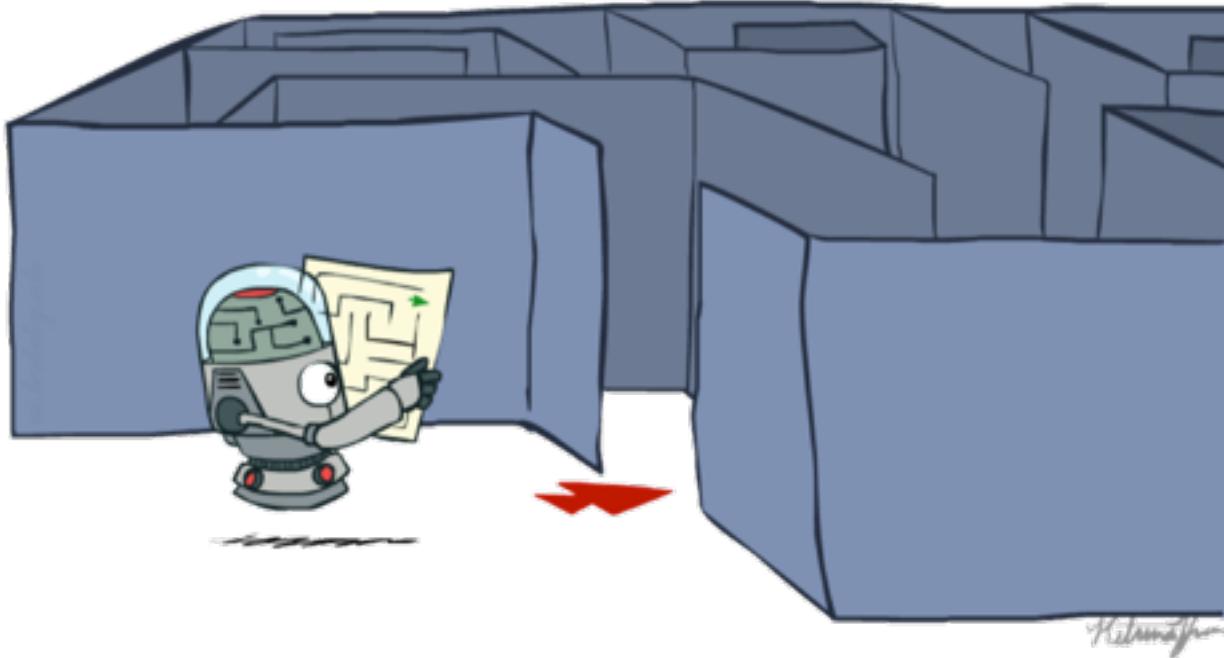


# Inteligencia Artificial

## Búsqueda



Instructor: Michael Jalkio

[mjalkio@gmail.com](mailto:mjalkio@gmail.com)

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All materials available at <http://ai.berkeley.edu>.]

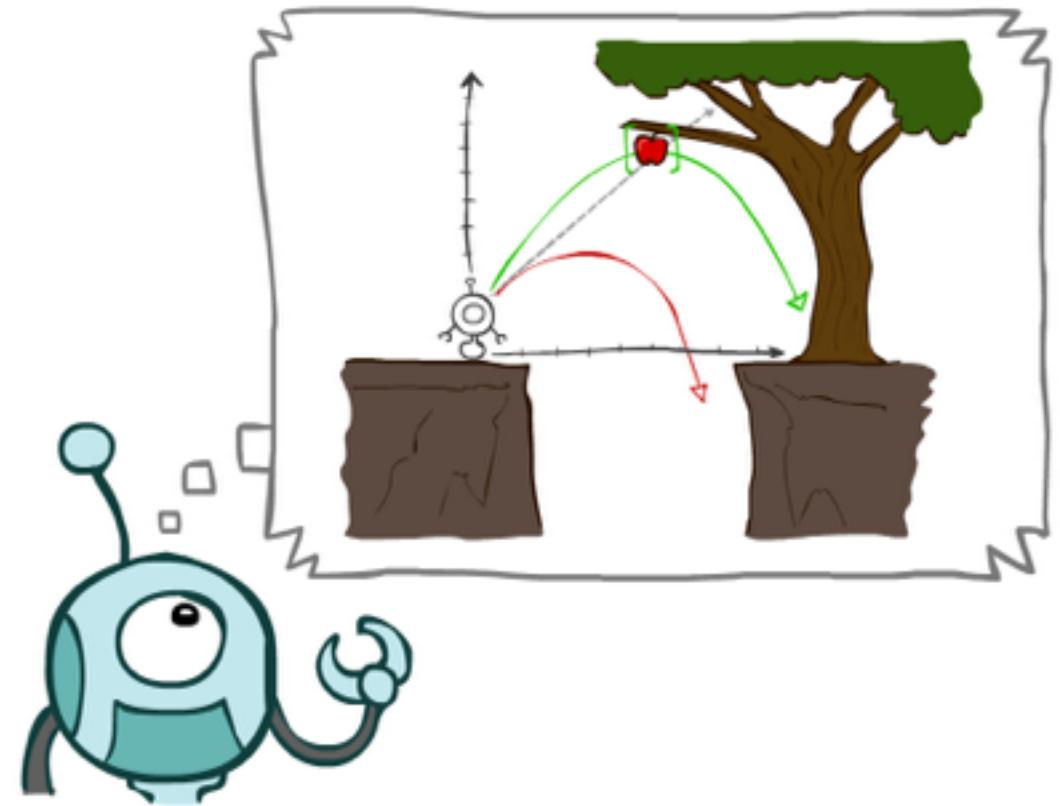
# Hoy

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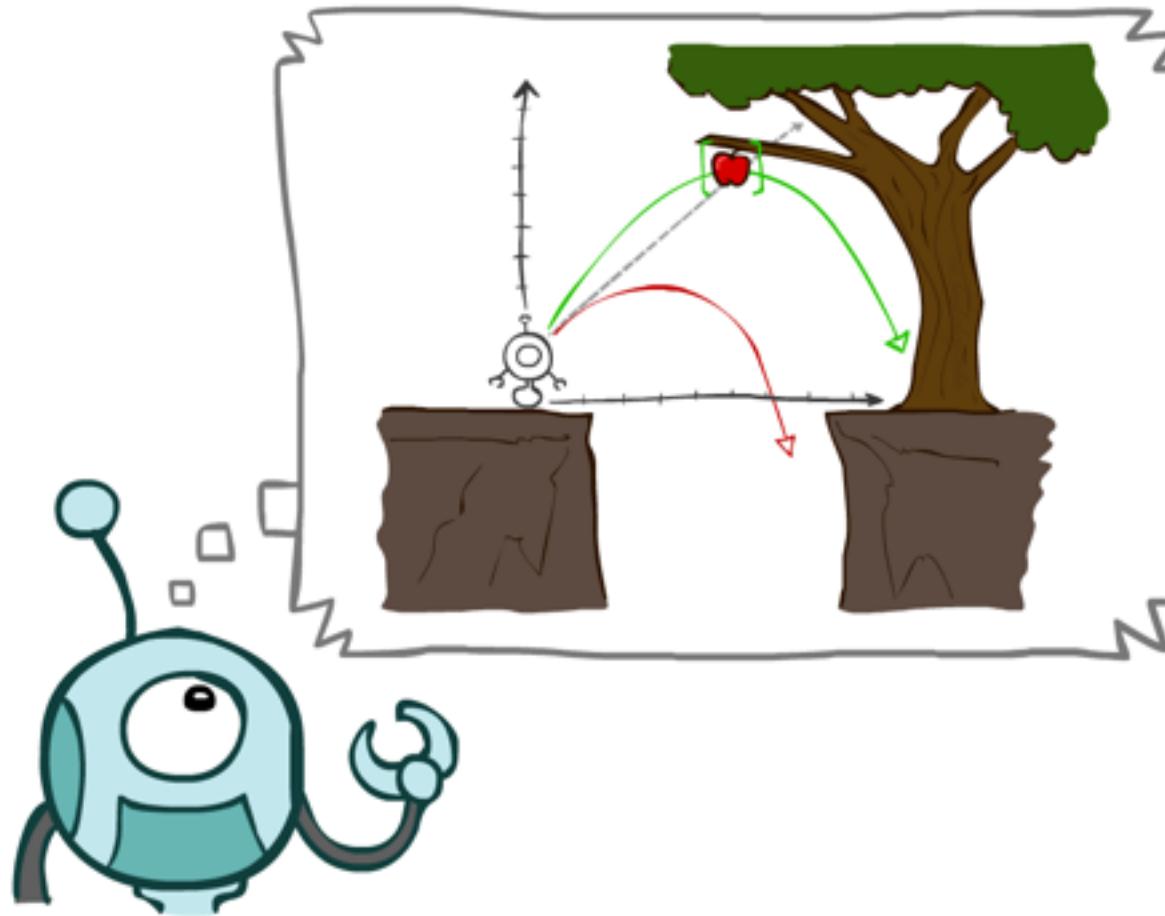
- Mis diapositivas estarán en Inglés porque Jorge está aquí.
- Dime si esto es peor que las diapositivas traducidas
- Voy a ir a través de las diapositivas, que son una introducción a buscar problemas
- Después, usted debe tener tiempo para trabajar en el proyecto
- Probablemente no va a terminar el proyecto durante la clase
- Haga preguntas sobre el grupo de Facebook y el domingo voy a preguntar si deberíamos ir sobre el proyecto en la siguiente clase

# Intro to Search

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

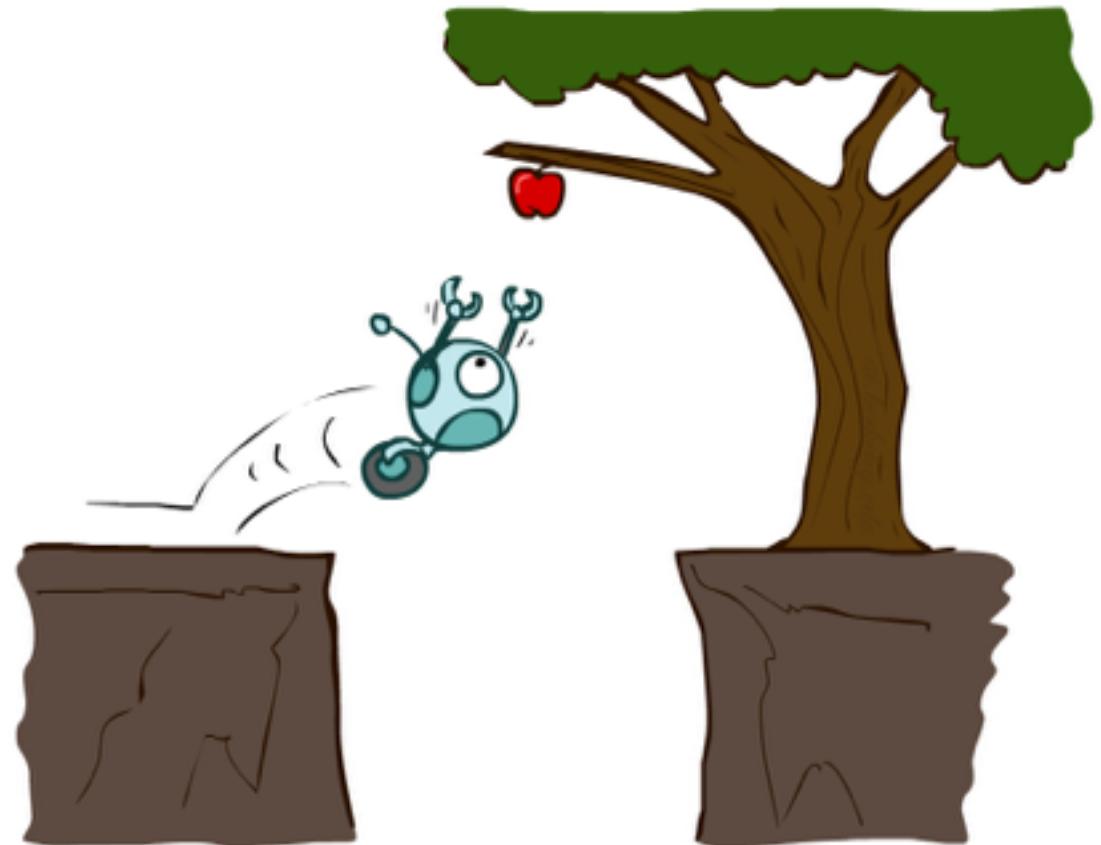


# Agents that Plan



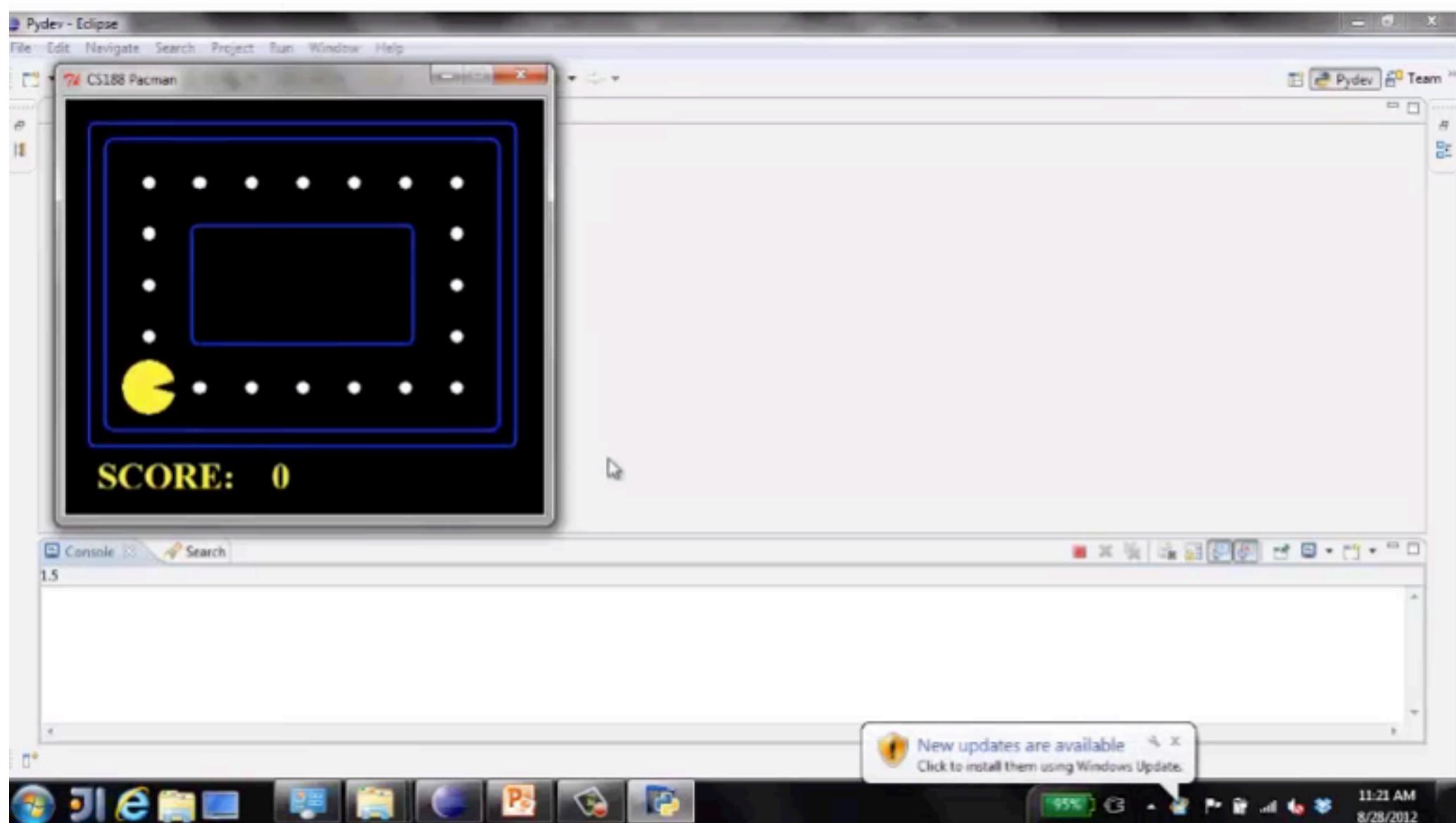
# Reflex Agents

- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS
- Can a reflex agent be rational?

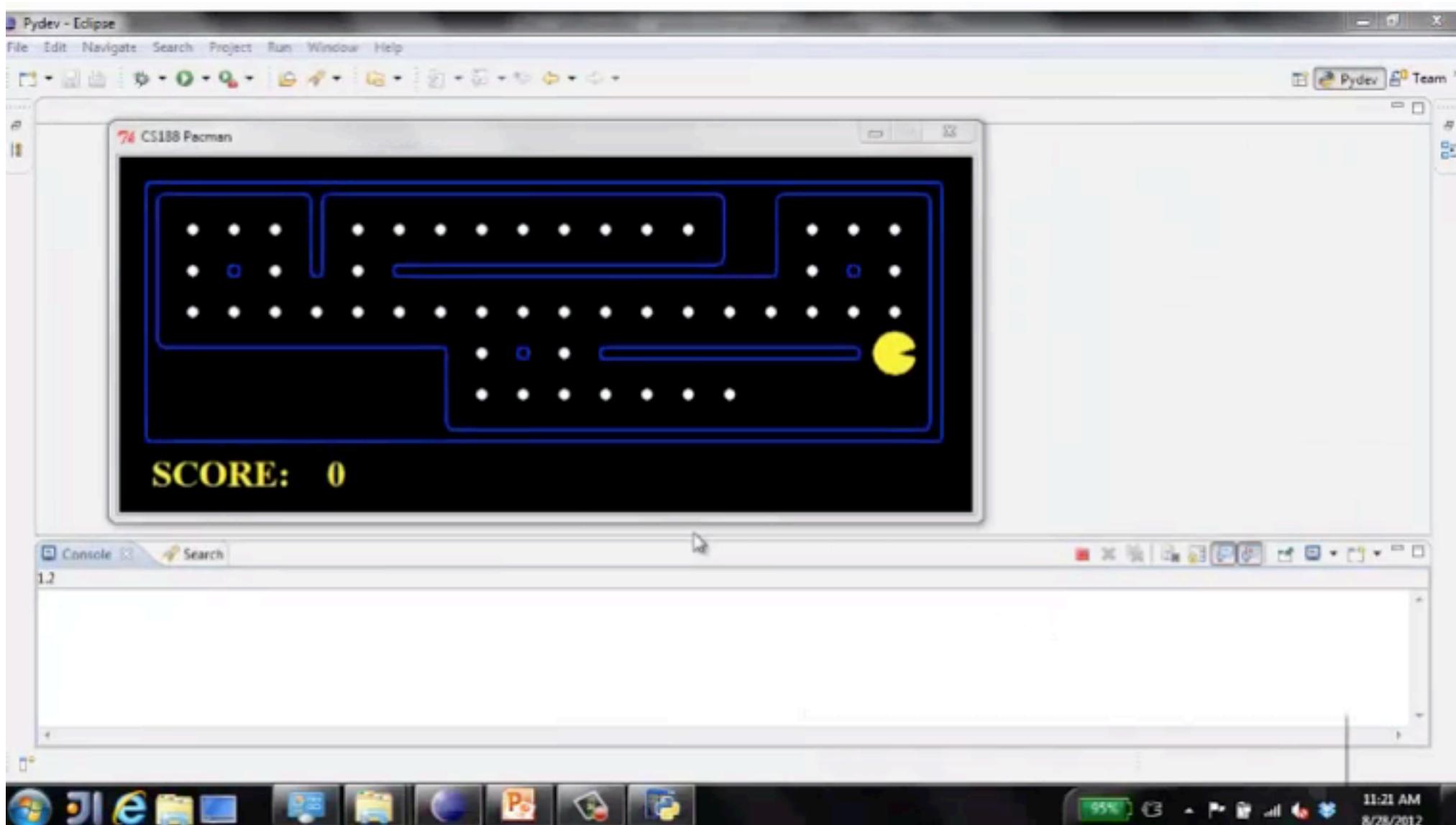


[Demo: reflex optimal (L2D1)]  
[Demo: reflex optimal (L2D2)]

# Video of Demo Reflex Optimal

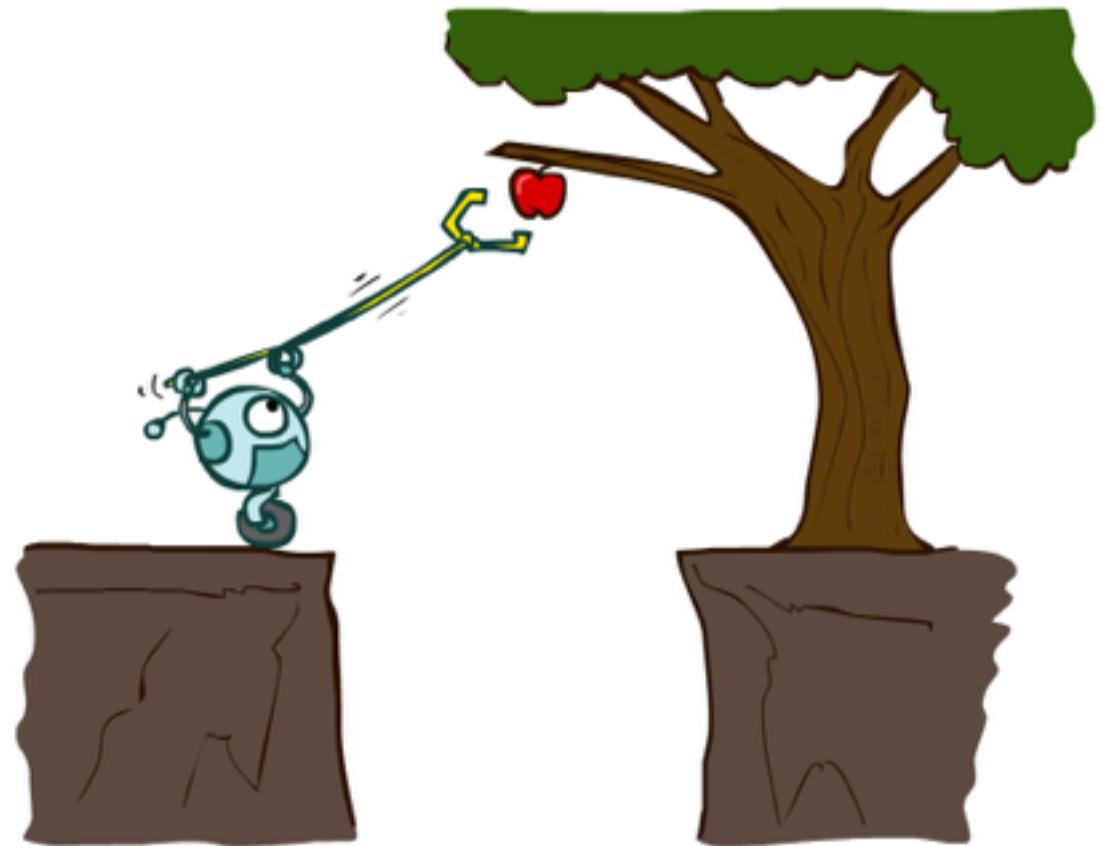


# Video of Demo Reflex Odd



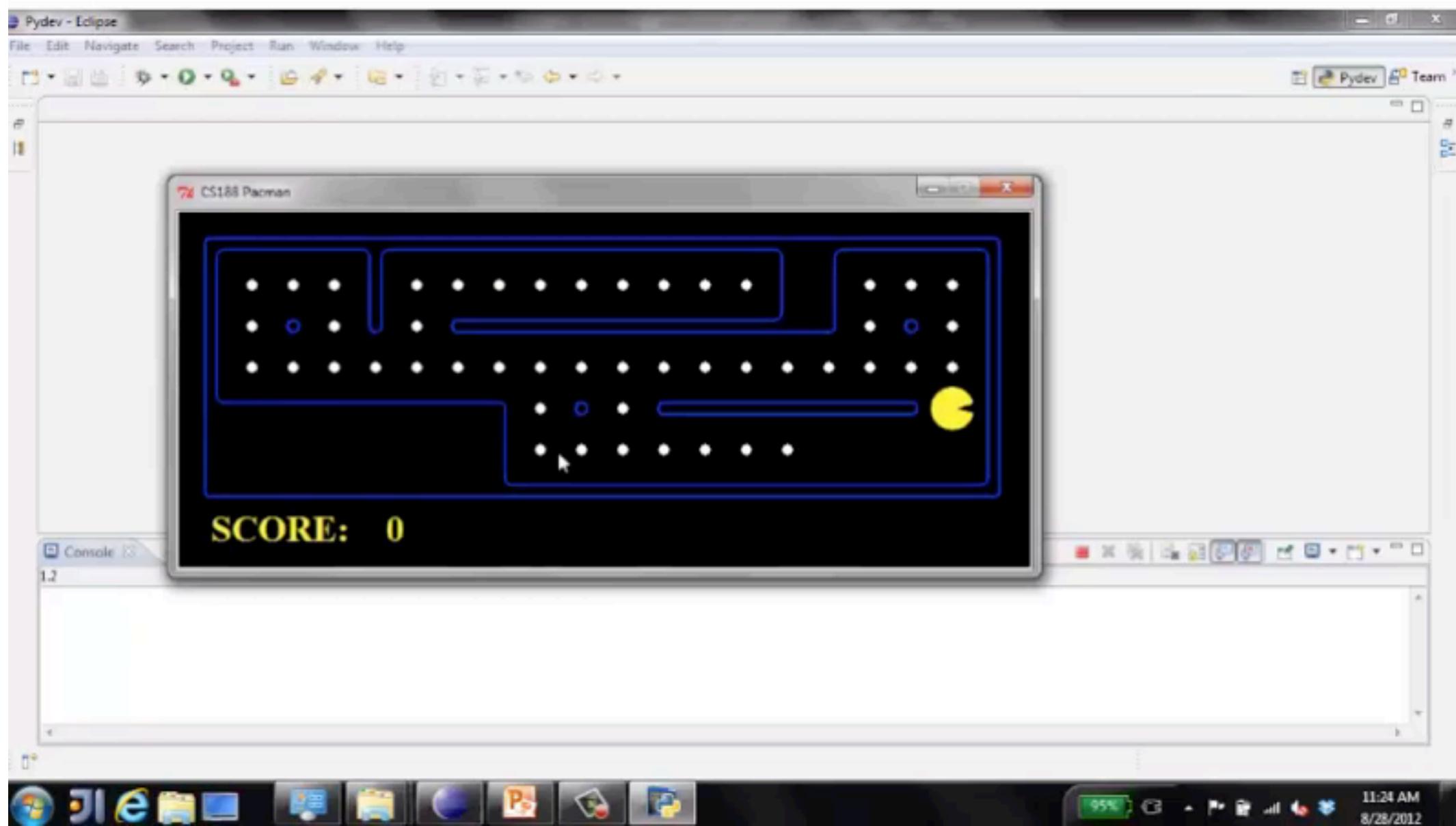
# Planning Agents

- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world **WOULD BE**

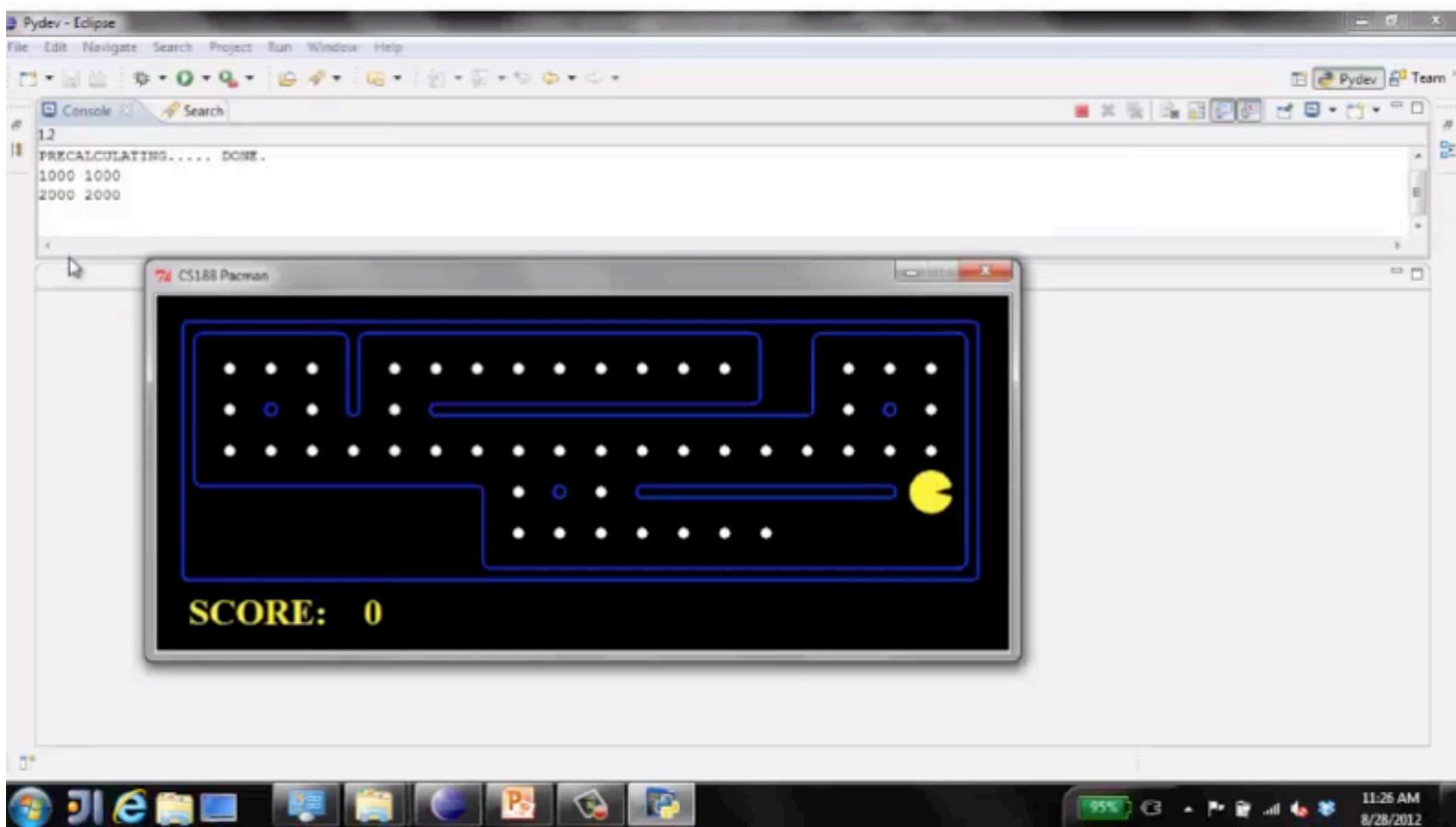


[Demo: replanning (L2D3)]  
[Demo: mastermind (L2D4)]

