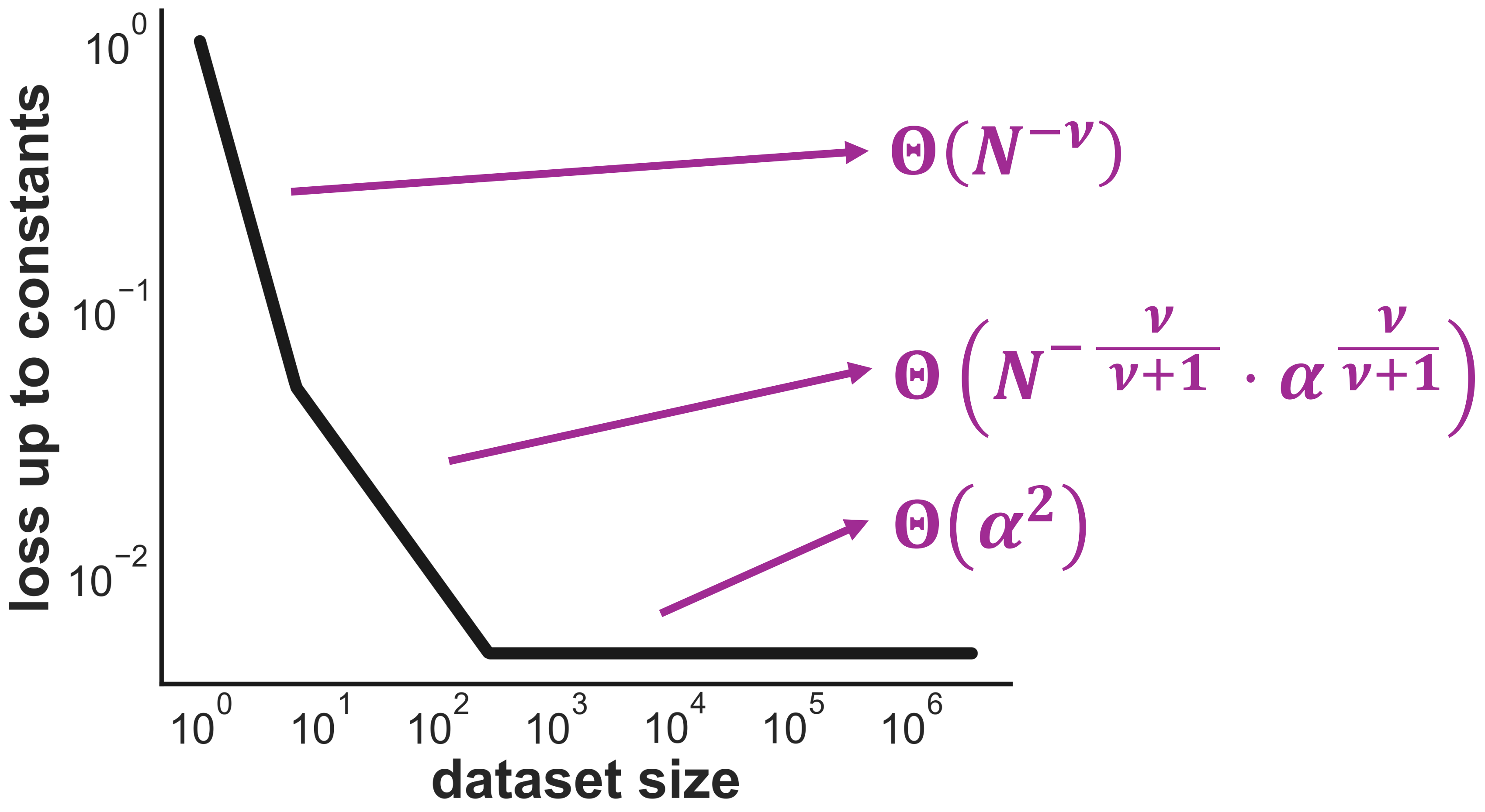

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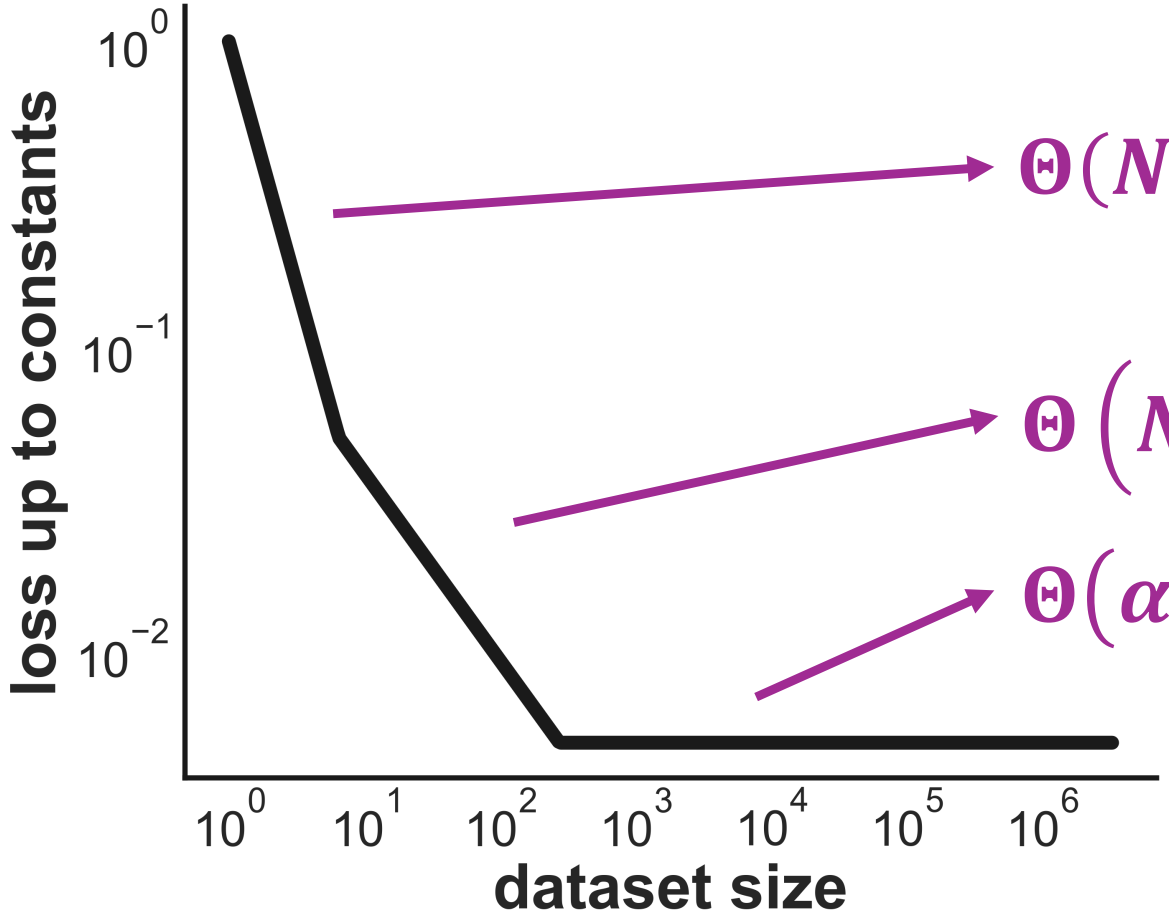

$\Theta(N^{-\nu})$

$\Theta\left(N^{-\frac{\nu}{\nu+1}} \cdot \alpha^{\frac{\nu}{\nu+1}}\right)$

$\Theta(\alpha^2)$

Key insight: *multi-objective* data efficiency decreases as the data size N increases

In comparison: *single-objective* data efficiency is constant in N

e.g., Cui et al., '21, Wei et al., '22, Bach '23

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Excess loss subtracts out the infinite-data performance with data mixture α.

