Problem Set 7: Computational Methods for the Economists

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Let us consider an individual who is choosing between current consumption and future spending on education of his children.
Preference: $\sum_{t=0}^{T} \beta^t ln(c_t)$
The degree of altruism shown towards his children is given by the discount factor β. The value of β lies between 0 and 1.
Endowment: The individual has an education level at initial period of time $e0 > 0$ and thus an annual wage of person with e years of education is given by $w(e0)$.
State variables: c_0 and e_0
Control variables: c_t and e_{t+1}

Therefore, the problem which defines the optimal individual consumption and the education level choice for the descendants is:

$$\begin{aligned} \max_{c_t, e_{t+1}} \sum_{t=0}^{T} \beta^t ln(c_t) \\ \text{s.t.} i) w(e_t) = c_t + g(e_{t+1}) \\ ii)\ c_t \geq 0 \\ iii)\ u'(c) > 0, u''(c) < 0 \end{aligned} \tag{1}$$

The lagrange function for this problem can be written as follows:

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t ln(c_t) + \lambda_t \sum_{t=0}^{T} [w(e_t) - c_t + g(e_{t+1})] + +\phi_t \sum_{t=0}^{T} c_t \tag{2}$$

The first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \frac{1}{c_t} - \lambda_t + \phi_t = 0 \\ \frac{\partial \mathcal{L}}{\partial e_{t+1}} = \lambda_t g(e_{t+1}) - \lambda_{t+1} w(e_{t+1}) = 0 \end{aligned} \tag{3}$$

Since our utility function is a logarithmic function, due to Innada condition $\phi_t = 0$

From the FOCs we get,

$$\begin{aligned} \beta^t \frac{1}{c_t} &= \lambda_t \\ \beta^{t+1} \frac{1}{c_{t+1}} &= \lambda_{t+1} \\ \frac{\lambda_{t+1}}{\lambda_t} = \beta^t \frac{c_t}{c_{t+1}} &= \frac{w(e_{t+1})}{g(e_{t+1}} \end{aligned} \tag{4}$$

This is an Euler equation which shows the equality between the inter-temporal marginal rate of substitution and the marginal real cost of education. It means that the willingness to substitute current consumption for future consumption must be equal to the net gain of increasing the level of the off-springs' education.

We can write the Bellman equation associated with the above problem as:

$$v(e_0) = \max_e lnw(e_0) - g(e) + \beta v(e) \tag{5}$$

To solve the model I need to specify the wage function $w(e)$ and education expenditure function $g(e)$. For simplicity I assume a linear returns to education $w(e) = m + n * e$ and linear education expenditure function $g(e) = k * e$ where k is the average annual education expenditure per unit of education level. Also, we assume that the lower boundary of education is 0 and the upper boundary of education is 22.

Below is the plotted value function for the model:

Figure 1: Value Function