# Video of Demo Replanning



# Video of Demo Mastermind



# Search Problems

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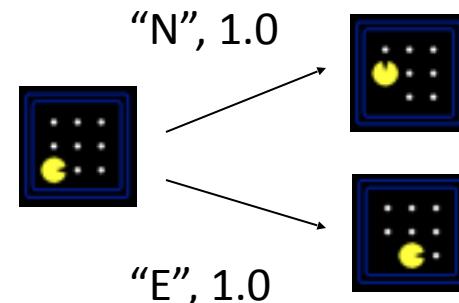
# Search Problems

- A search problem consists of:

- A state space



- A successor function  
(with actions, costs)

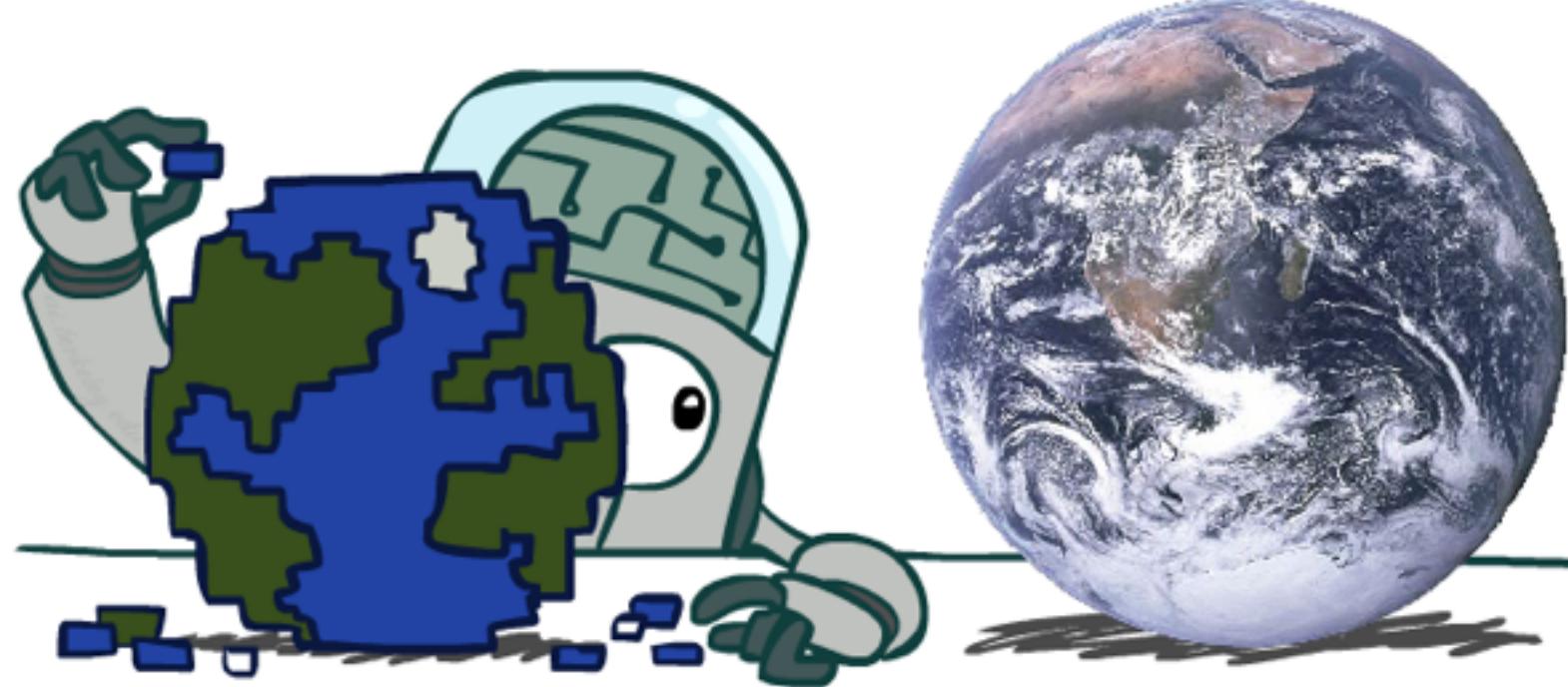


- A start state and a goal test

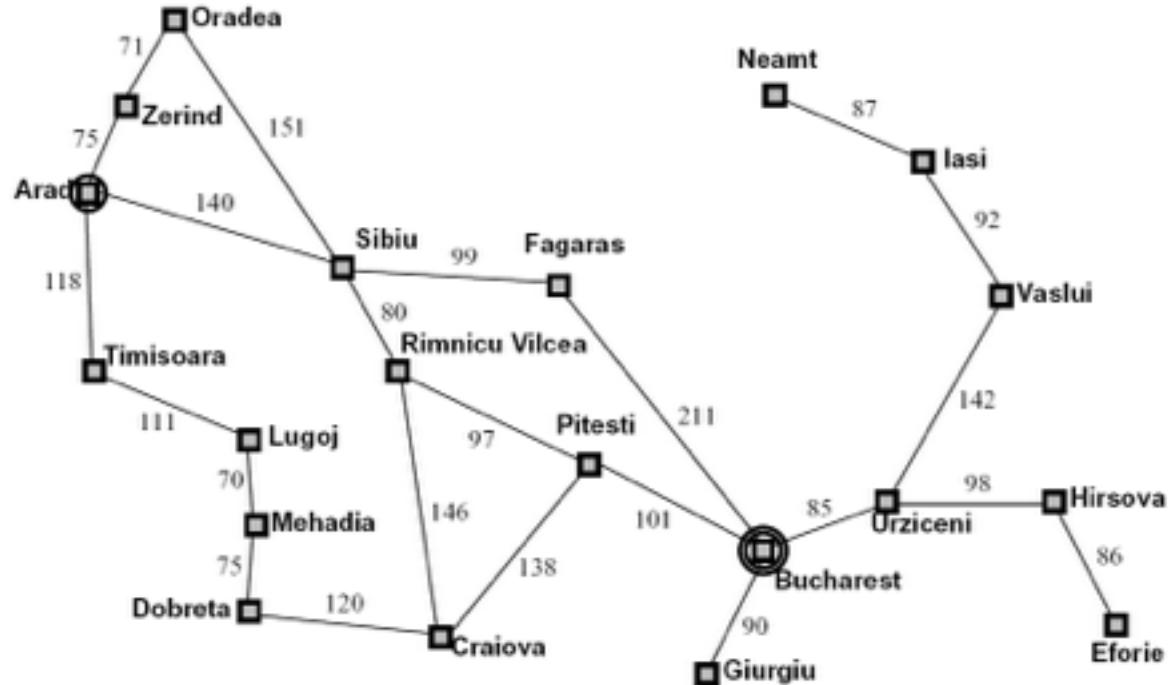
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

# Search Problems Are Models

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# Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

# What's in a State Space?

The **world state** includes every last detail of the environment



A **search state** keeps only the details needed for planning (abstraction)

- **Problem: Pathing**

- States:  $(x,y)$  location
- Actions: NSEW
- Successor: update location only
- Goal test: is  $(x,y)=\text{END}$

- **Problem: Eat-All-Dots**

- States:  $\{(x,y), \text{dot booleans}\}$
- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false

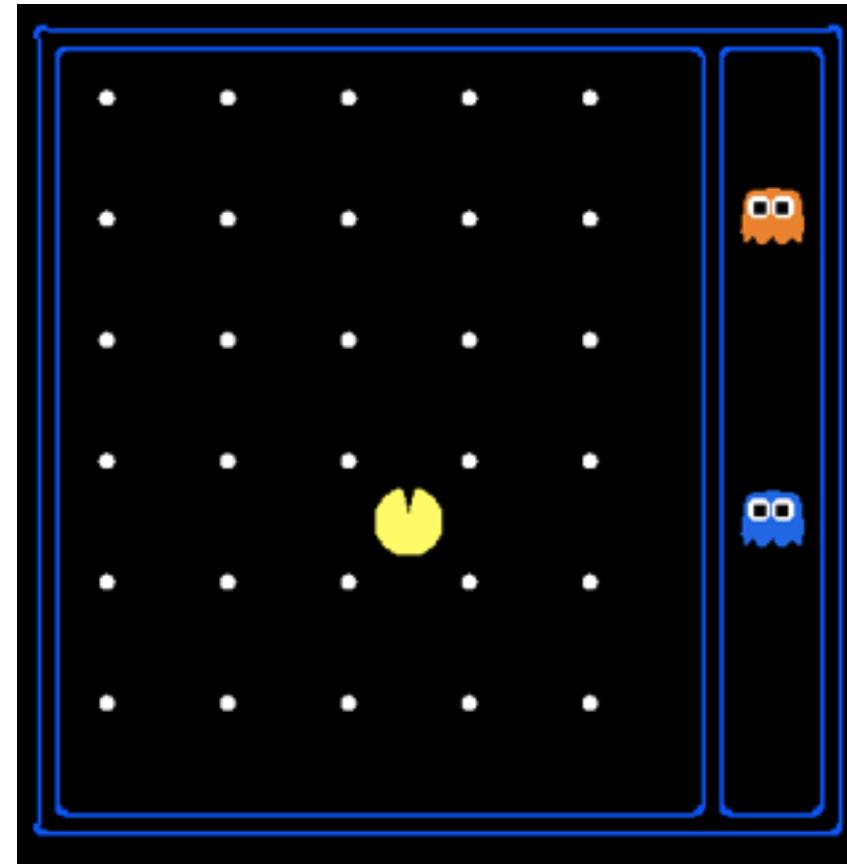
# State Space Sizes?

- World state:

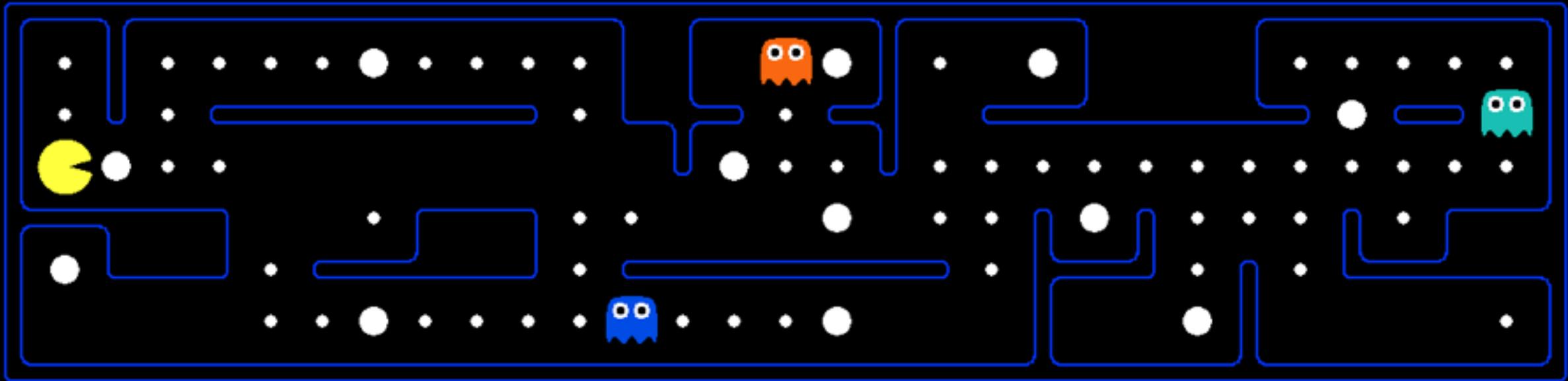
- Agent positions: 120
- Food count: 30
- Ghost positions: 12
- Agent facing: NSEW

- How many

- World states?  
 $120 \times (2^{30}) \times (12^2) \times 4$
- States for pathing?  
120
- States for eat-all-dots?  
 $120 \times (2^{30})$



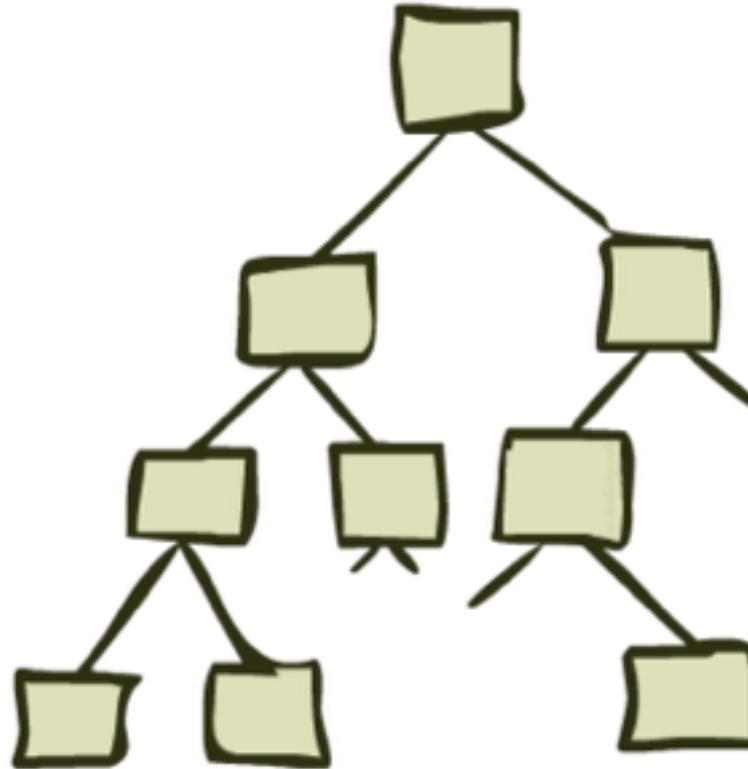
# Quiz: Safe Passage



- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

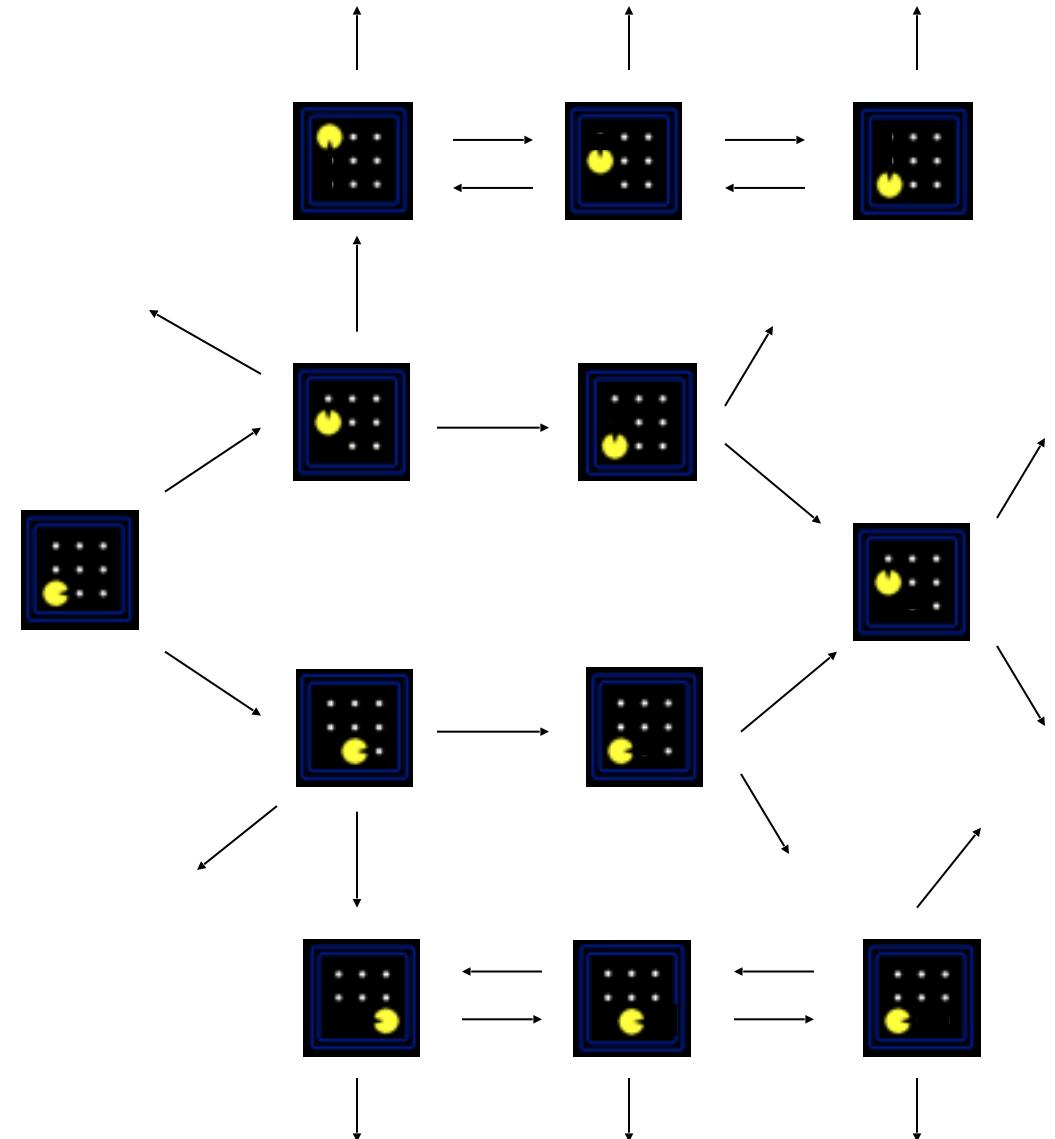
# State Space Graphs and Search Trees

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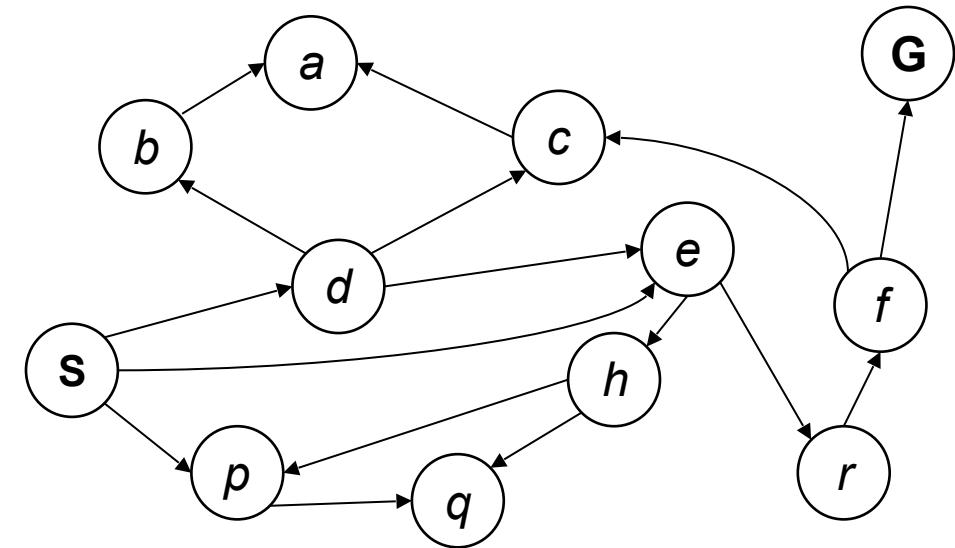
# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



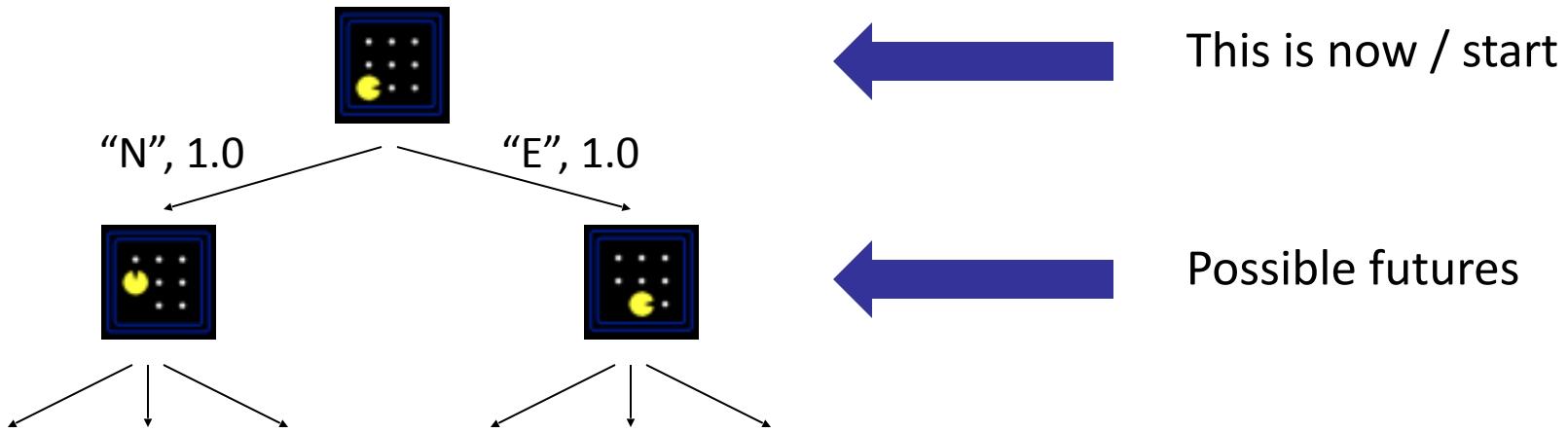
# State Space Graphs

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*Tiny search graph for a tiny search problem*

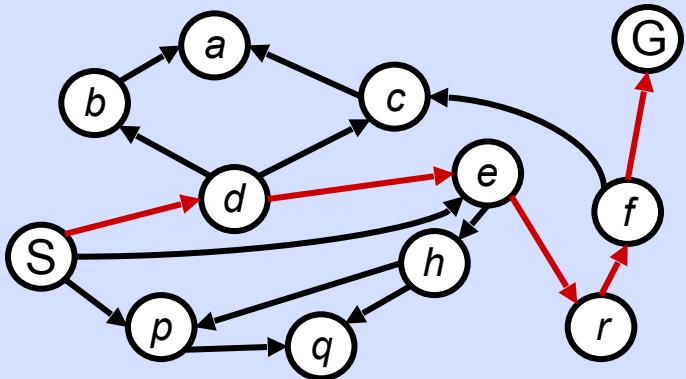
# Search Trees



- A search tree:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

# State Space Graphs vs. Search Trees

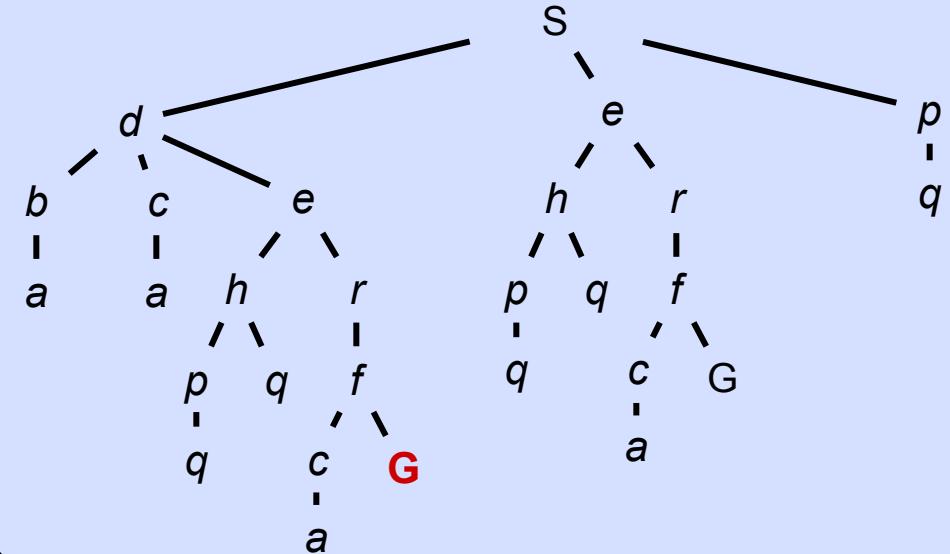
State Space Graph



Each NODE in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.