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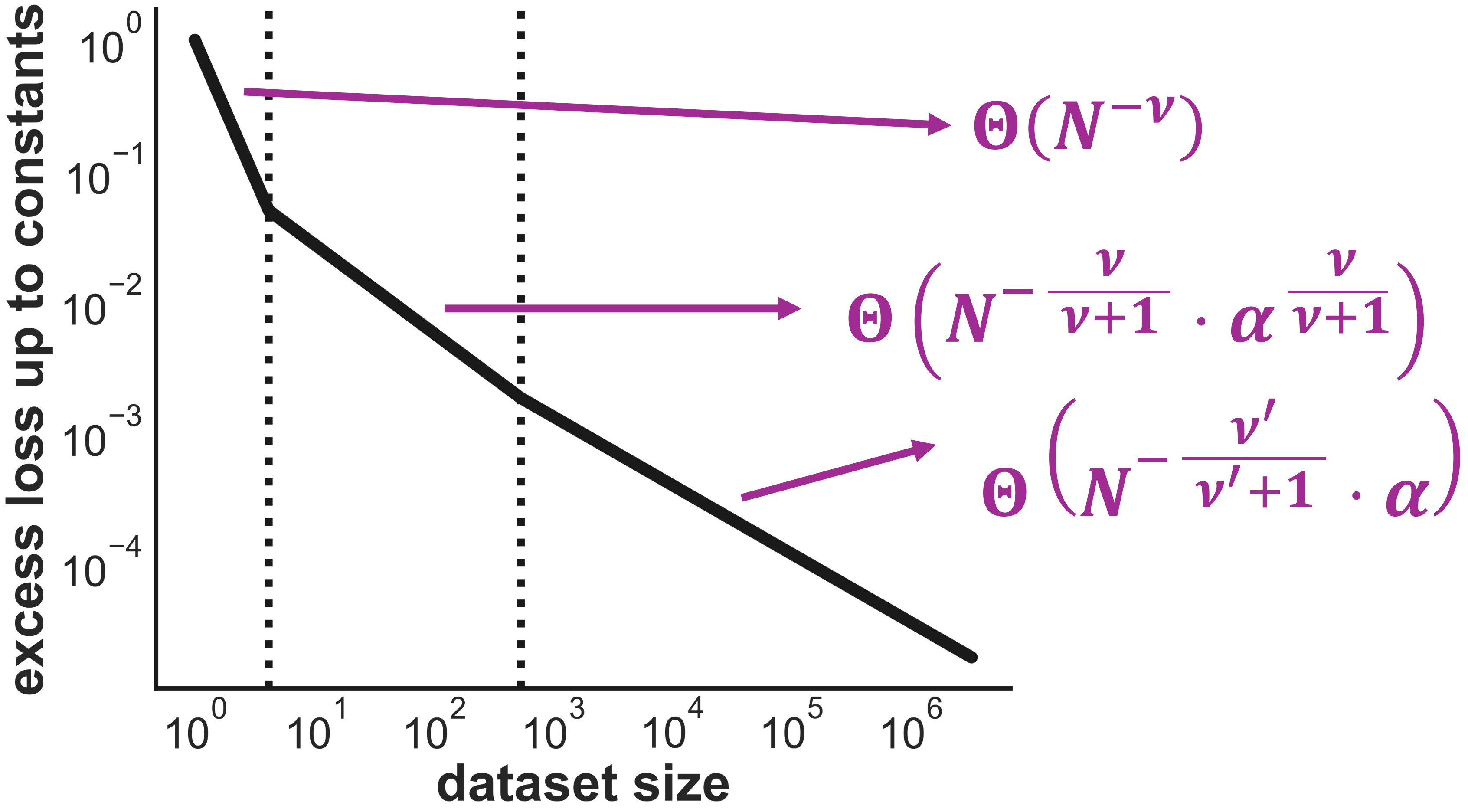