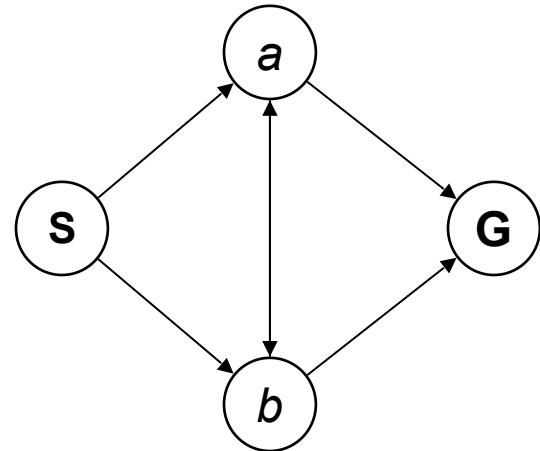
Search Tree



# Quiz: State Space Graphs vs. Search Trees

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Consider this 4-state graph:



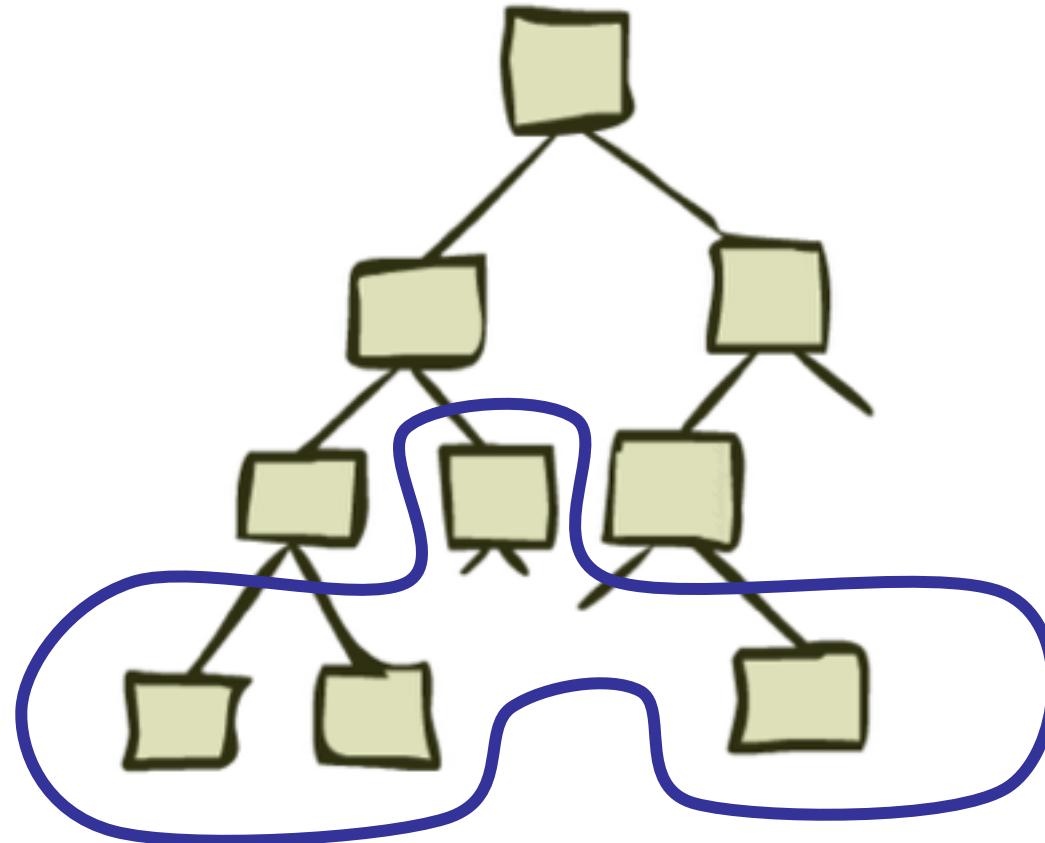
How big is its search tree (from S)?



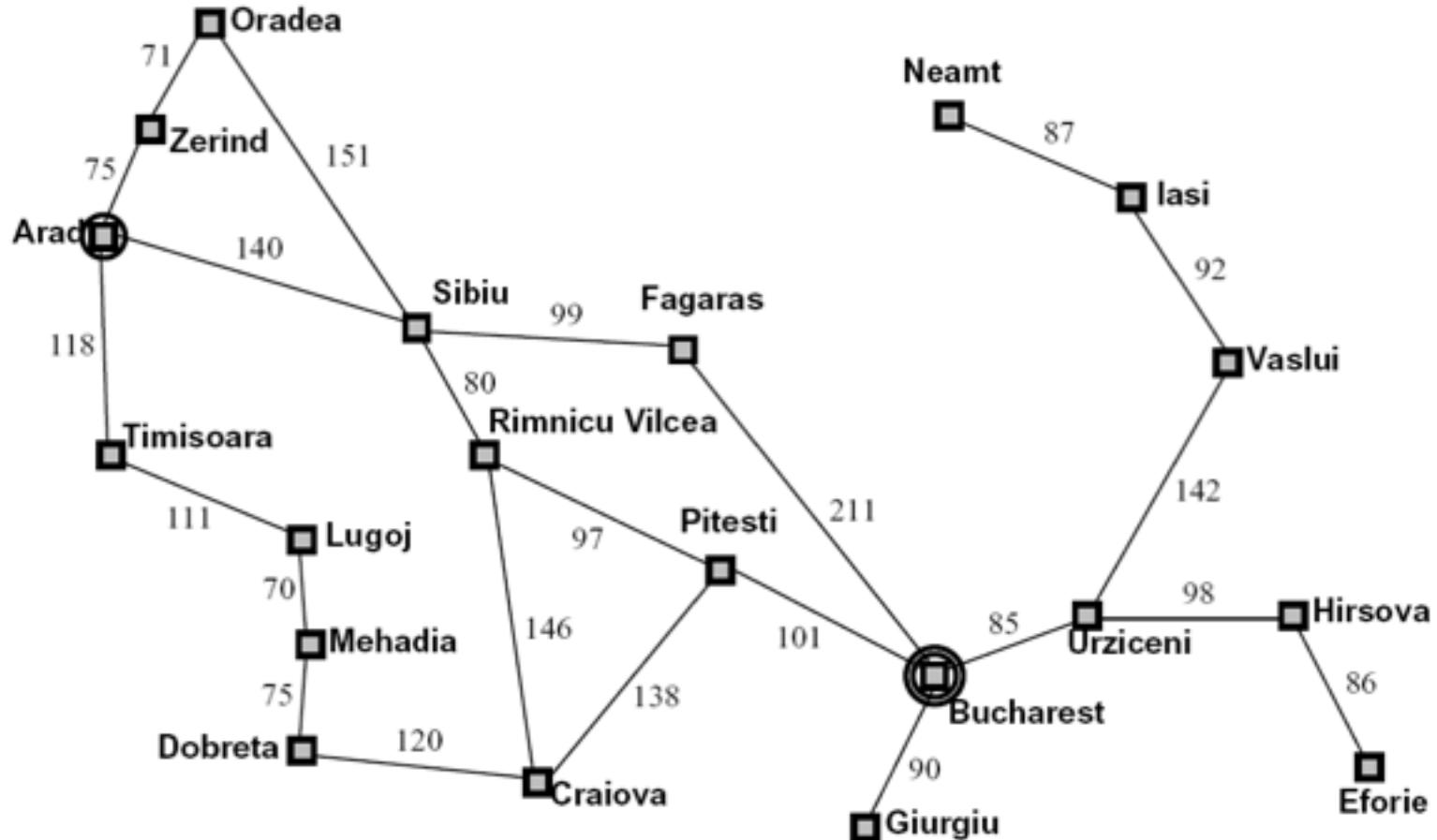
Important: Lots of repeated structure in the search tree!

# Tree Search

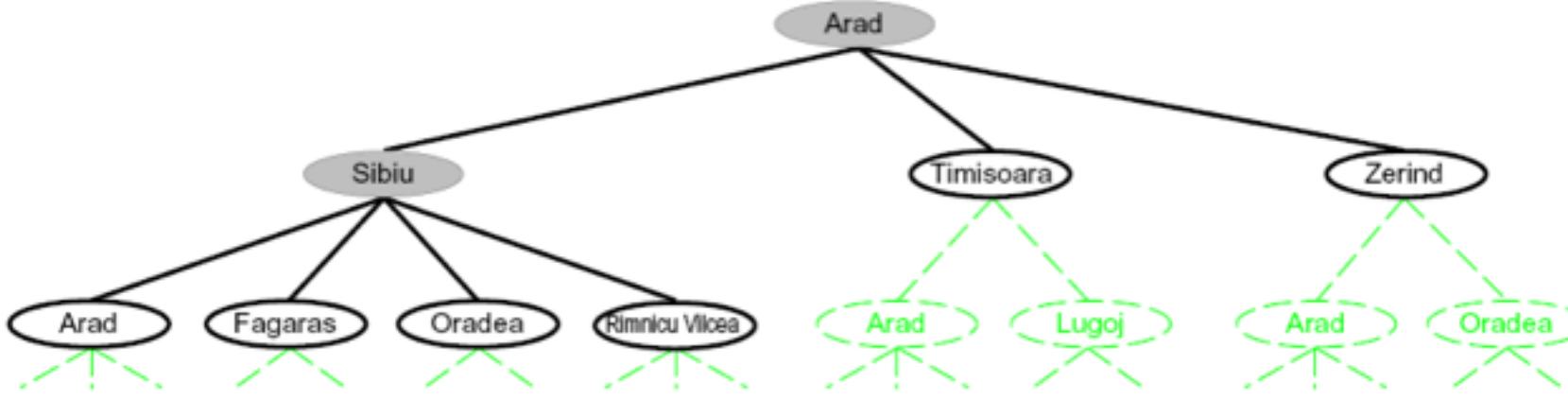
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# Search Example: Romania



# Searching with a Search Tree



- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a **fringe** of partial plans under consideration
  - Try to expand as few tree nodes as possible

# General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Depth-First Search

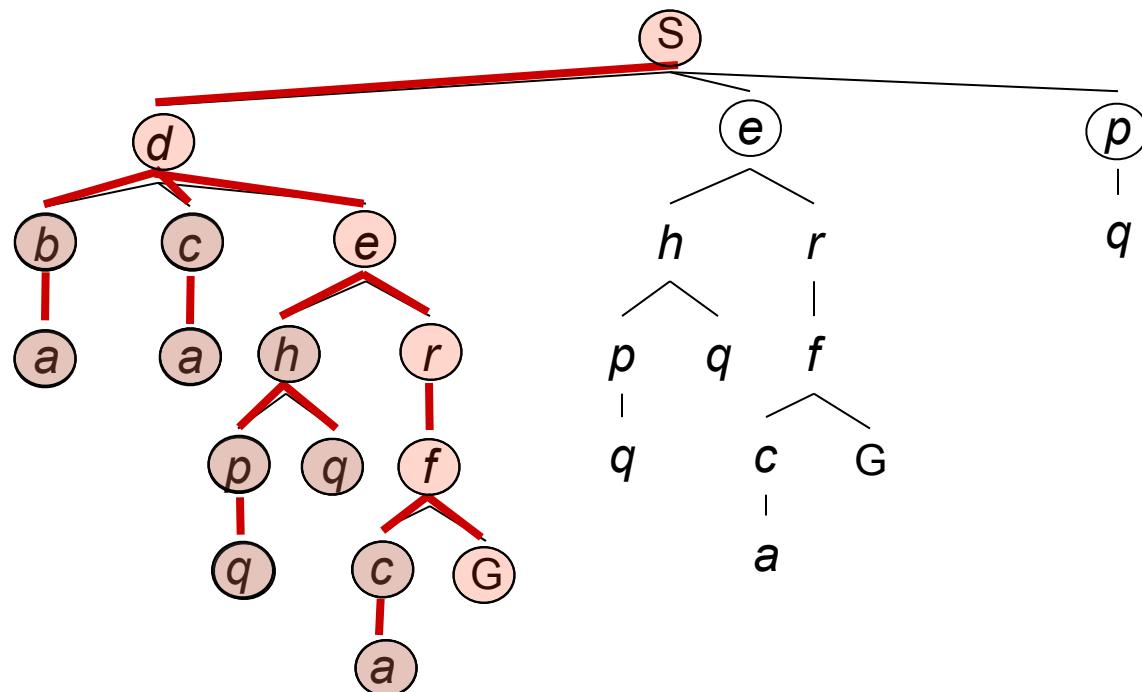
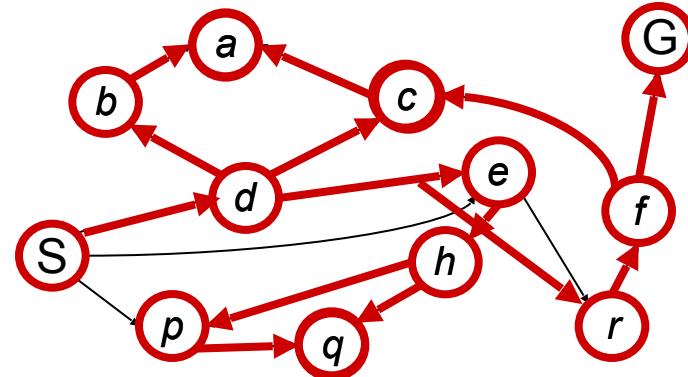
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# Depth-First Search

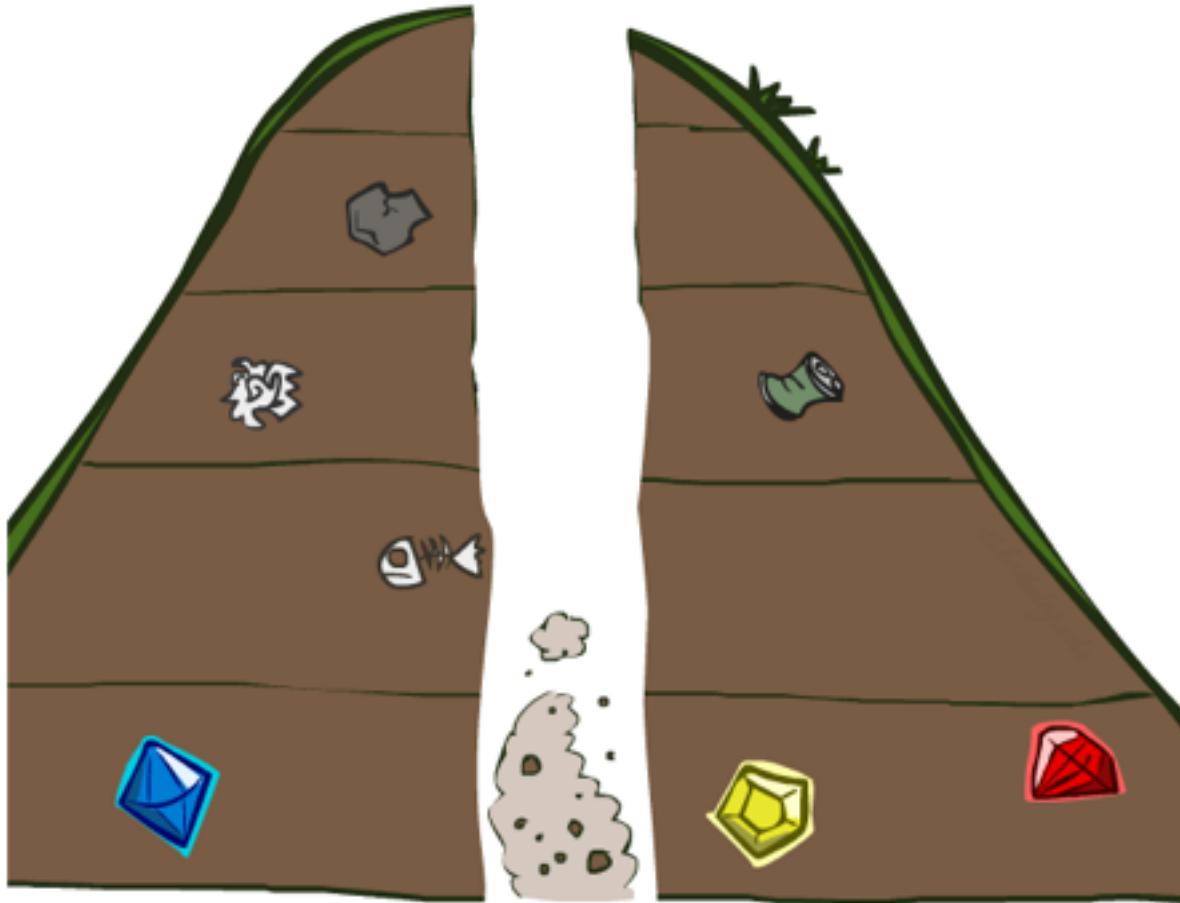
Strategy: expand a deepest node first

Implementation: Fringe is a LIFO stack



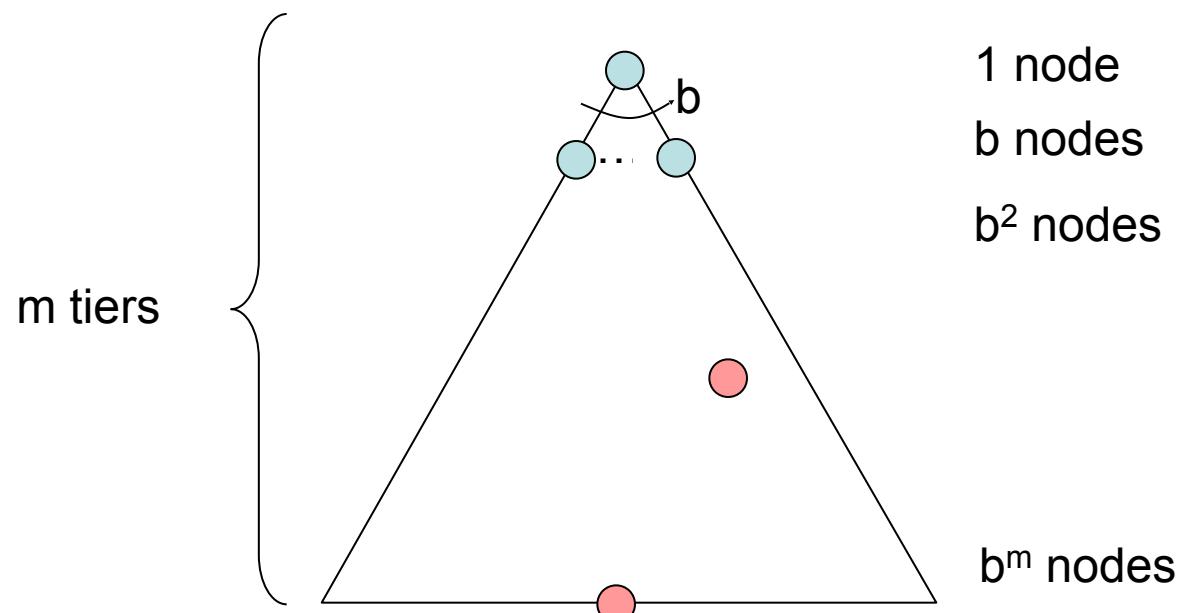
# Search Algorithm Properties

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# Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - $b$  is the branching factor
  - $m$  is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + \dots + b^m = O(b^m)$



# Depth-First Search (DFS) Properties

- What nodes DFS expand?

- Some left prefix of the tree.
- Could process the whole tree!
- If  $m$  is finite, takes time  $O(b^m)$

- How much space does the fringe take?

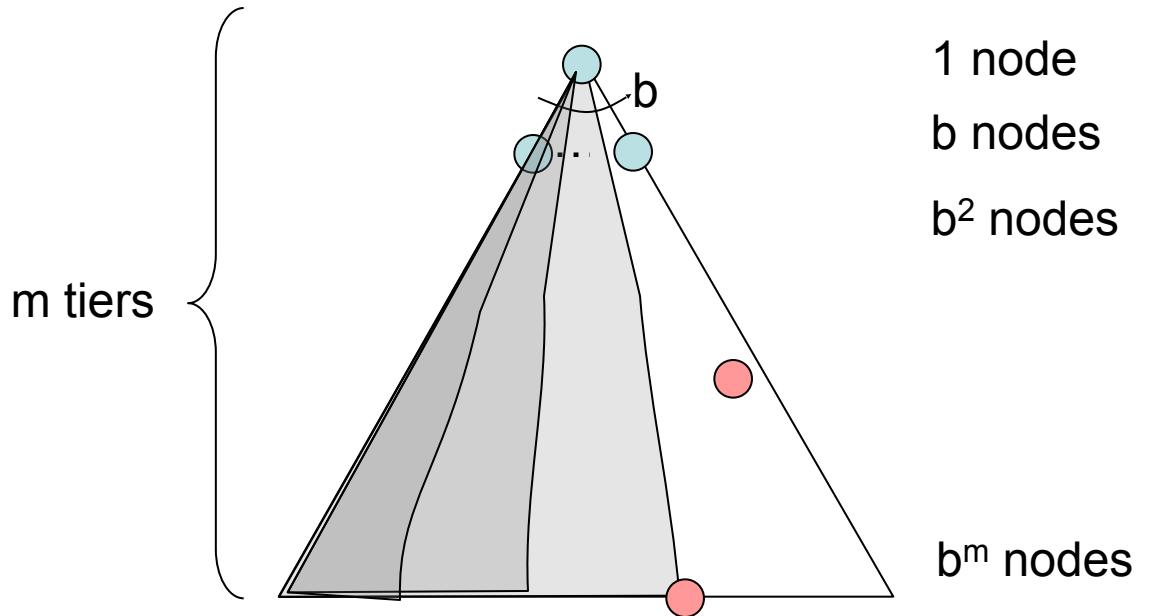
- Only has siblings on path to root, so  $O(bm)$

- Is it complete?

- $m$  could be infinite, so only if we prevent cycles  
(more later)

- Is it optimal?

- No, it finds the “leftmost” solution, regardless of depth or cost



# Breadth-First Search

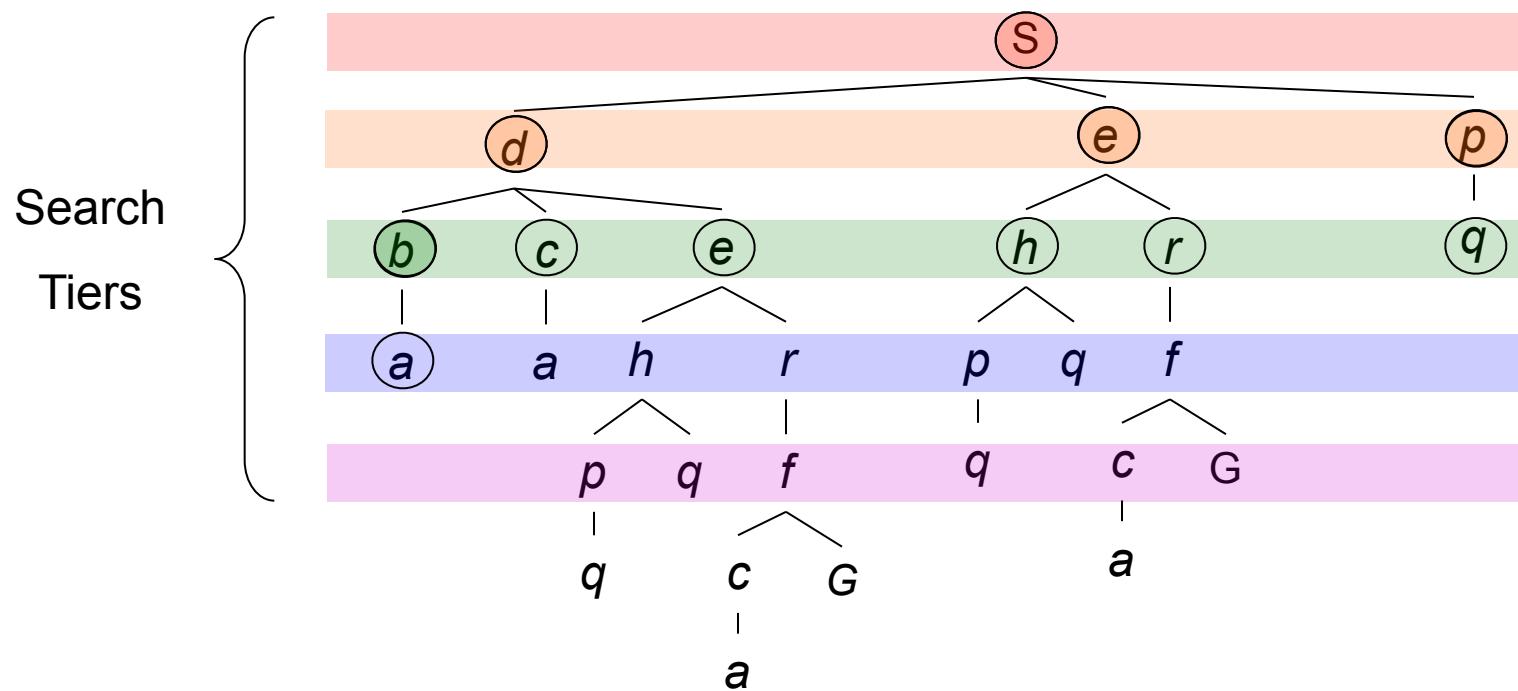
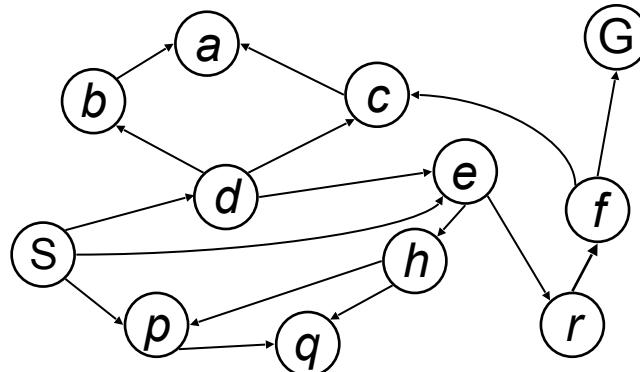
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# Breadth-First Search

*Strategy: expand a shallowest node first*

*Implementation: Fringe is a FIFO queue*



# Breadth-First Search (BFS) Properties

- What nodes does BFS expand?

- Processes all nodes above shallowest solution
- Let depth of shallowest solution be  $s$
- Search takes time  $O(b^s)$

- How much space does the fringe take?

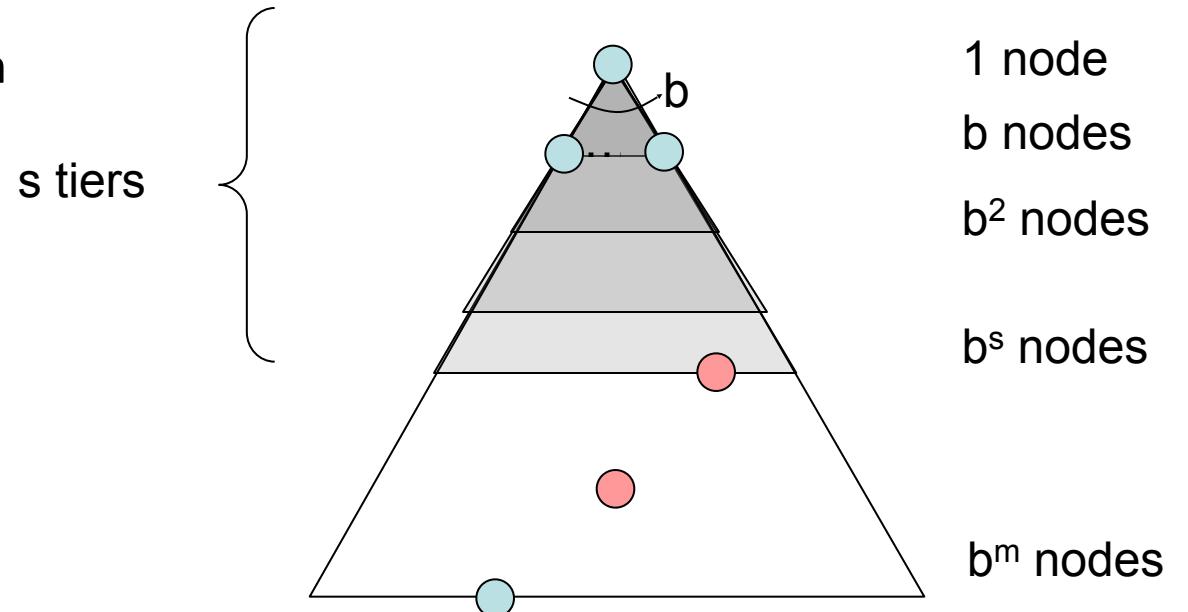
- Has roughly the last tier, so  $O(b^s)$

- Is it complete?

- $s$  must be finite if a solution exists, so yes!

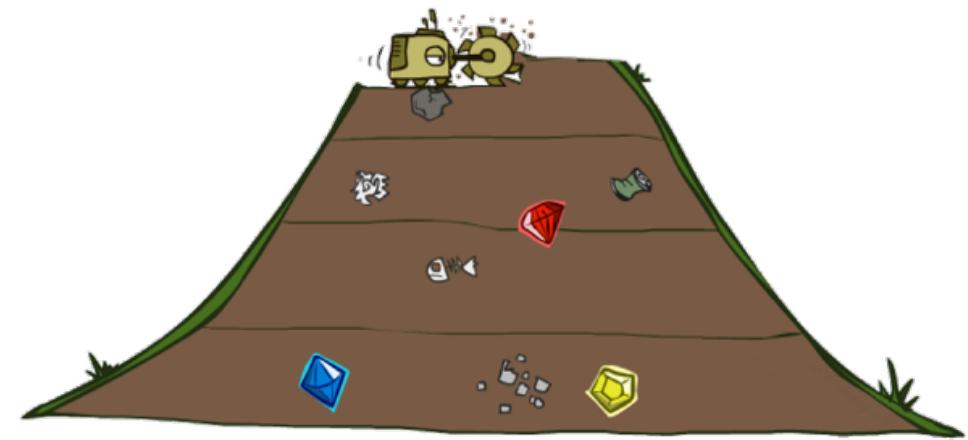
- Is it optimal?

- Only if costs are all 1 (more on costs later)



# Quiz: DFS vs BFS

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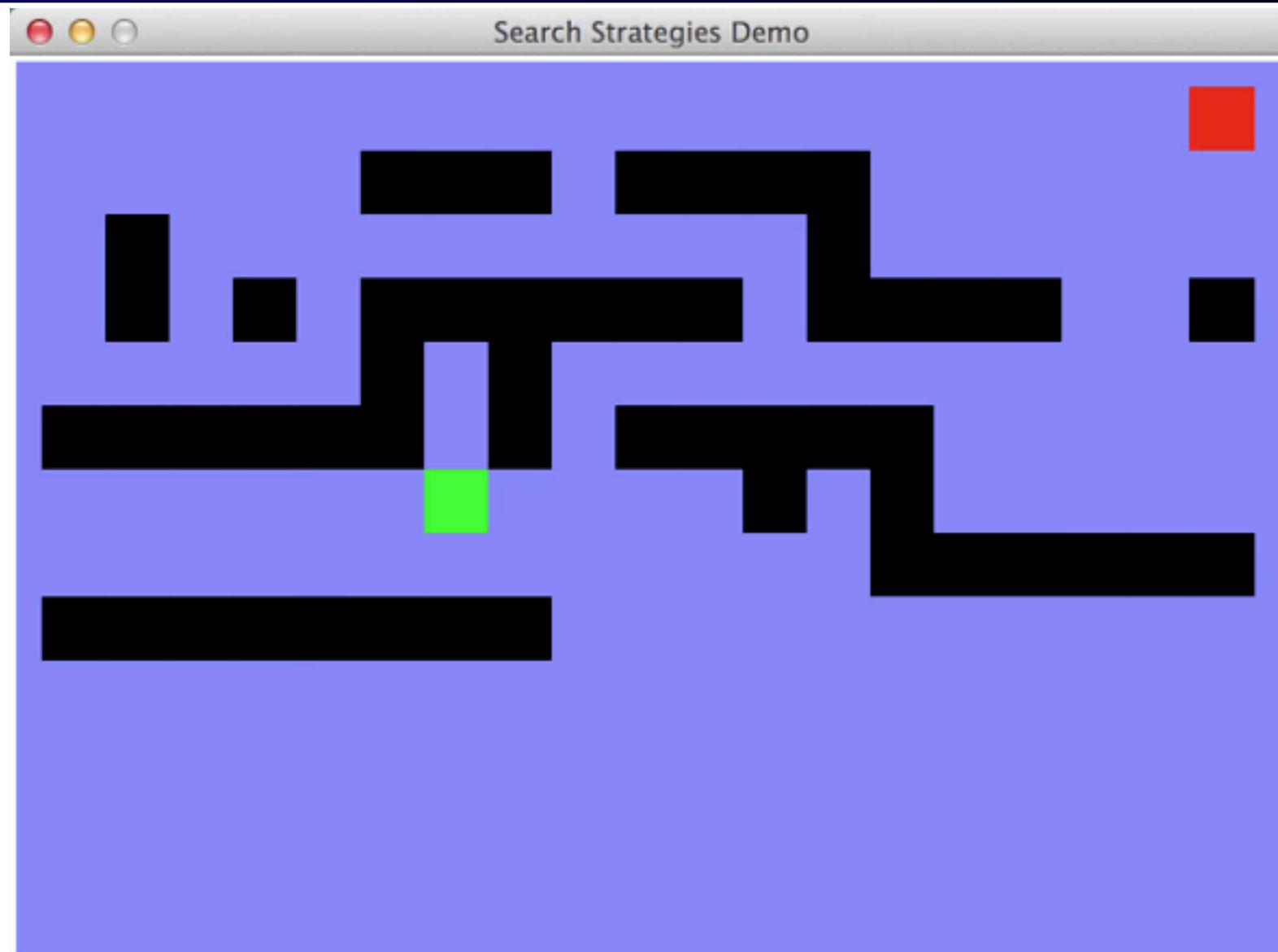


# Quiz: DFS vs BFS

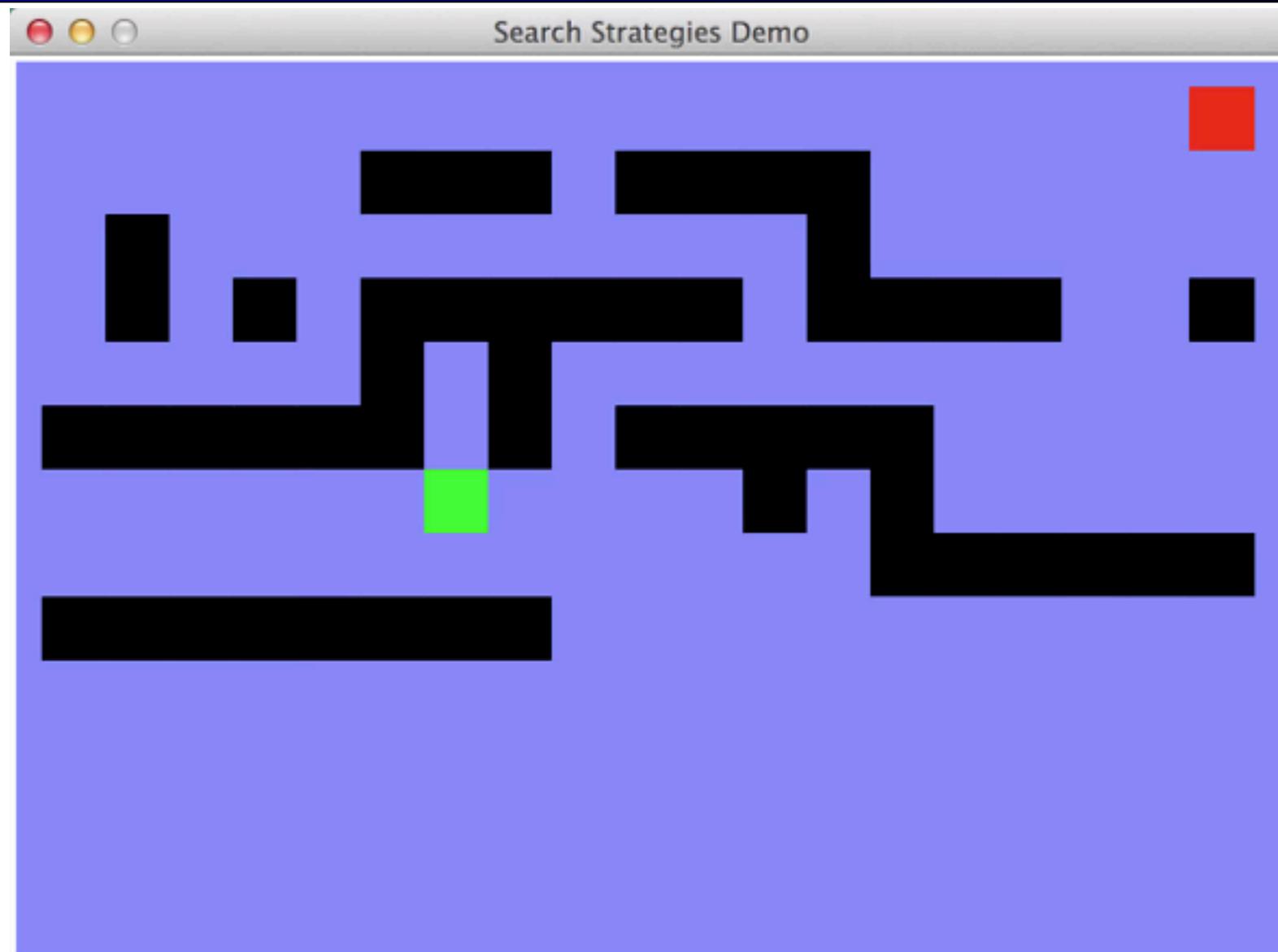
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- When will BFS outperform DFS?
- When will DFS outperform BFS?

# Video of Demo Maze Water DFS/BFS (part 1)

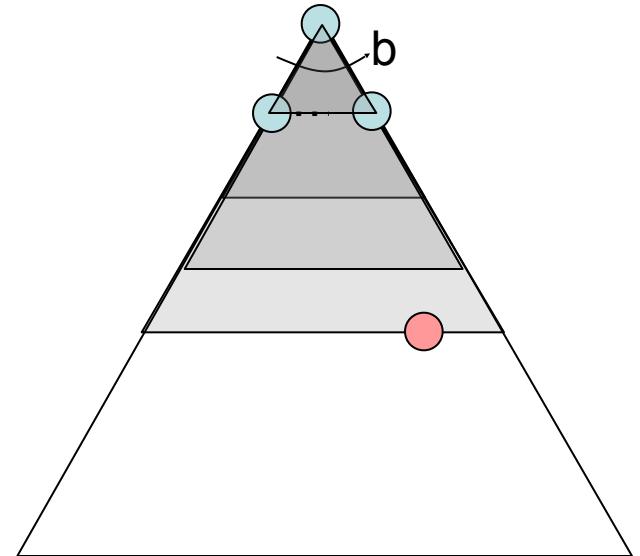


# Video of Demo Maze Water DFS/BFS (part 2)

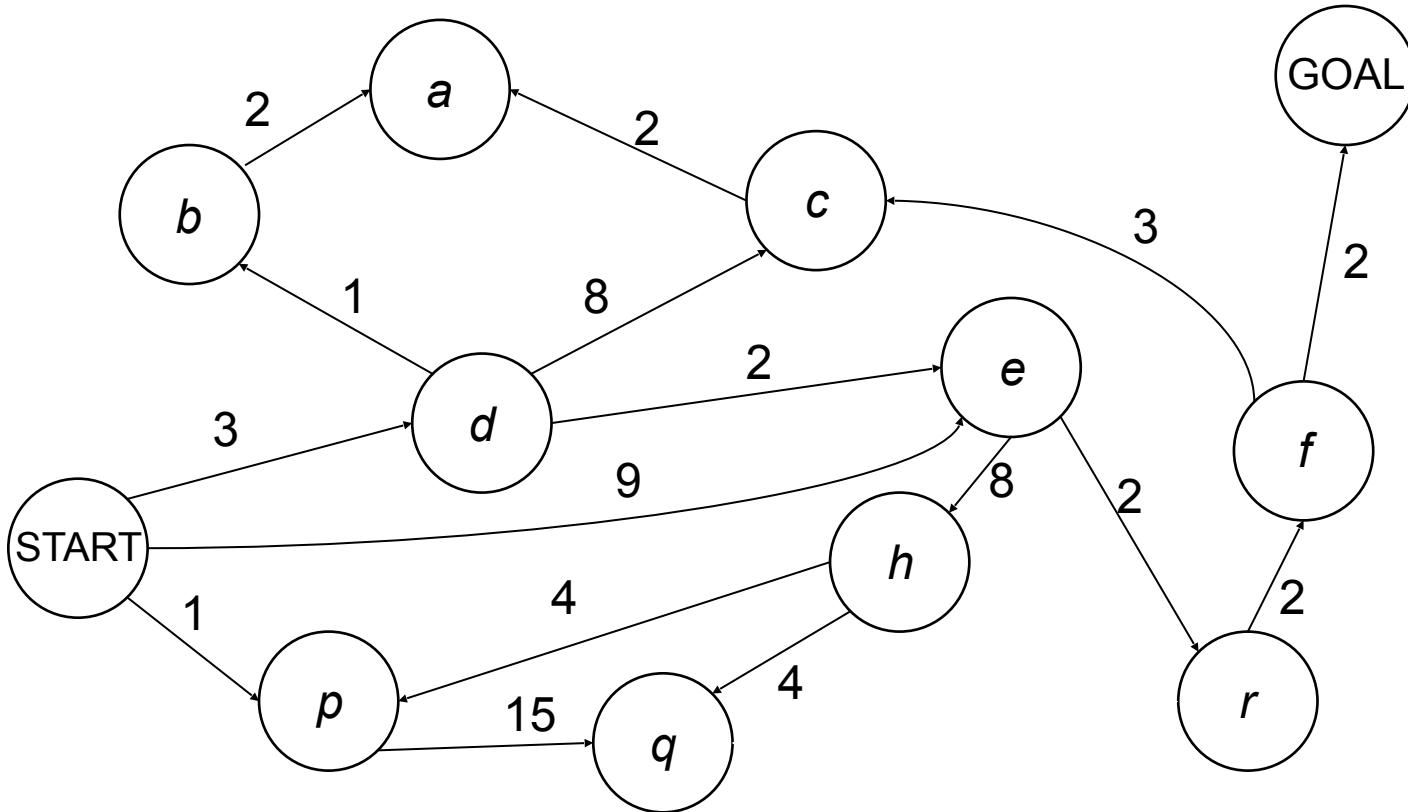


# Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. .....
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!



# Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.  
It does not find the least-cost path. We will now cover  
a similar algorithm which does find the least-cost path.

# Uniform Cost Search

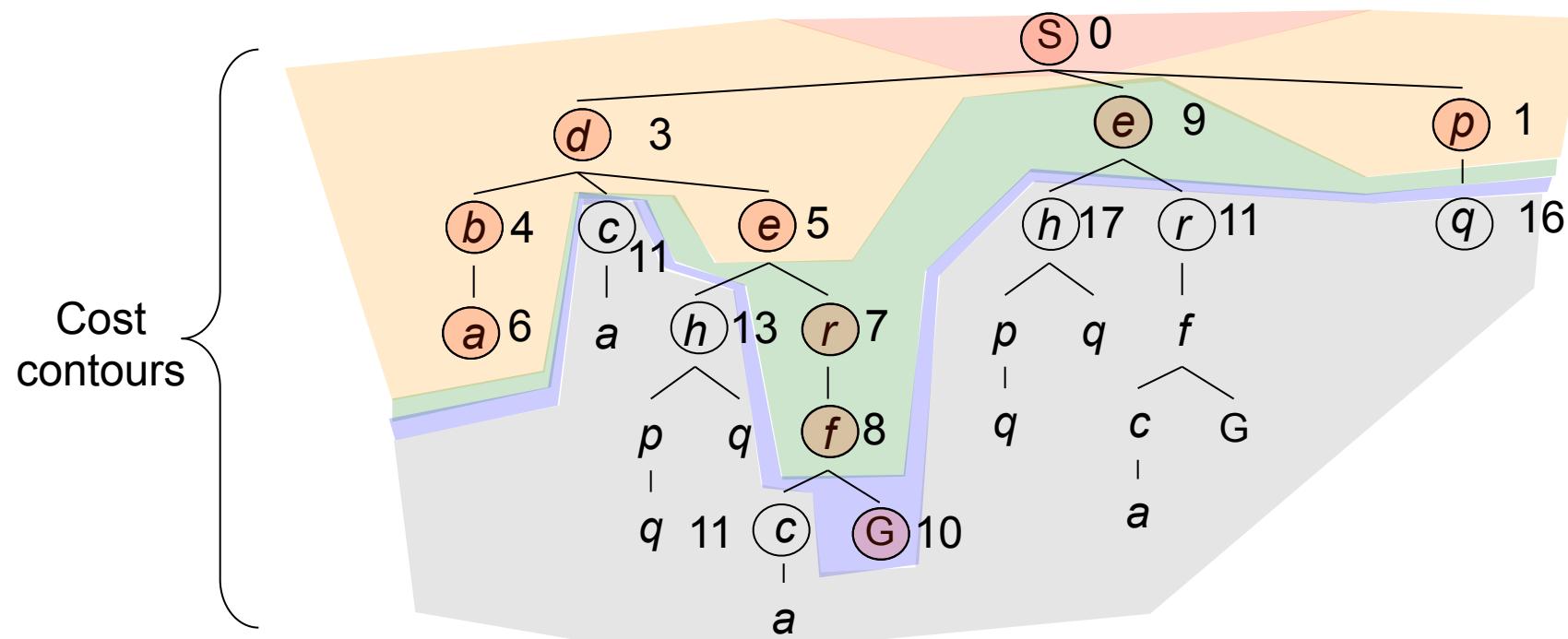
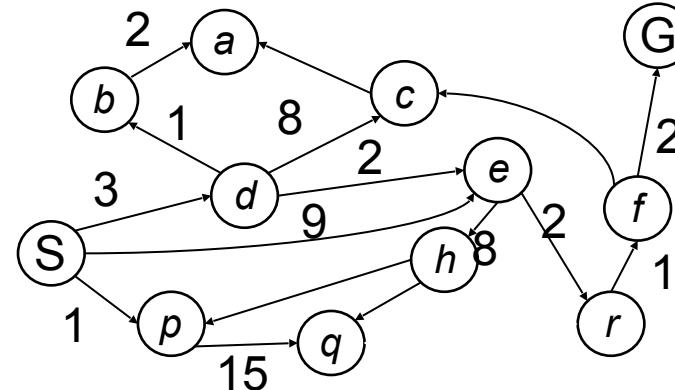
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# Uniform Cost Search

Strategy: expand a cheapest node first:

Fringe is a priority queue  
(priority: cumulative cost)



# Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?

- Processes all nodes with cost less than cheapest solution!
- If that solution costs  $C^*$  and arcs cost at least  $\epsilon$ , then the “effective depth” is roughly  $C^*/\epsilon$
- Takes time  $O(b^{C^*/\epsilon})$  (exponential in effective depth)

- How much space does the fringe take?

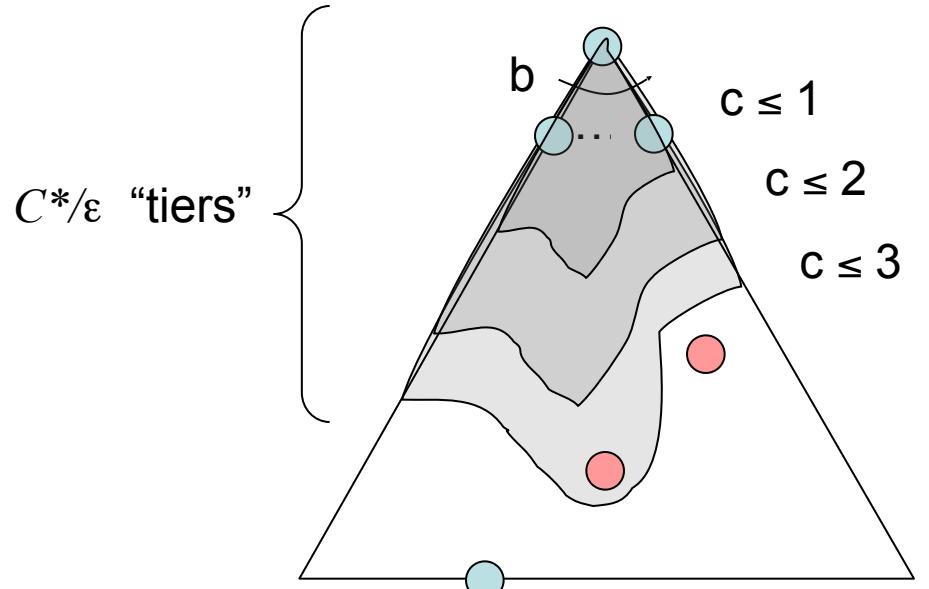
- Has roughly the last tier, so  $O(b^{C^*/\epsilon})$

- Is it complete?

- Assuming best solution has a finite cost and minimum arc cost is positive, yes!

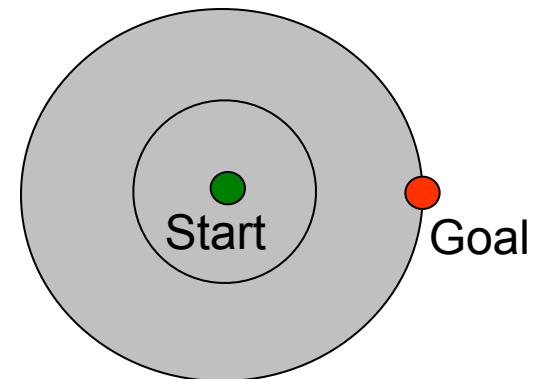
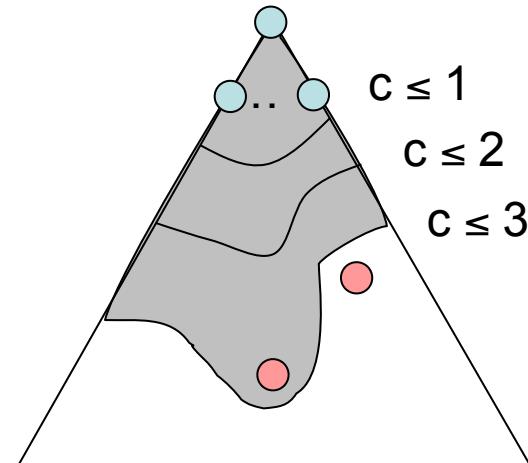
- Is it optimal?

- Yes!



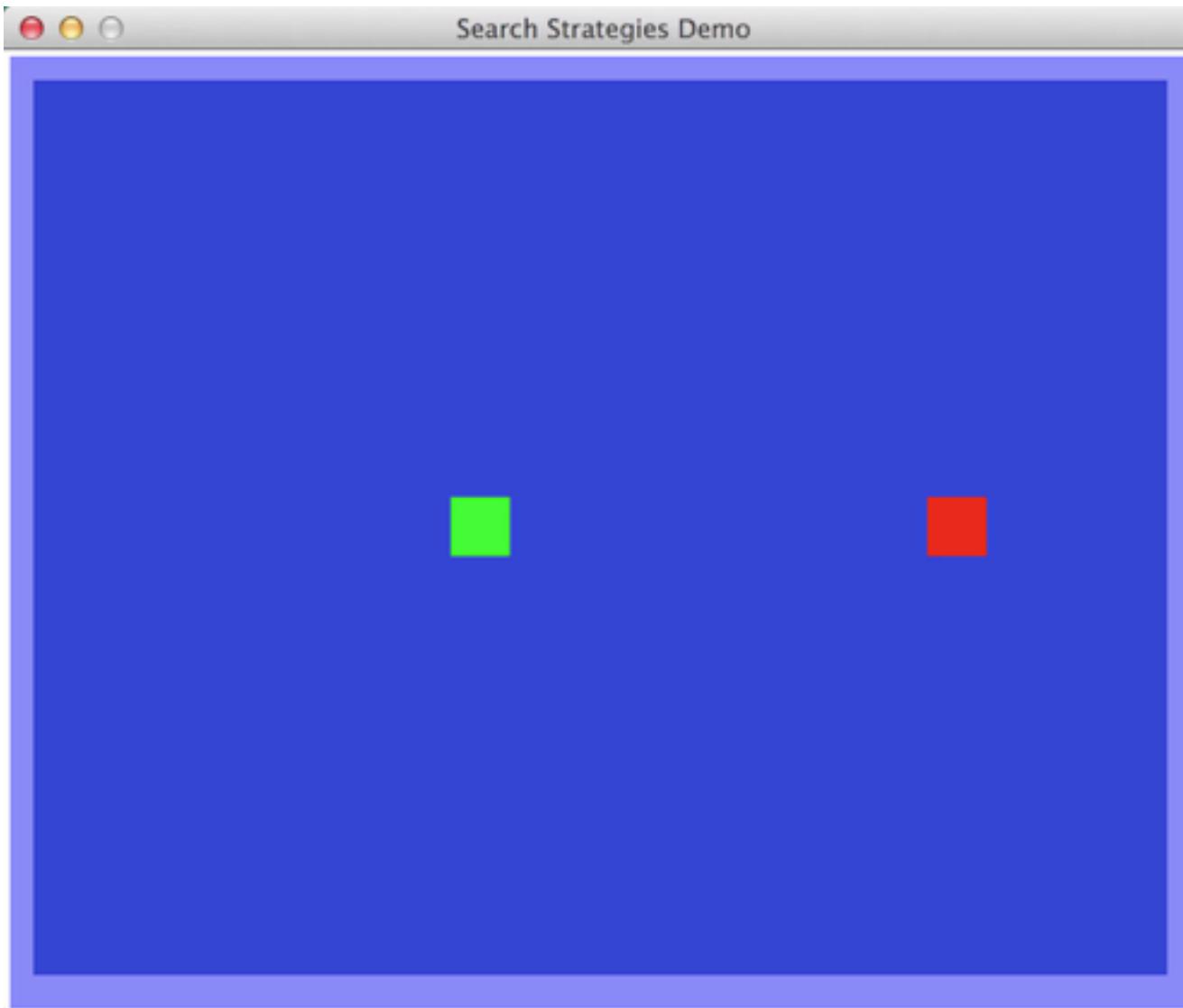
# Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location
- We’ll fix that soon!