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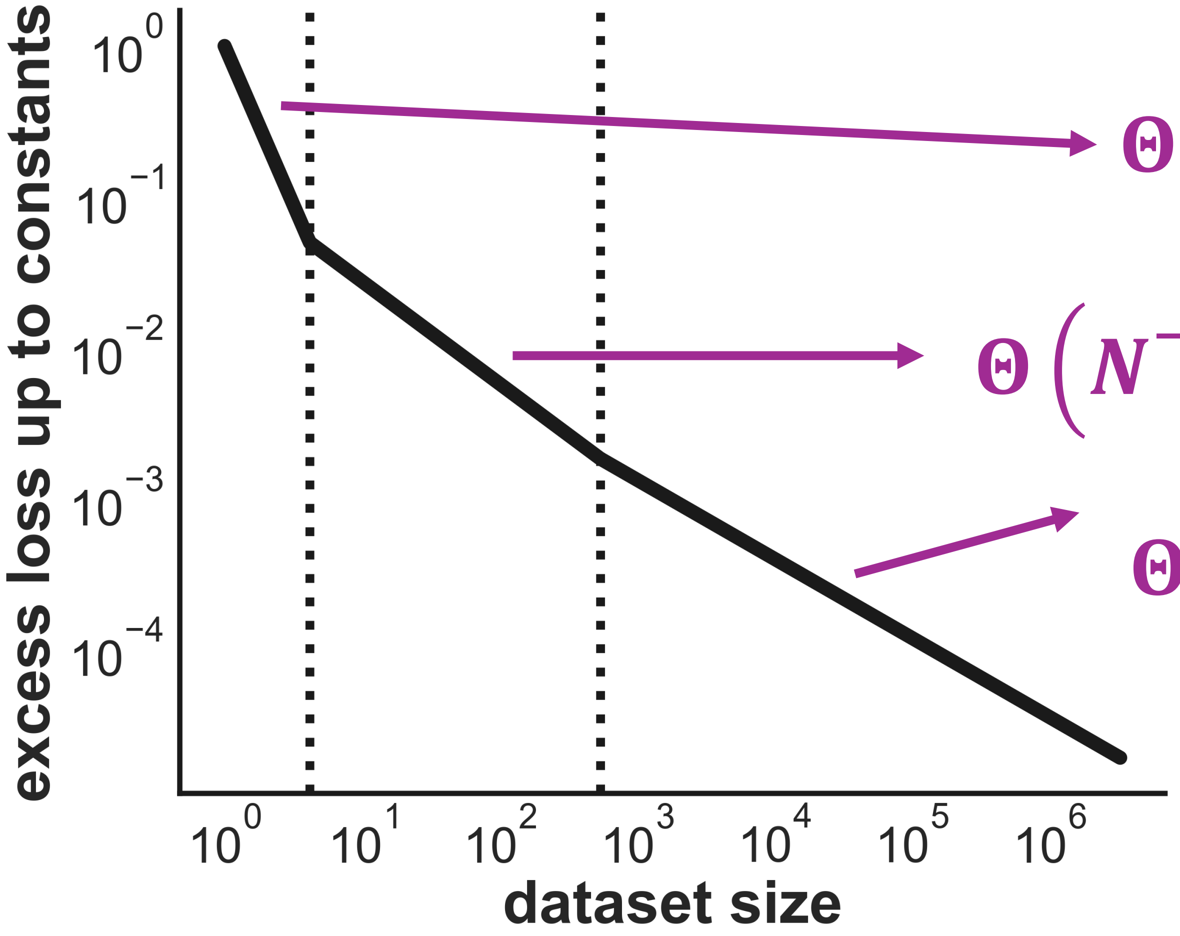

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Need **tight bounds** on the **loss** $\mathbf{E}_{x \sim D}\left[\left(\langle \hat{\beta}, x \rangle - \langle \beta_1, x \rangle\right)^2\right]$ of ridge regression

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Key idea: Use random matrix theory to characterize the loss

- Derive a **deterministic equivalent** using the Marčenko-Pastur law
- Characterize loss under the **power law decay assumptions**
- Analyze scaling behavior under **optimal regularization**

Bounds on the loss for multi-objective regression

Setup: training data size N, data mixture level α, regularization level λ

Lemma (Informal): The loss $\mathbf{E}_{x\sim D}\left[\left(\langle\hat{\beta}, x\rangle - \langle\beta_1, x\rangle\right)^2\right]$ is approximately equal to:

$$\underbrace{\max\left(\lambda^{\frac{\nu}{1+\gamma}}, N^{-\nu}\right)}_{\text{Finite data error}} + \alpha \cdot Q \cdot \underbrace{\frac{\min\left(N,\ \lambda^{-\frac{1}{1+\gamma}}\right)}{N}}_{\text{Overfitting error}} + \alpha^2 \cdot \underbrace{Q}_{\text{Optimal infinite data loss}} + \alpha \cdot Q \underbrace{\max(\lambda^{\frac{\nu'}{1+\gamma}}, N^{-\nu'})}_{\text{Extra (Mixture finite data error)}}$$

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Finite data error

Overfitting error

Optimal infinite data loss

Extra (Mixture finite data error)

<u>Implication</u>: must regularize to avoid overfitting, but this reduces data efficiency

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We studied data-driven barriers to market entry for companies training LLMs.

<u>This work</u>: a technical framework to quantify how much data a new company needs to enter the market

- *Model*: We modelled these markets within a multi-objective learning framework.
- *Technical tool*: multi-objective data scaling laws

***Key finding*: *Scrutiny of safety* often---but not always---enables *new LLM companies* to enter the market with less data than incumbents**

Broader direction: *how do **details of the ML pipeline** shape the **market of companies training ML models**?*

Training data

Evaluation metrics

Pretraining & finetuning