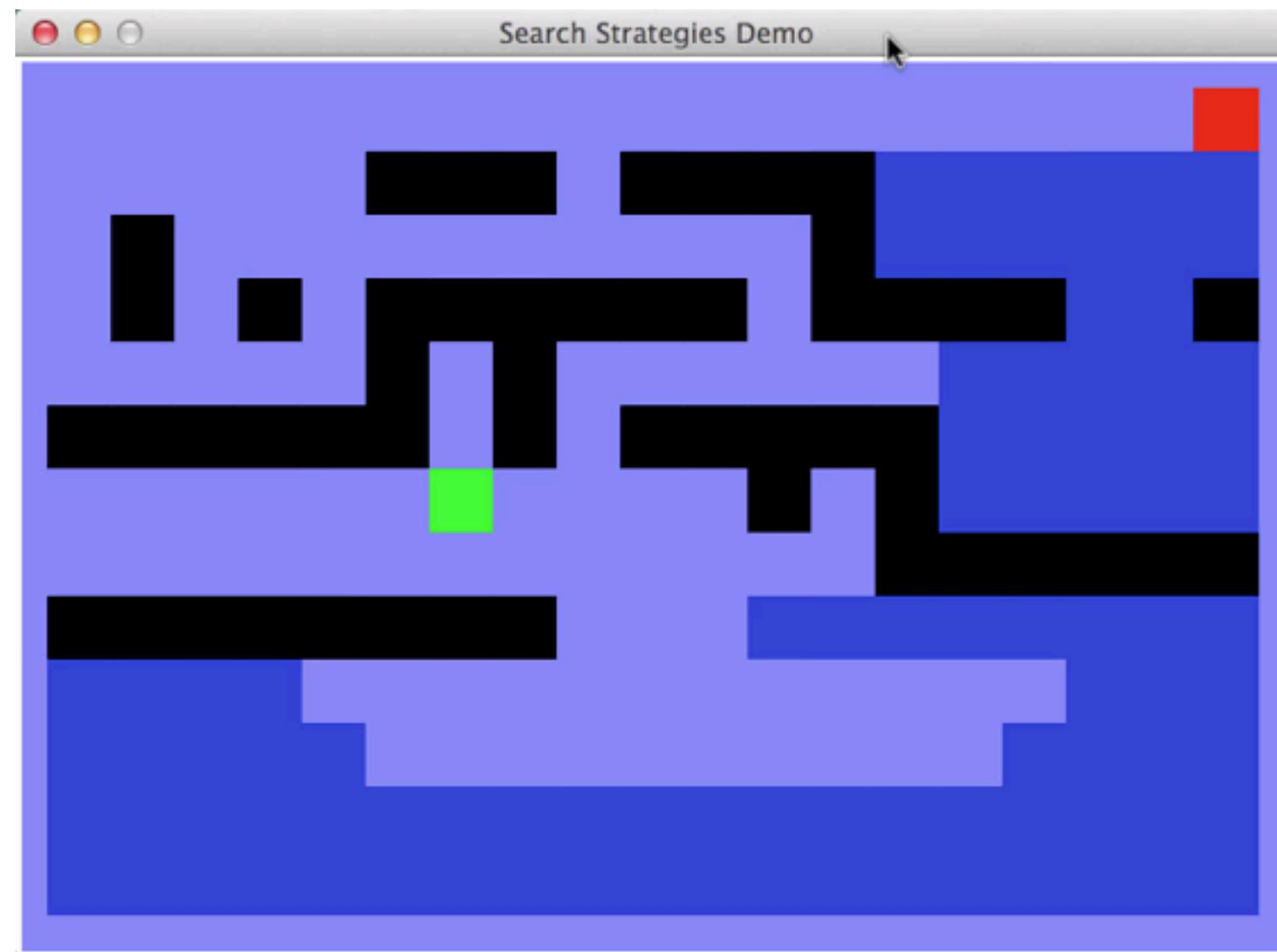


[Demo: empty grid UCS (L2D5)]  
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

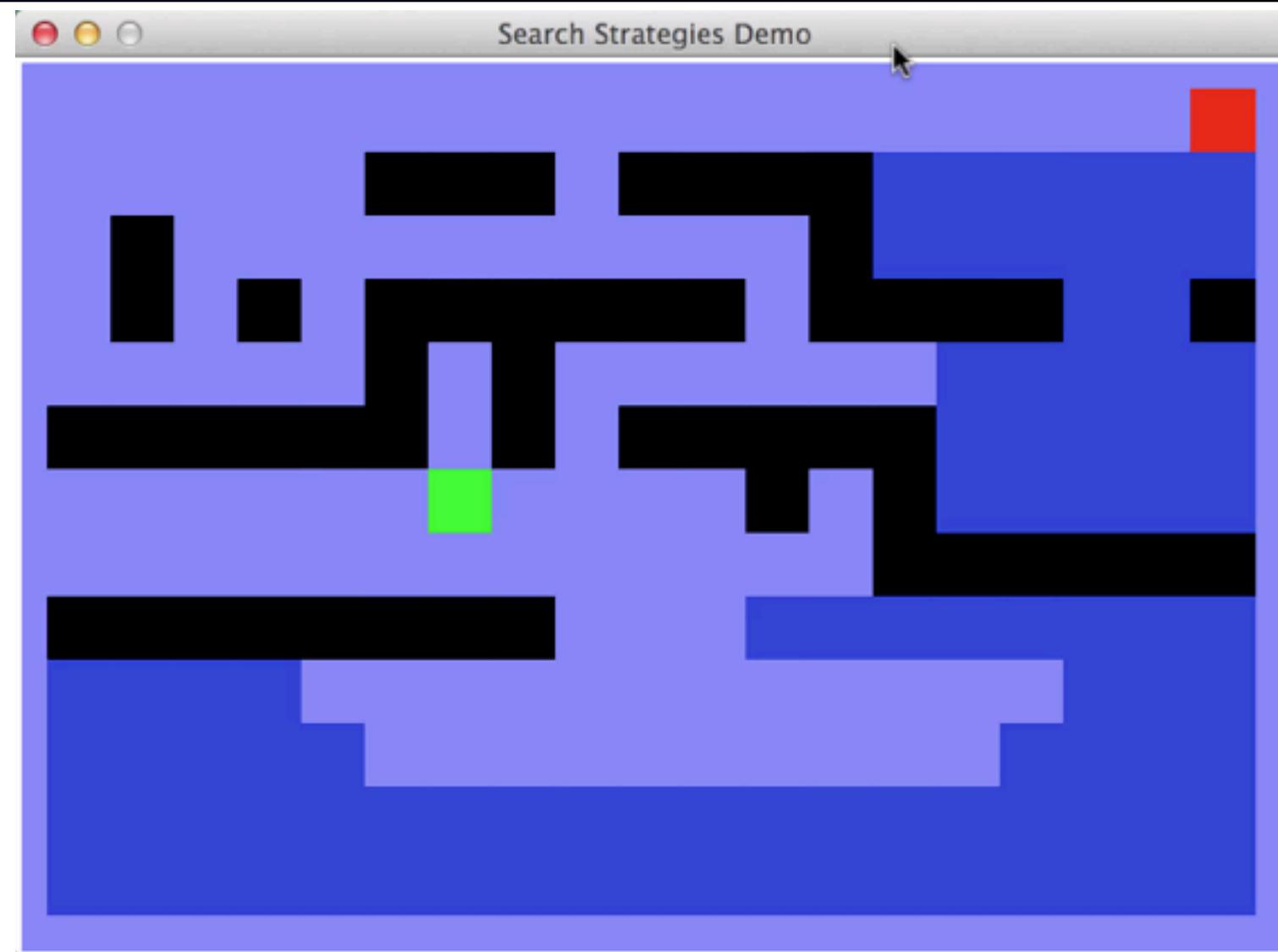
# Video of Demo Empty UCS



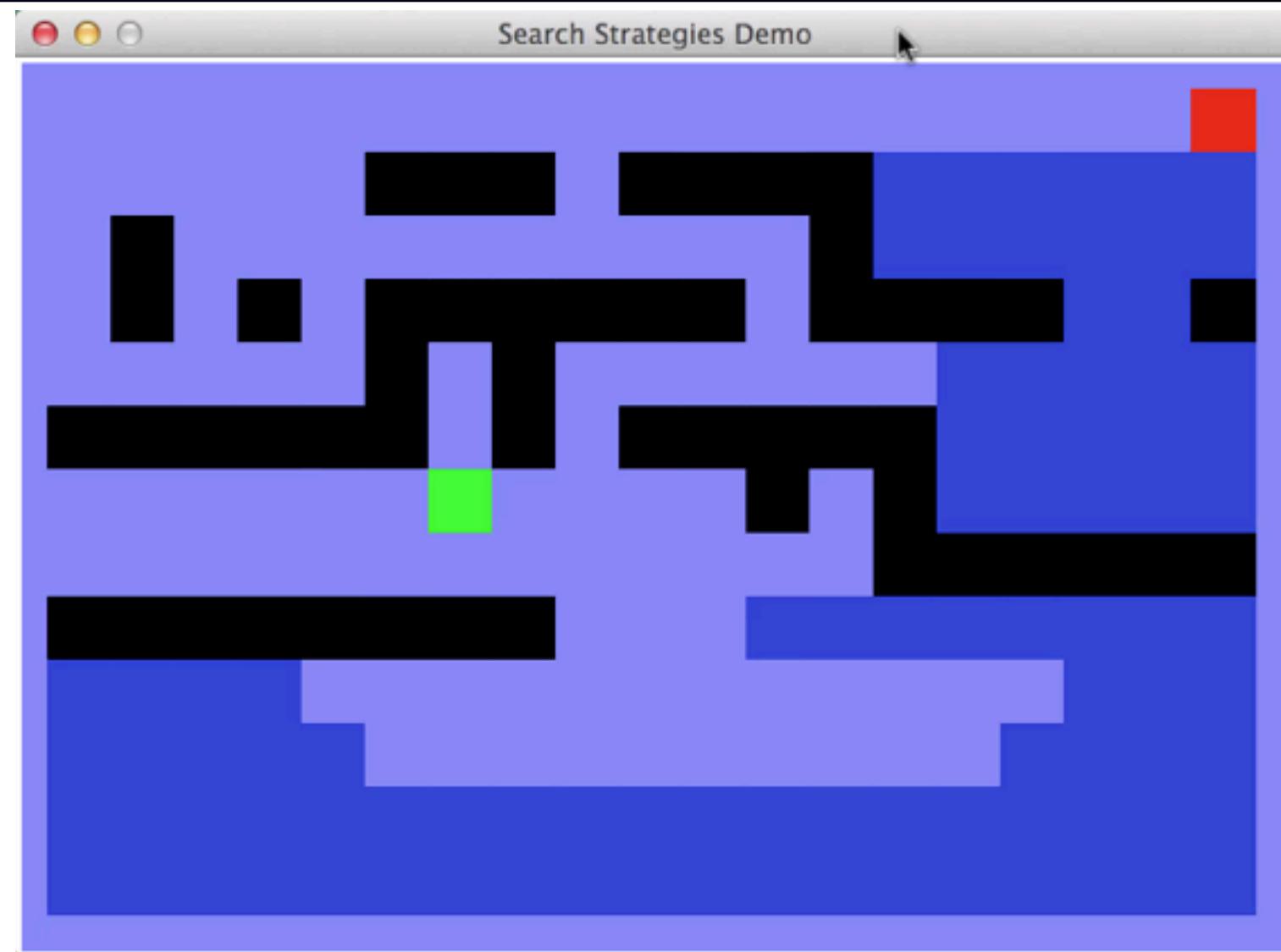
# Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)



## Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)

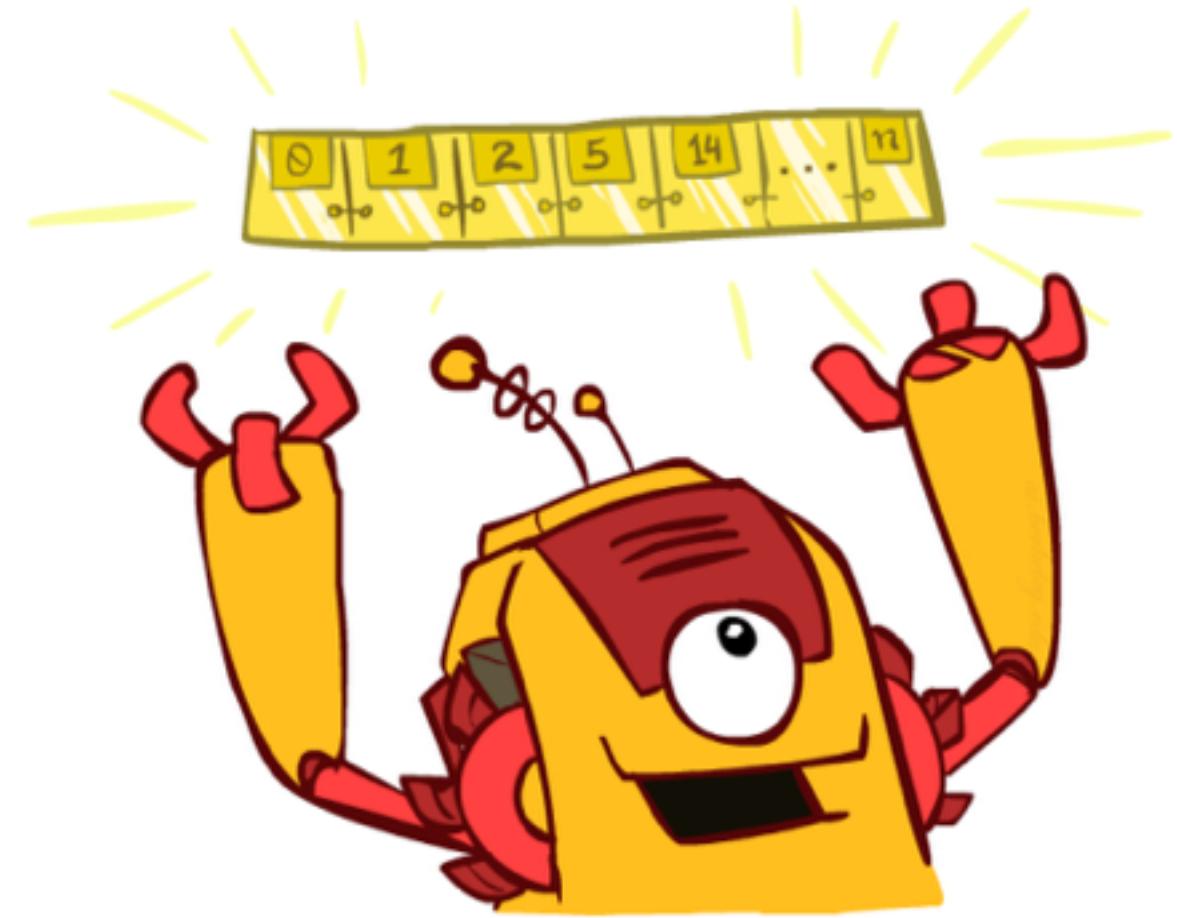


# Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)



# The One Queue

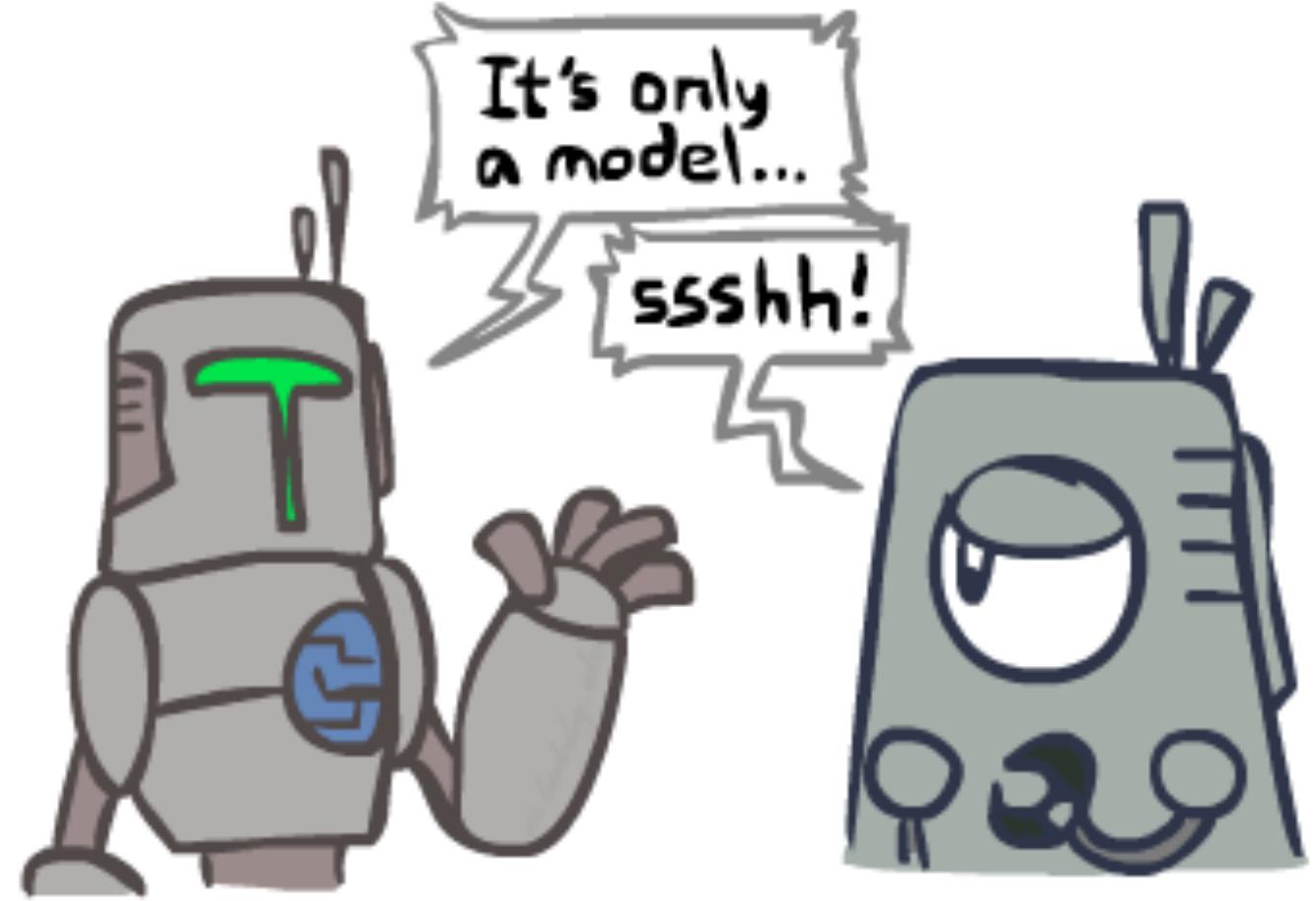
- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the  $\log(n)$  overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



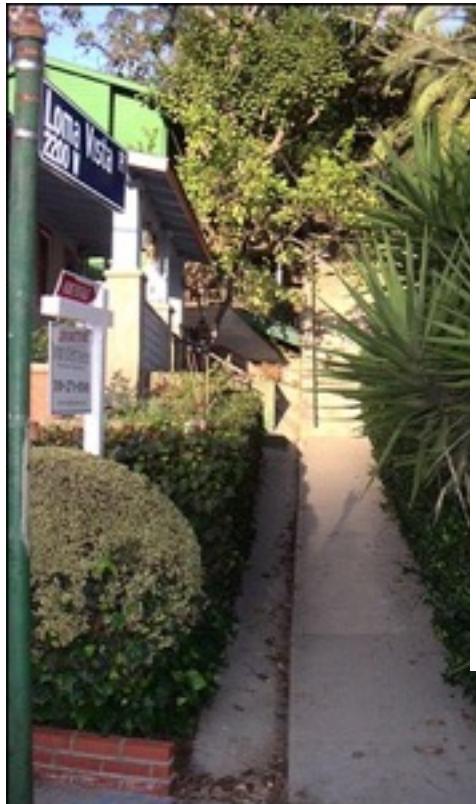
# Search and Models

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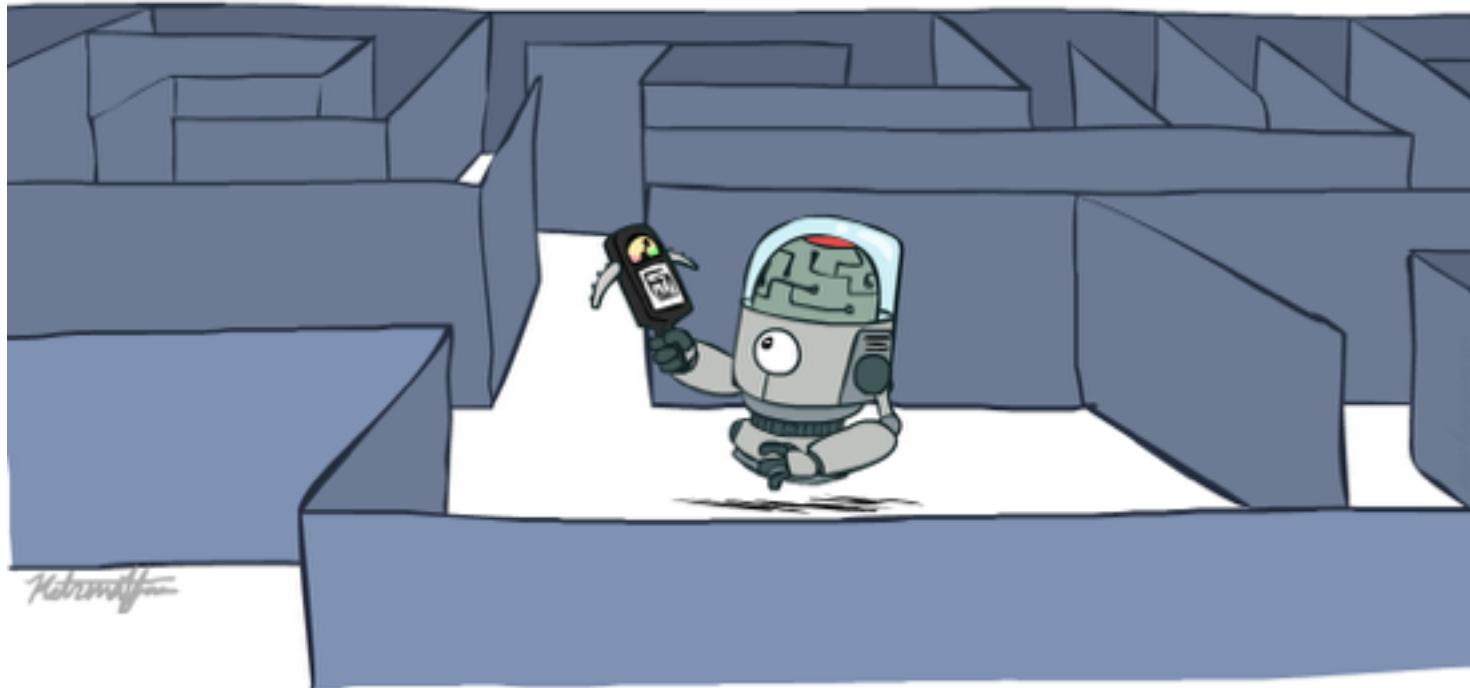
- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all “in simulation”
  - Your search is only as good as your models...



# Search Gone Wrong?



# Buscar informado



# Informed Search

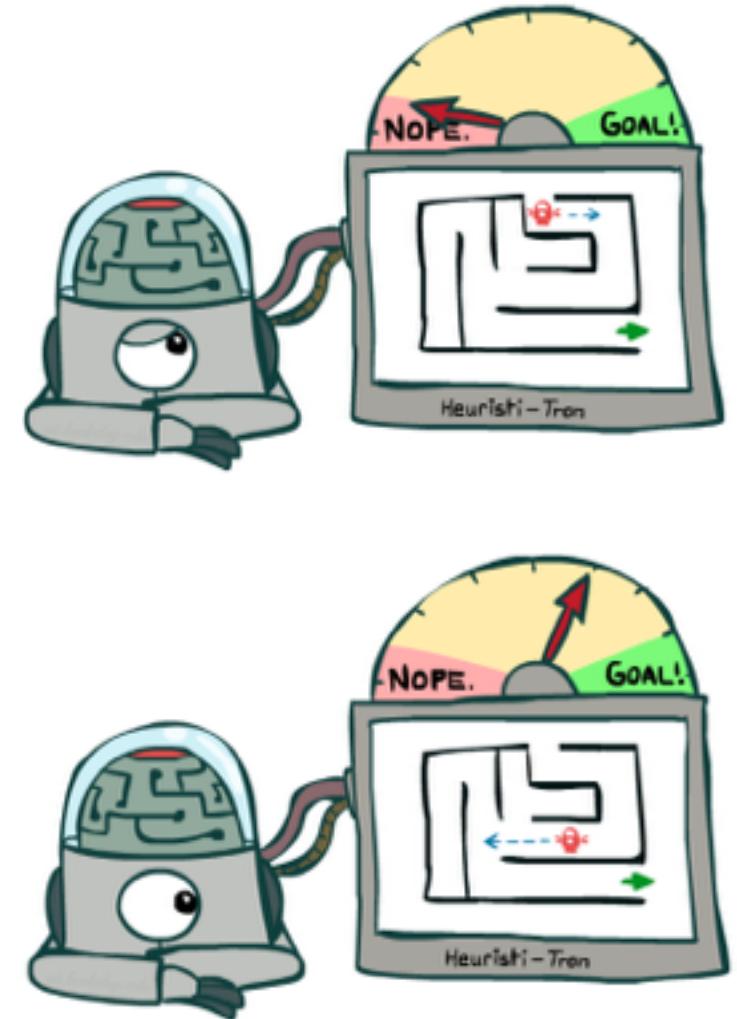
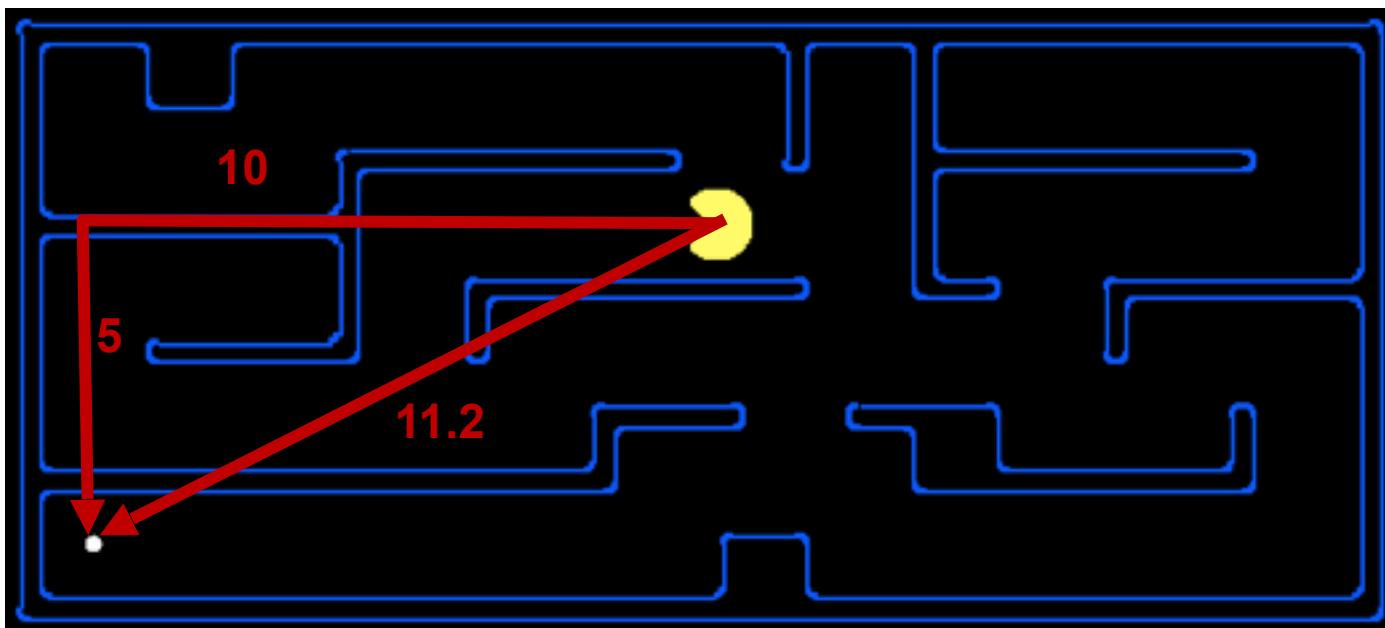
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- Informed Search
  - Heuristics
  - Greedy Search
  - A\* Search
- Graph Search

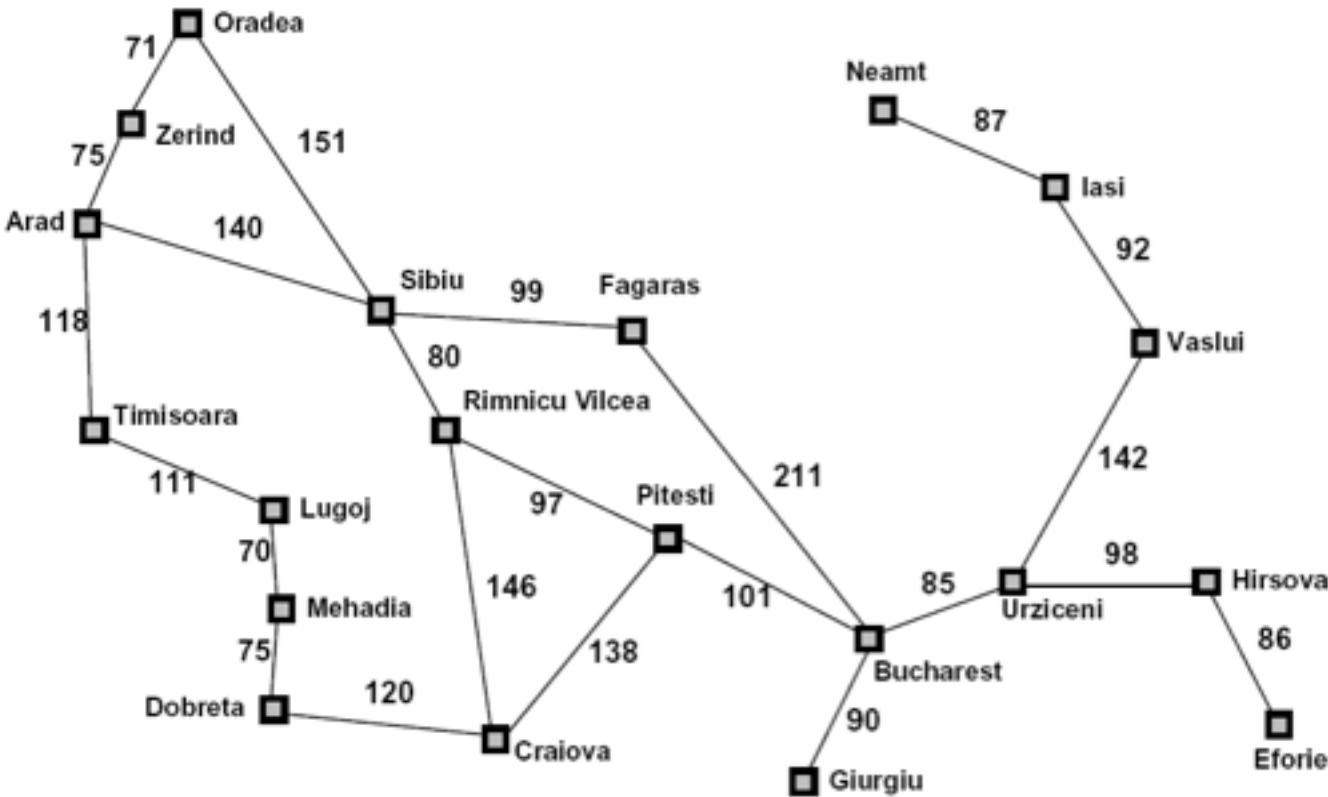


# Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing



# Example: Heuristic Function



$h(x)$

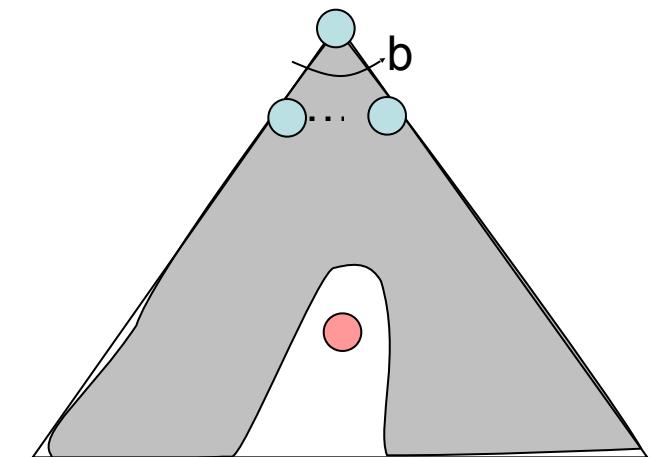
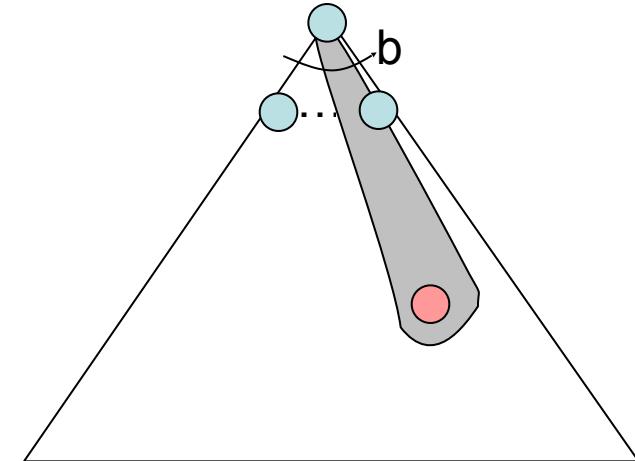
# Greedy Search

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# Greedy Search

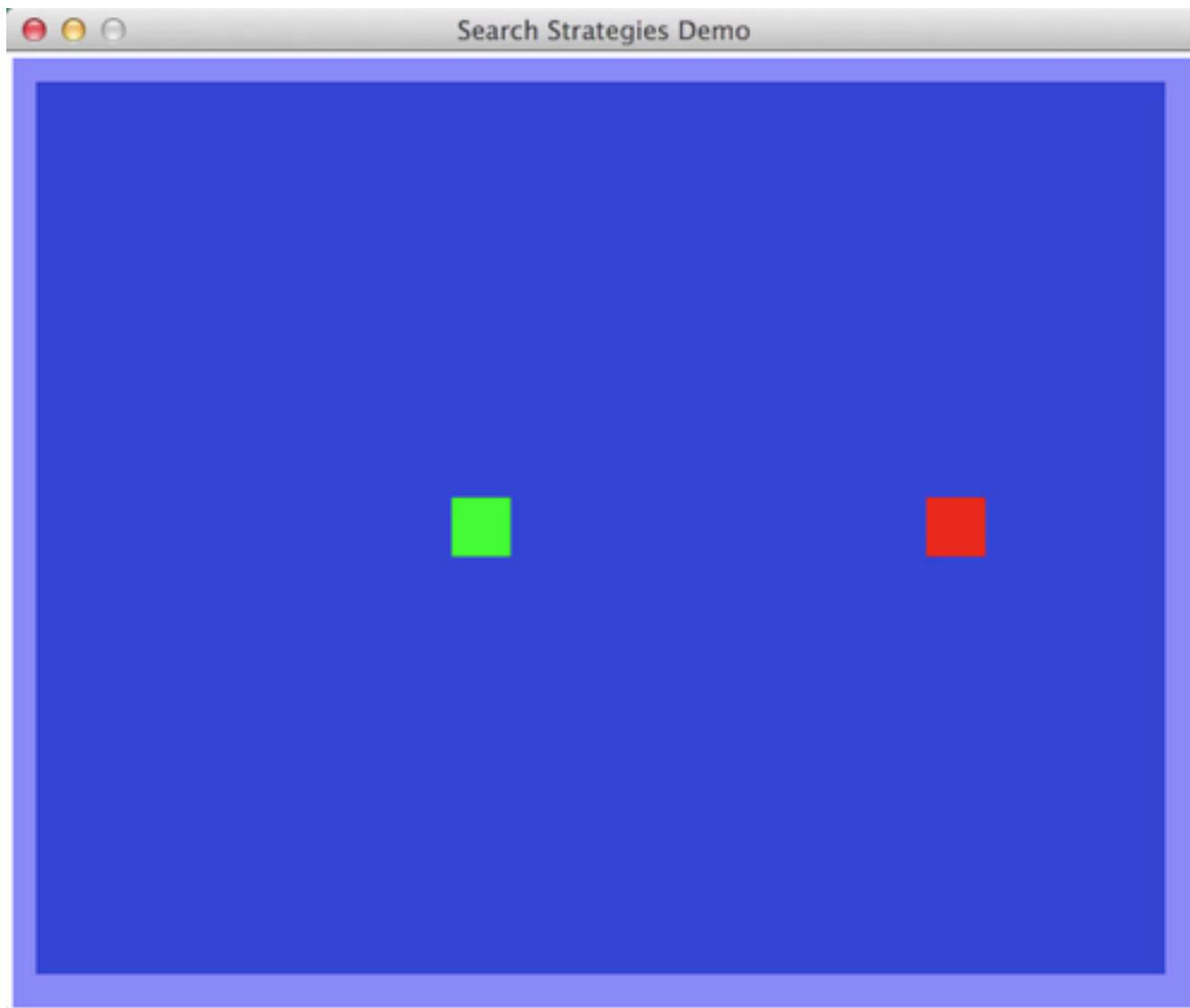
- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS



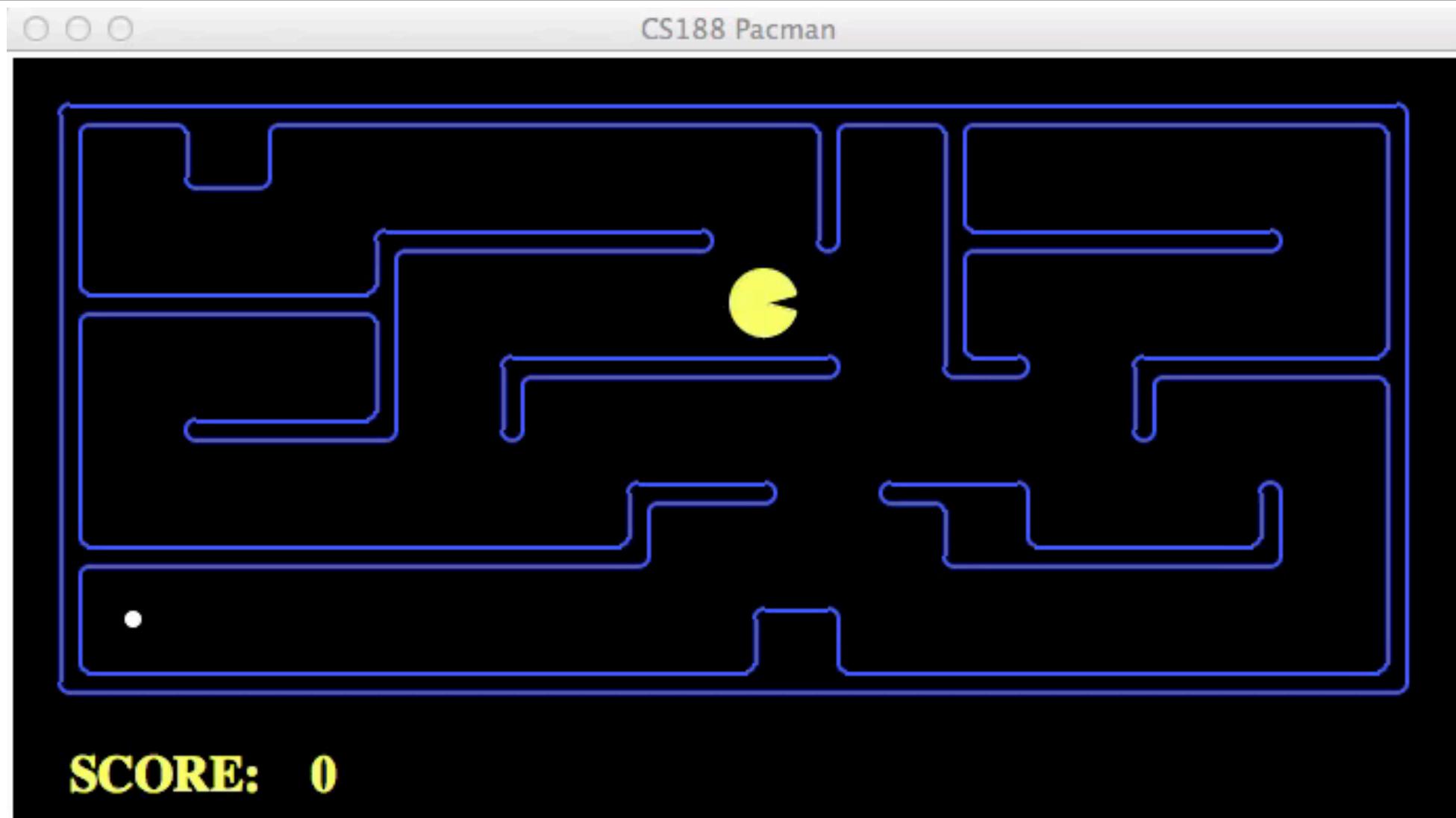
[Demo: contours greedy empty (L3D1)]

[Demo: contours greedy pacman small maze (L3D4)]

# Video of Demo Contours Greedy (Empty)



# Video of Demo Contours Greedy (Pacman Small Maze)



# A\* Search

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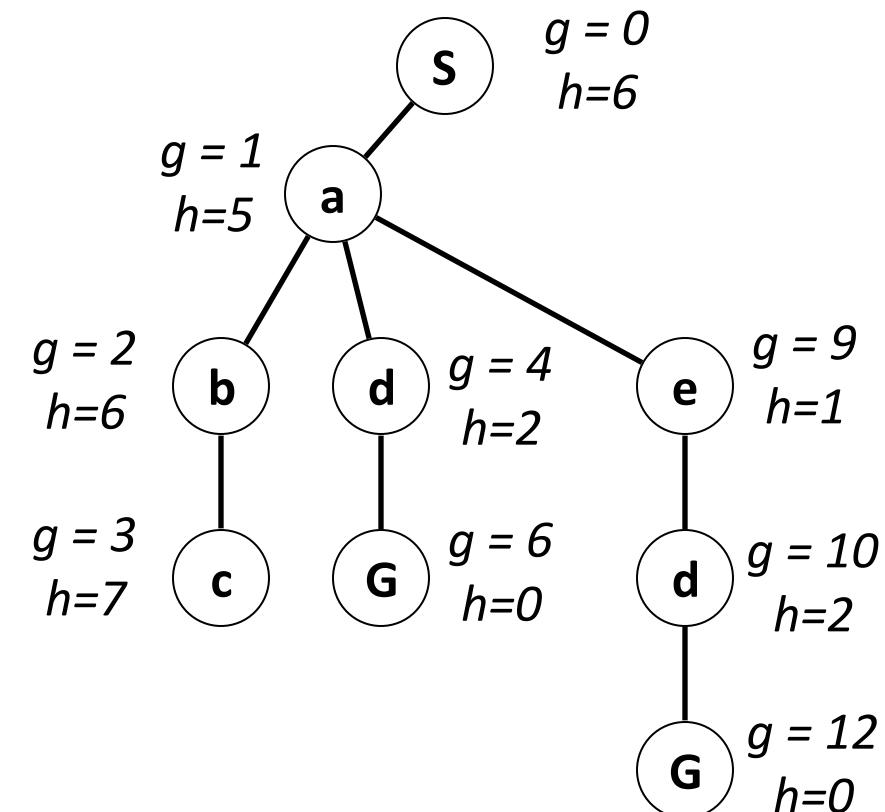
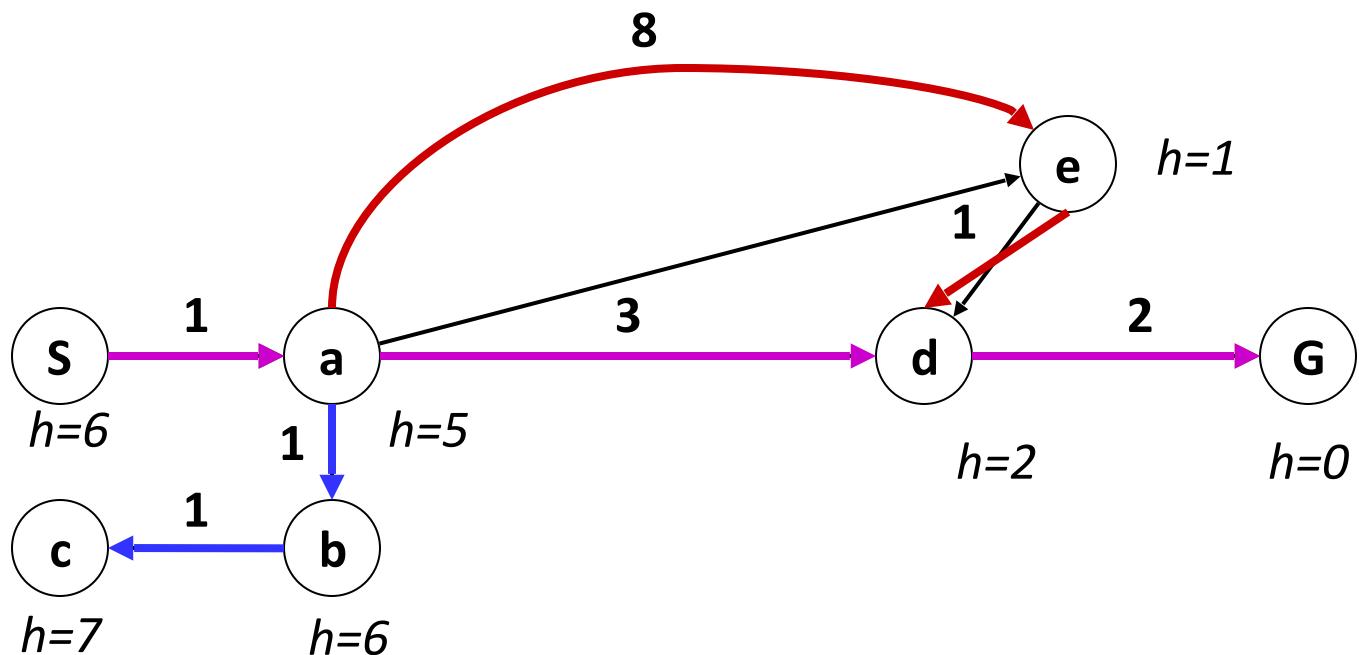
# A\* Search

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# Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost*  $g(n)$
- Greedy orders by goal proximity, or *forward cost*  $h(n)$

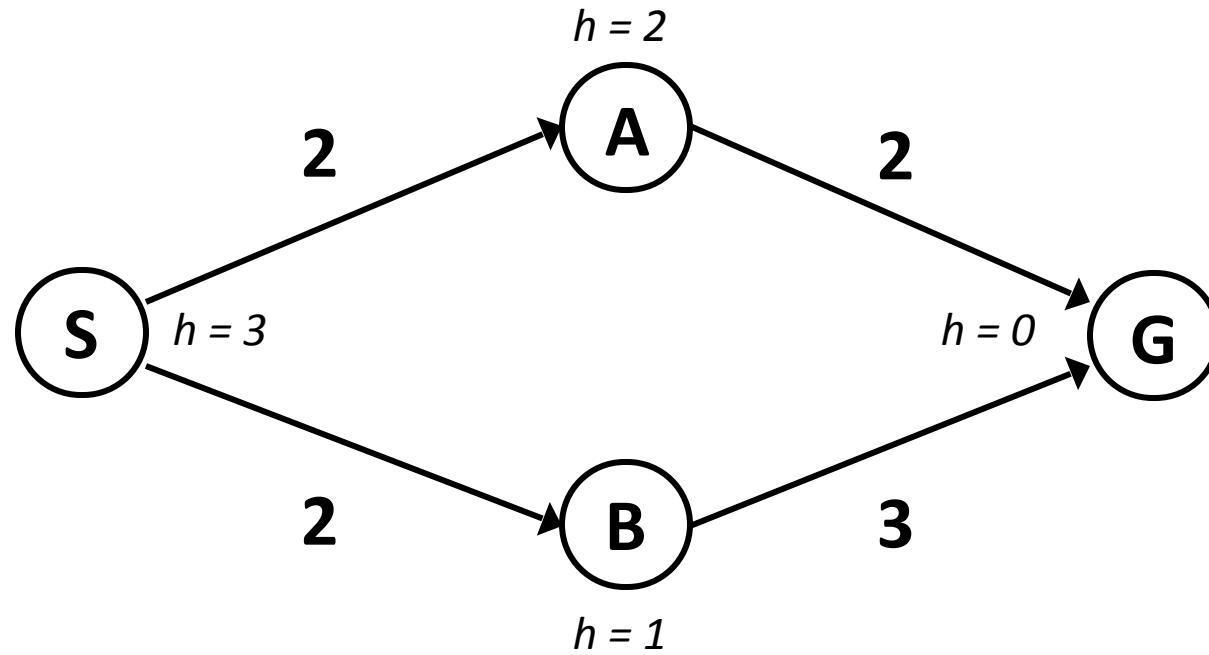


- A\* Search orders by the sum:  $f(n) = g(n) + h(n)$

Example: Teg Grenager

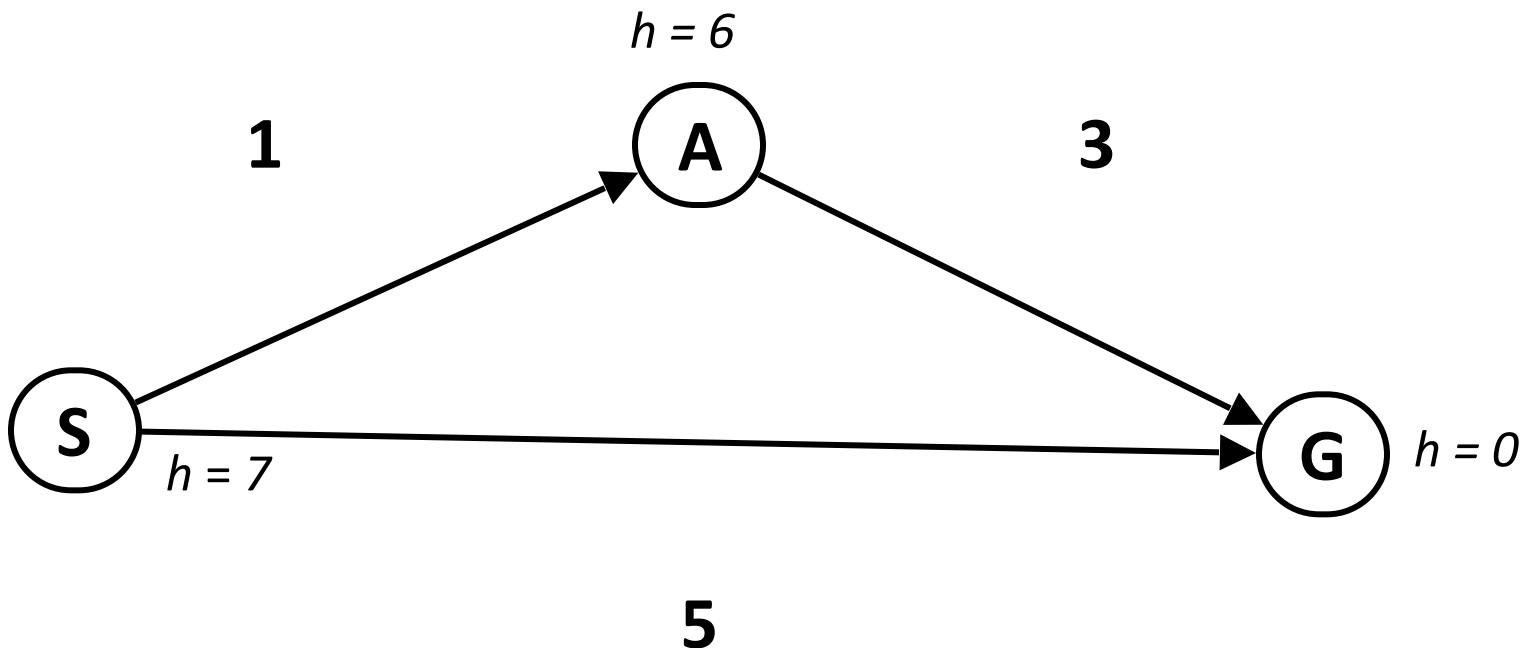
# When should A\* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

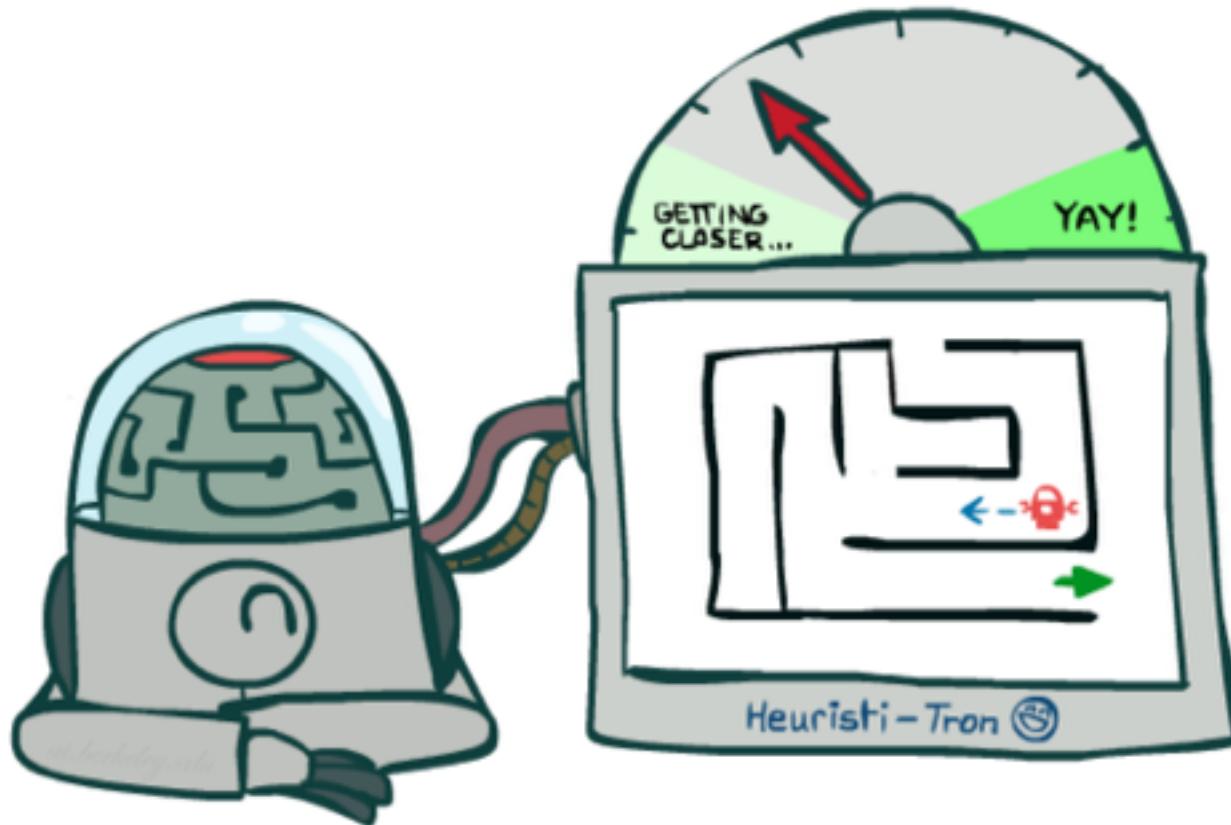
# Is A\* Optimal?



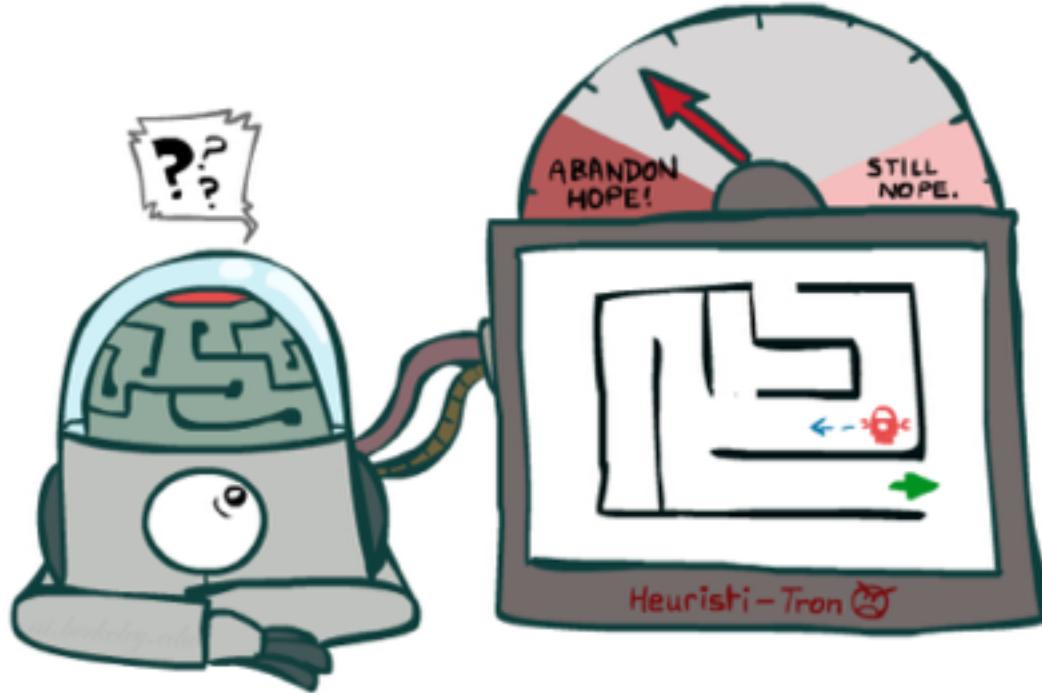
- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

# Admissible Heuristics

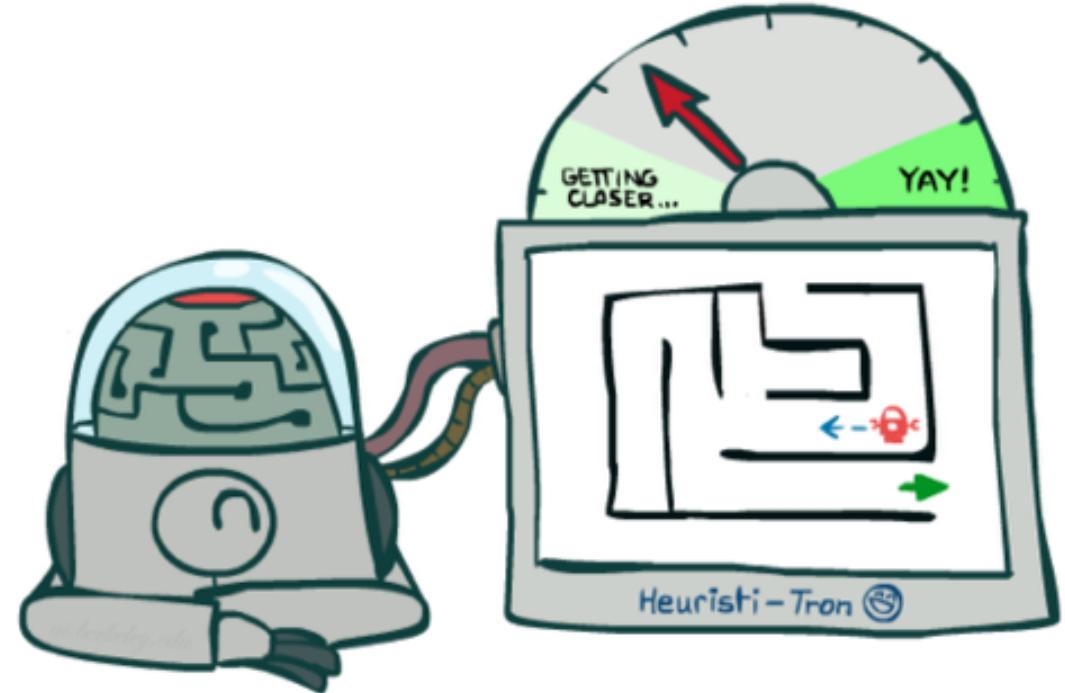
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# Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

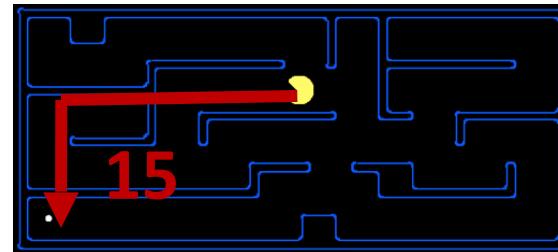
# Admissible Heuristics

- A heuristic  $h$  is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where  $h^*(n)$  is the true cost to a nearest goal

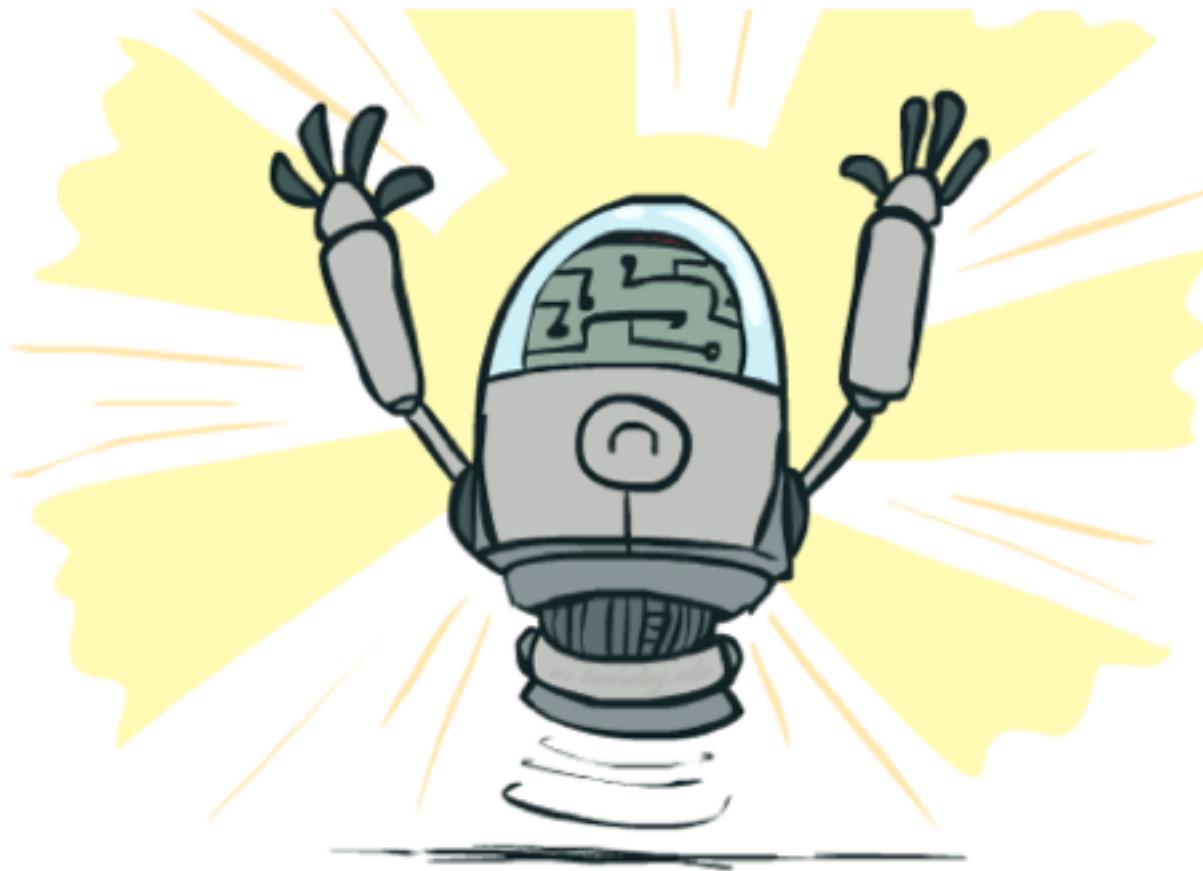
- Example:



- Coming up with admissible heuristics is most of what's involved in using A\* in practice.

# [We'll skip this] Optimality of A\* Tree Search

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# Optimality of A\* Tree Search

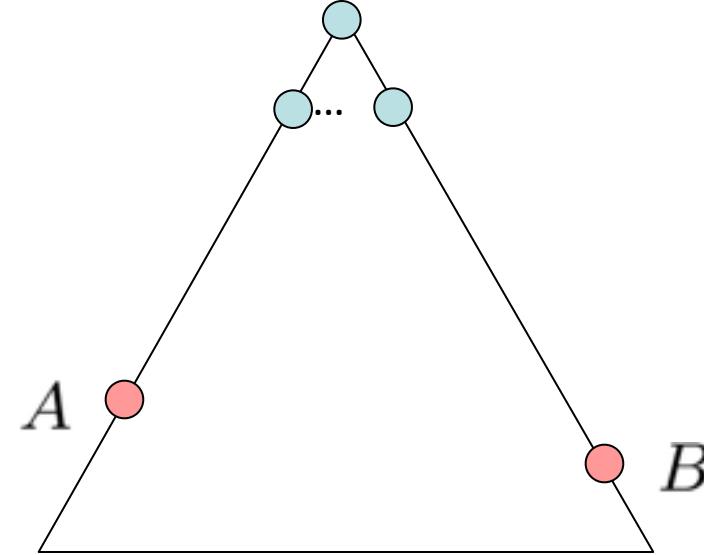
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Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- $h$  is admissible

Claim:

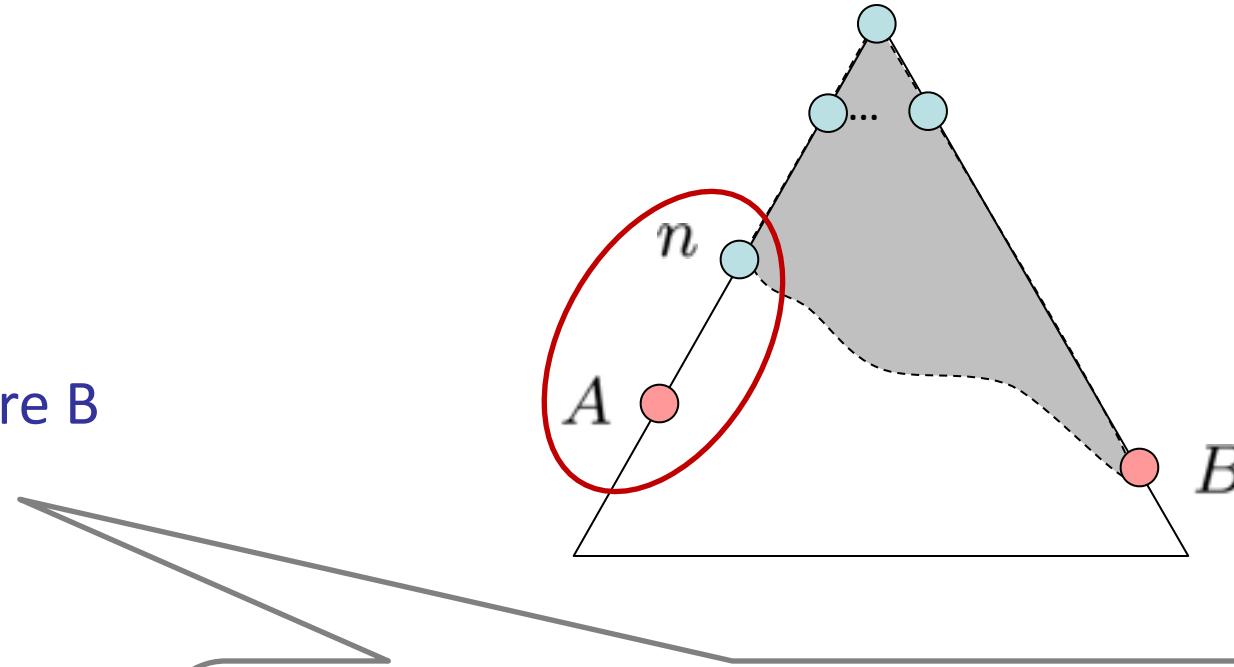
- A will exit the fringe before B



# Optimality of A\* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor  $n$  of A is on the fringe, too (maybe A!)
- Claim:  $n$  will be expanded before B
  1.  $f(n)$  is less or equal to  $f(A)$



$$f(n) = g(n) + h(n)$$

$$f(n) \leq g(A)$$

$$g(A) = f(A)$$

Definition of f-cost

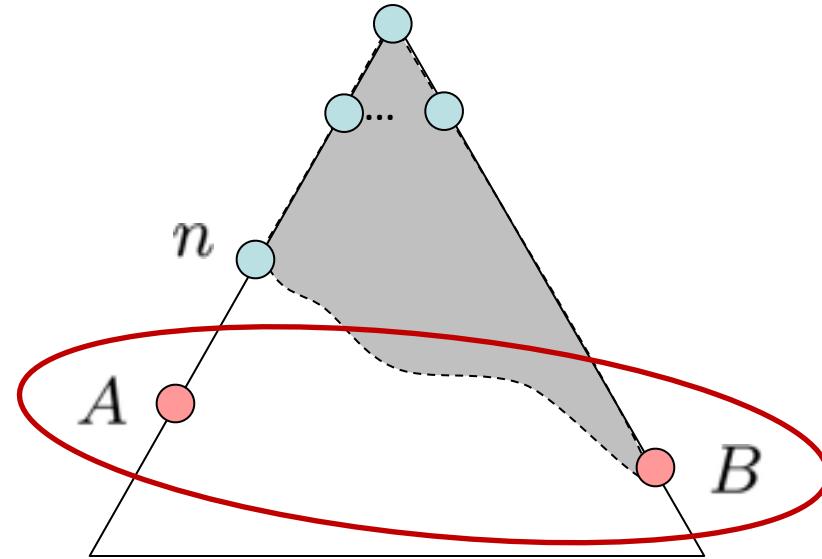
Admissibility of h

$h = 0$  at a goal

# Optimality of A\* Tree Search: Blocking

Proof:

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- Claim:  $n$  will be expanded before B
  1.  $f(n)$  is less or equal to  $f(A)$
  2.  $f(A)$  is less than  $f(B)$



$$g(A) < g(B)$$

$$f(A) < f(B)$$

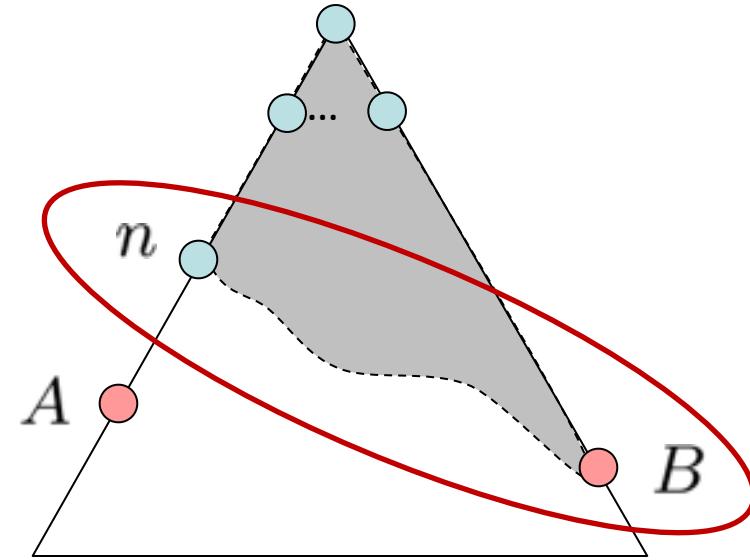
B is suboptimal

$h = 0$  at a goal

# Optimality of A\* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor  $n$  of A is on the fringe, too (maybe A!)
- Claim:  $n$  will be expanded before B
  1.  $f(n)$  is less or equal to  $f(A)$
  2.  $f(A)$  is less than  $f(B)$
  3.  $n$  expands before B
- All ancestors of A expand before B
- A expands before B
- A\* search is optimal



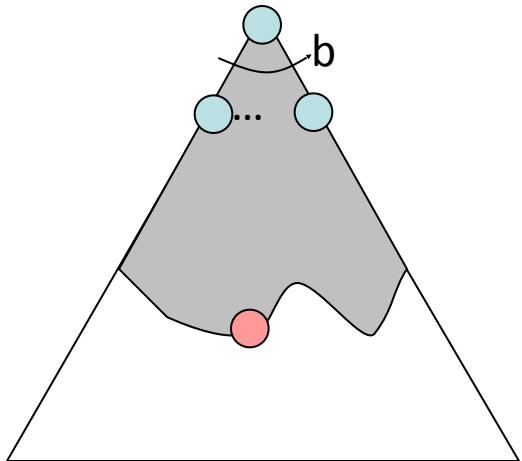
$$f(n) \leq f(A) < f(B)$$

# Properties of A\*

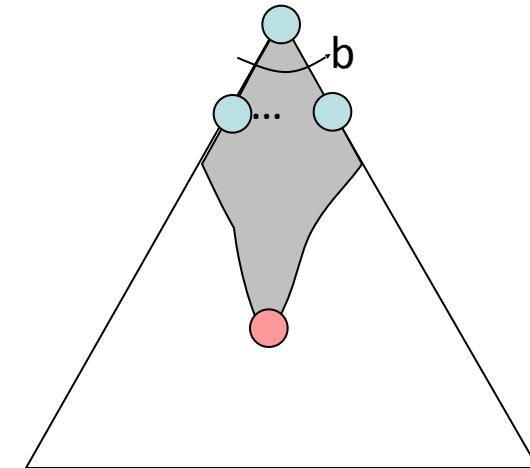
# Properties of A\*

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Uniform-Cost

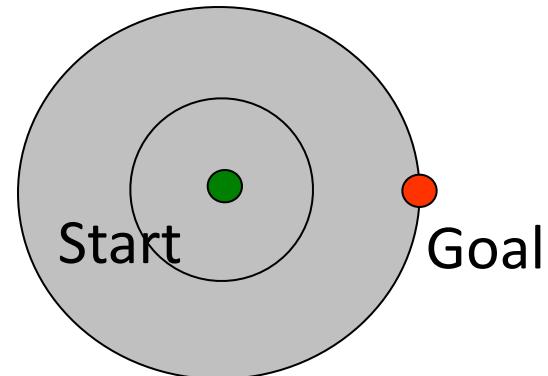


A\*

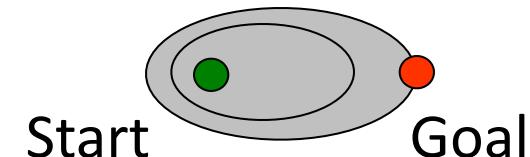


# UCS vs A\* Contours

- Uniform-cost expands equally in all “directions”



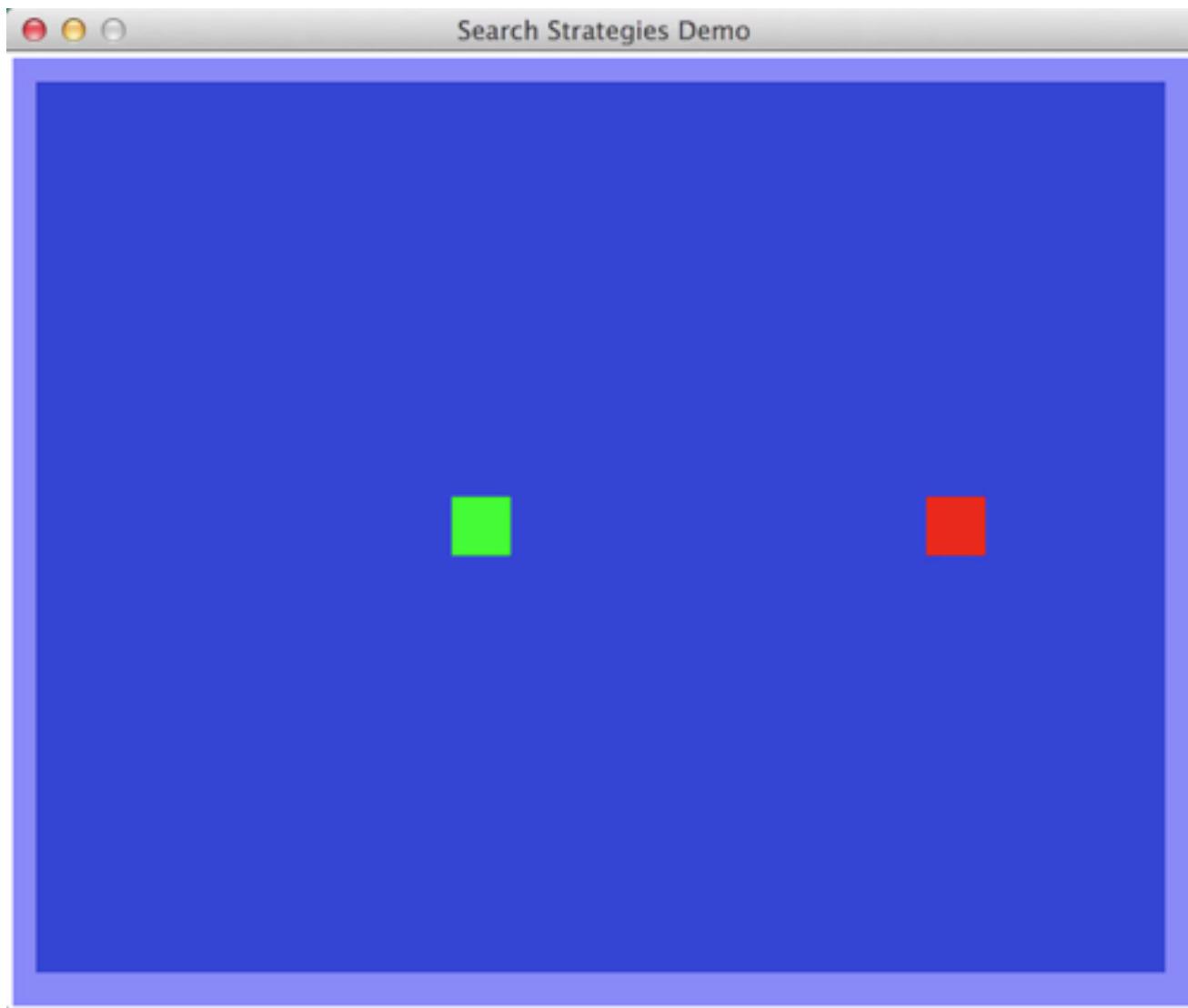
- A\* expands mainly toward the goal, but does hedge its bets to ensure optimality



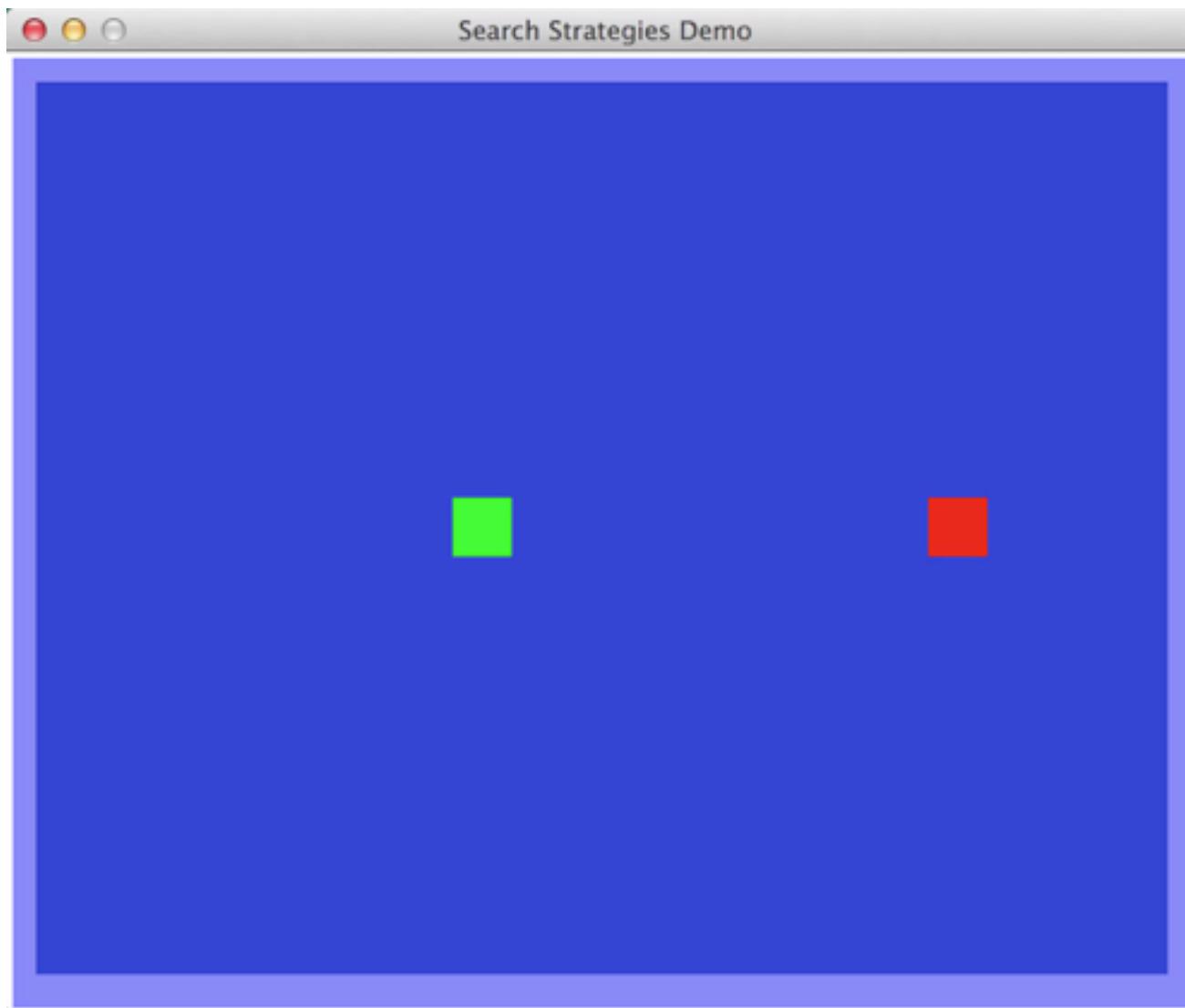
[Demo: contours UCS / greedy / A\* empty (L3D1)]

[Demo: contours A\* pacman small maze (L3D5)]

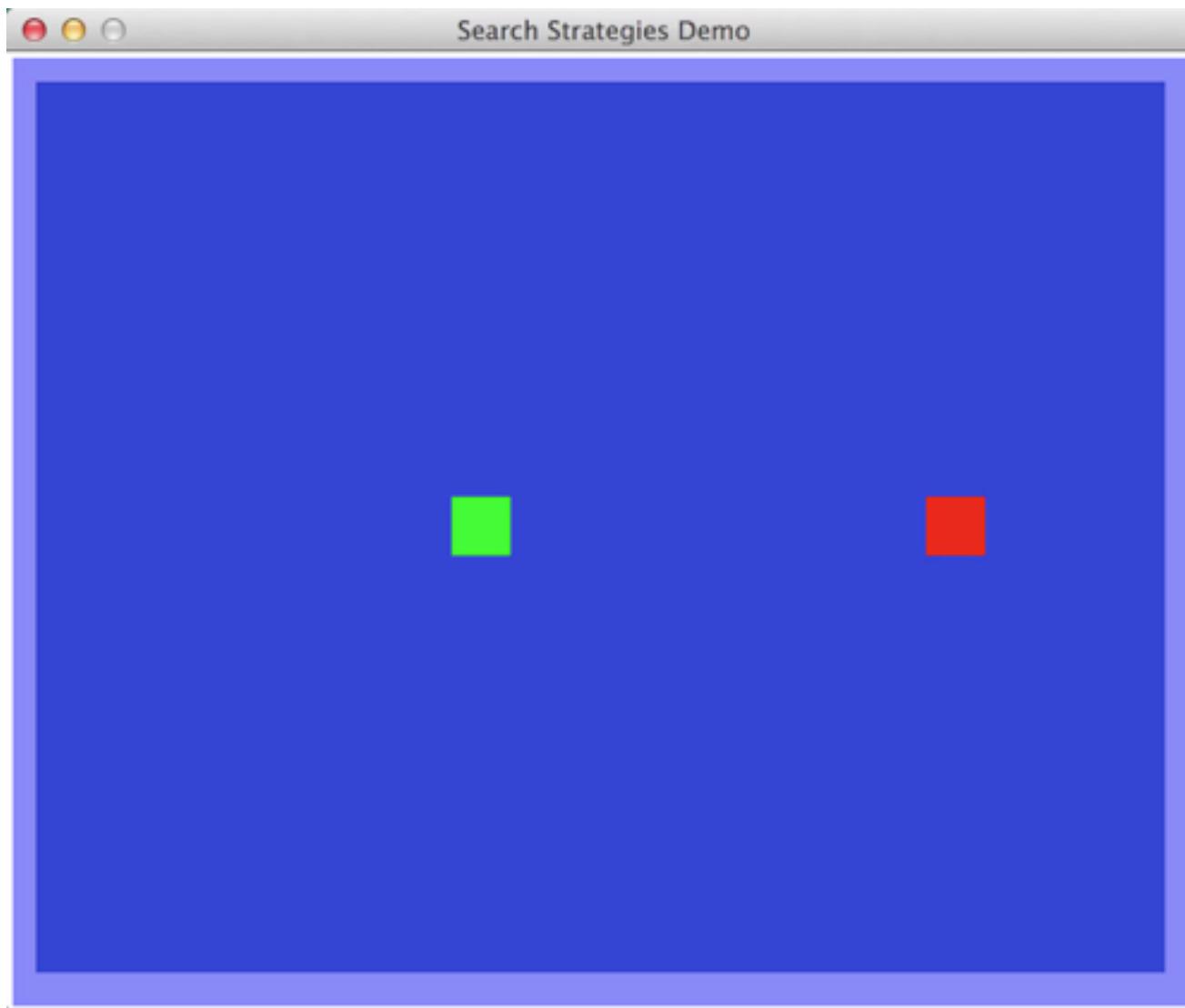
# Video of Demo Contours (Empty) -- UCS



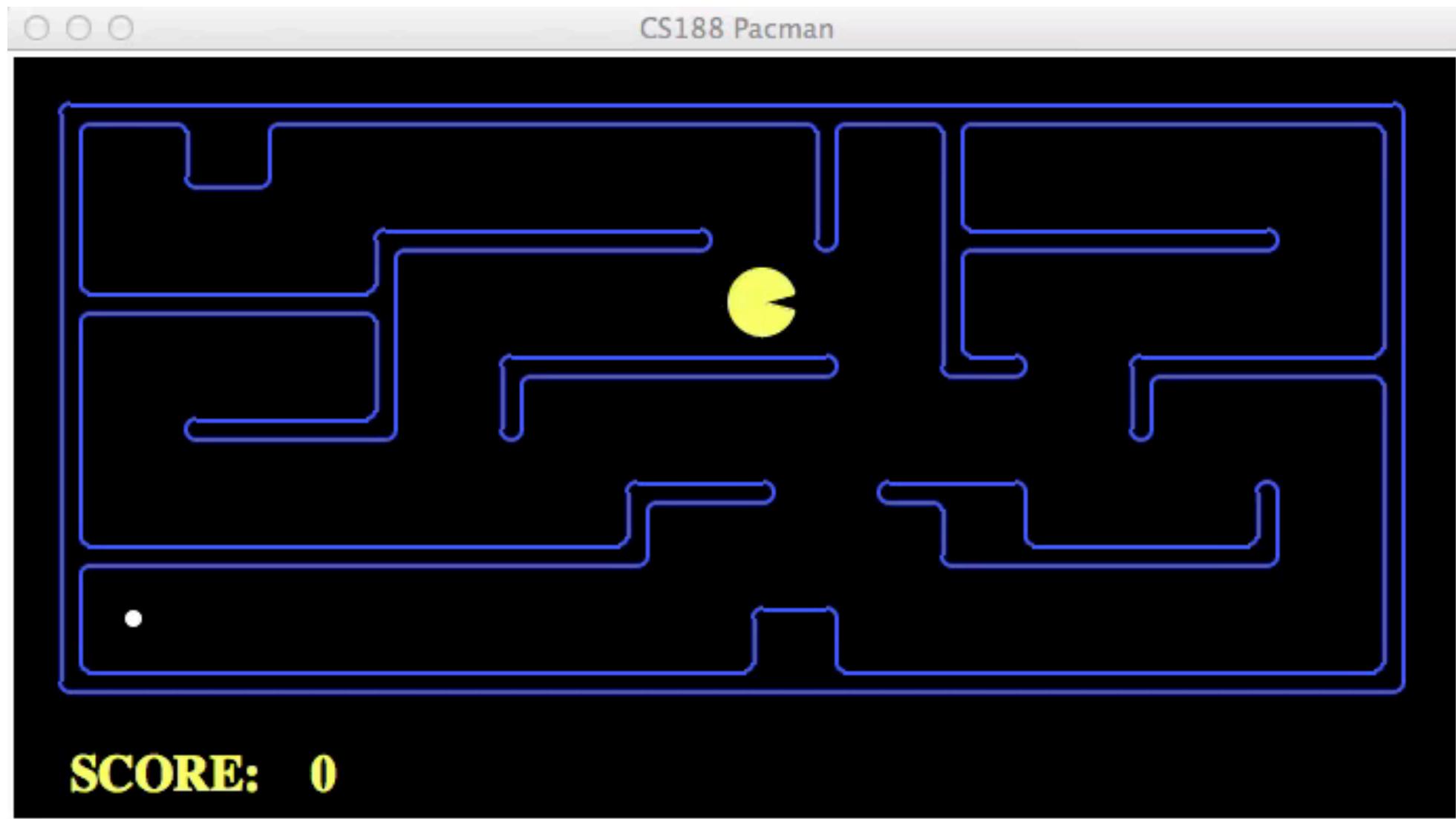
# Video of Demo Contours (Empty) -- Greedy



# Video of Demo Contours (Empty) – A\*



# Video of Demo Contours (Pacman Small Maze) – A\*



# Comparison



Greedy



Uniform Cost



A\*

# A\* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...



# Creating Heuristics

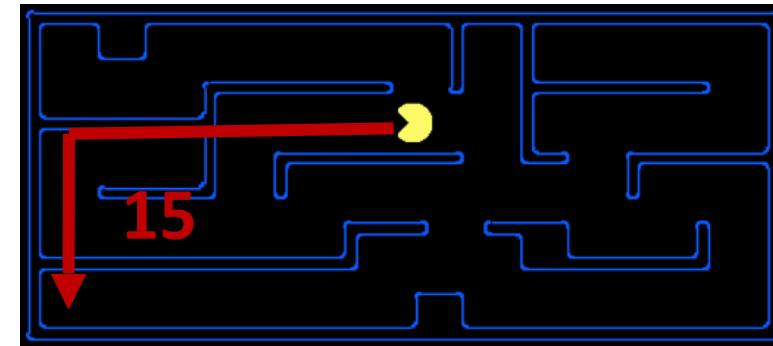
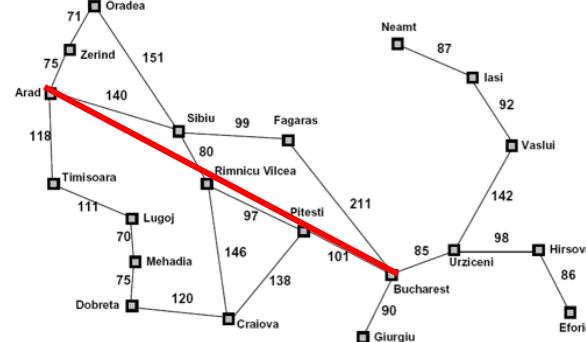
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# Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

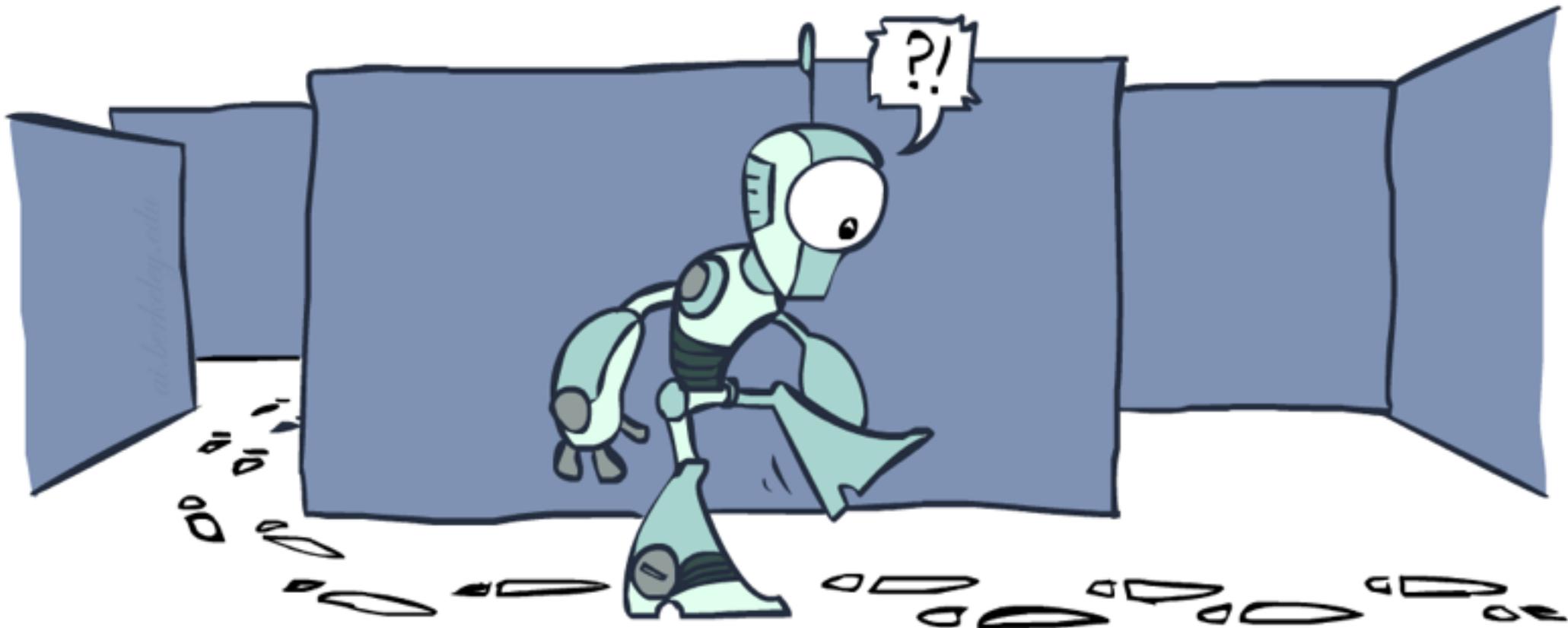
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- Inadmissible heuristics are often useful too

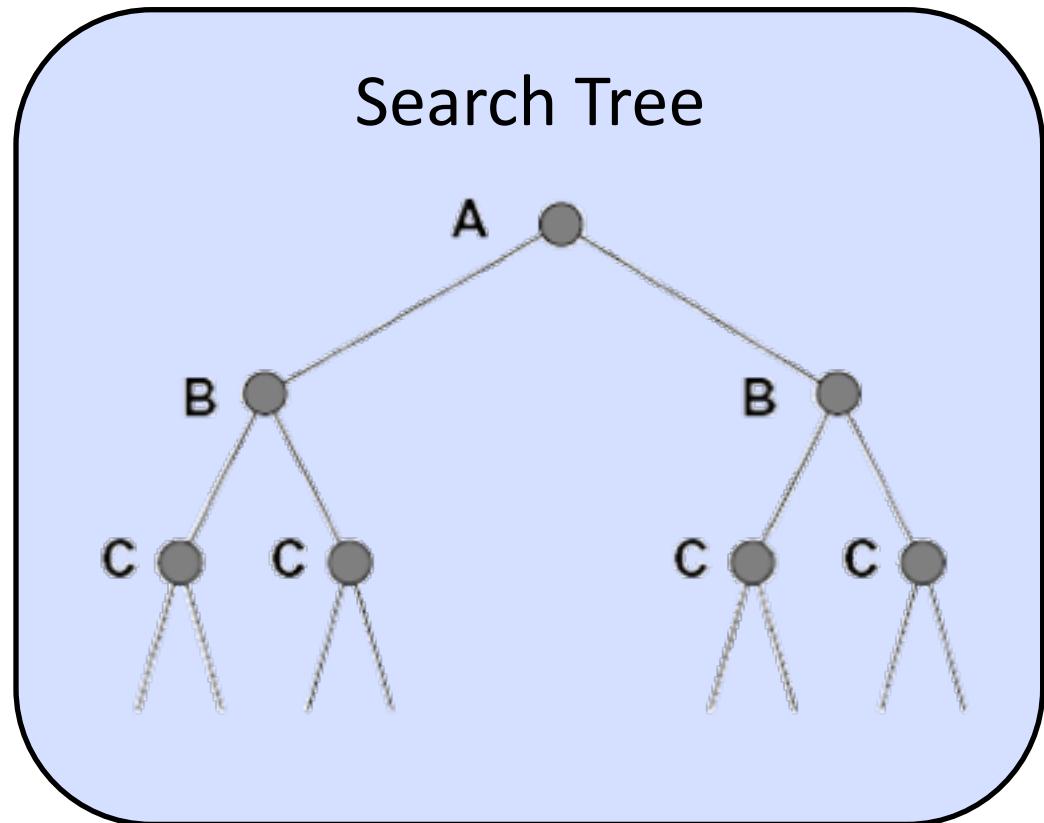
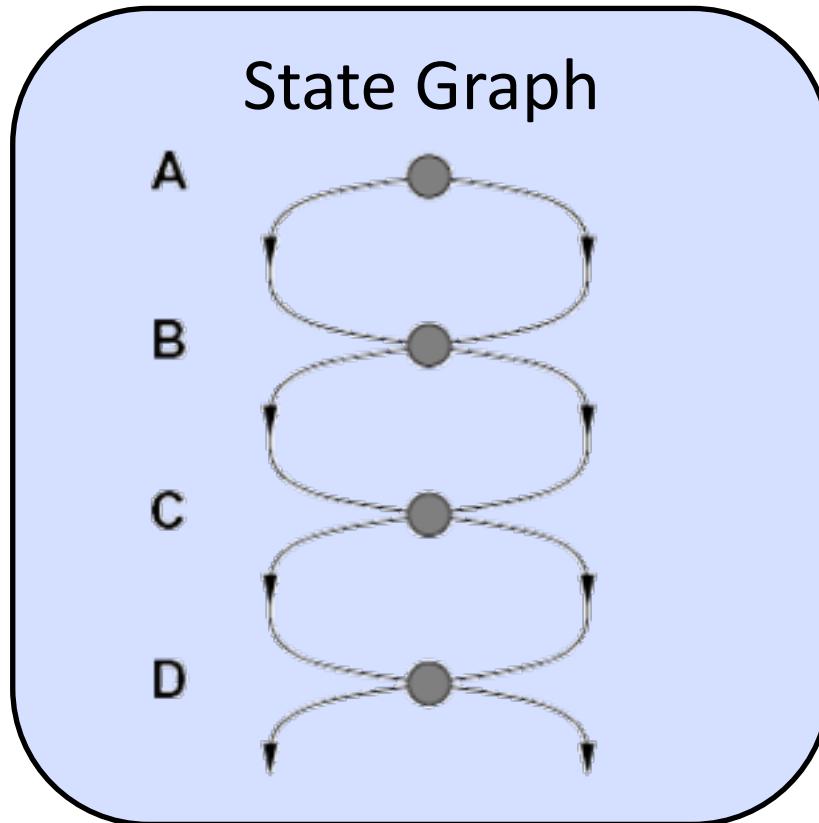
# Graph Search

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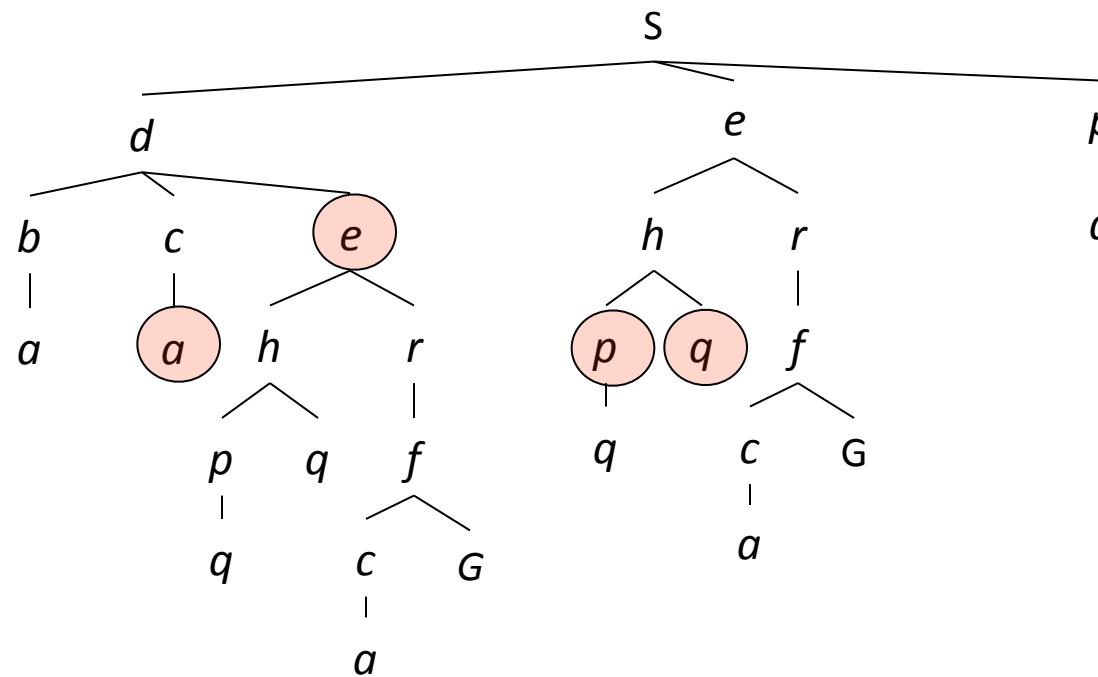
# Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



# Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



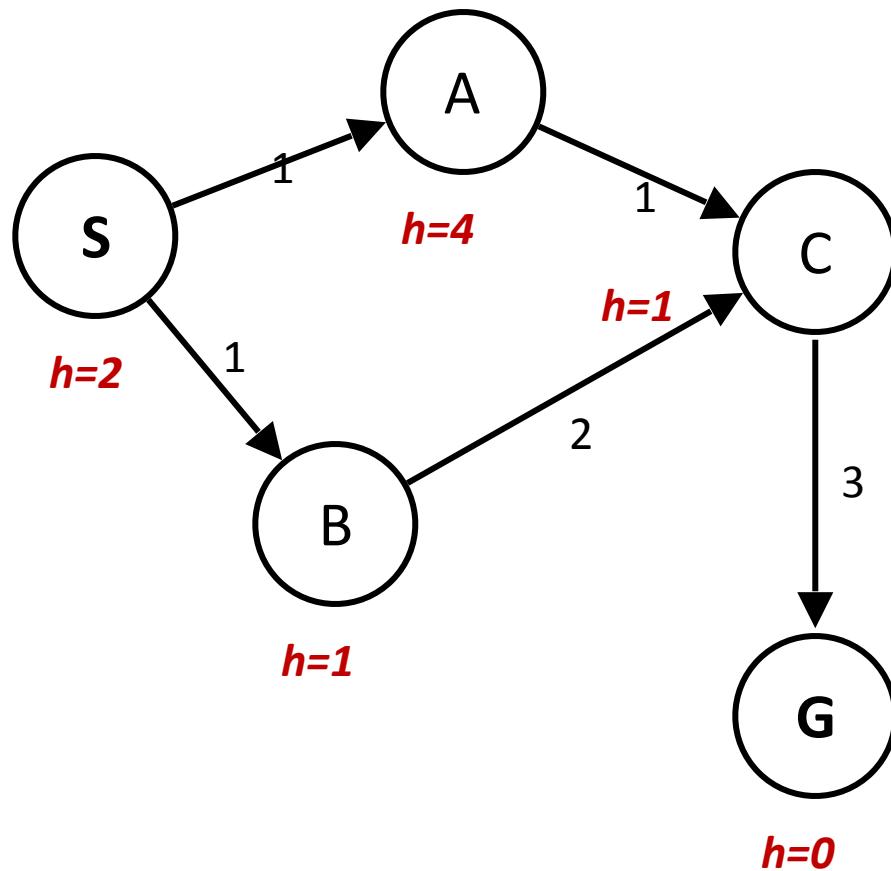
# Graph Search

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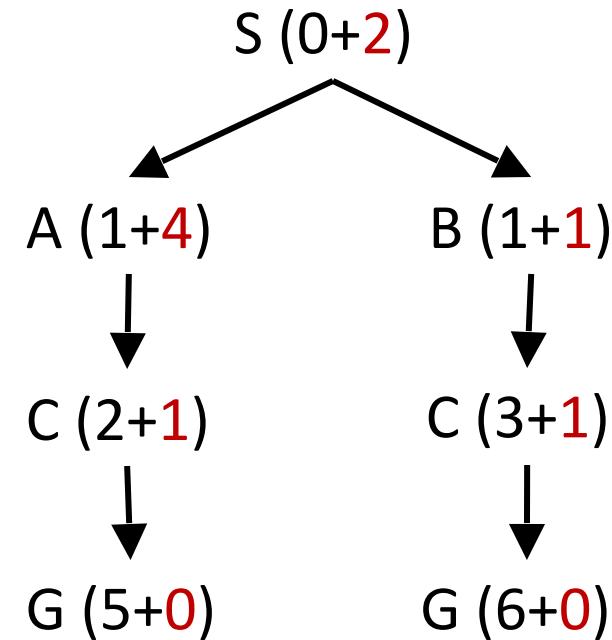
- Idea: never **expand** a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but...
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: **store the closed set as a set**, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

# A\* Graph Search Gone Wrong?

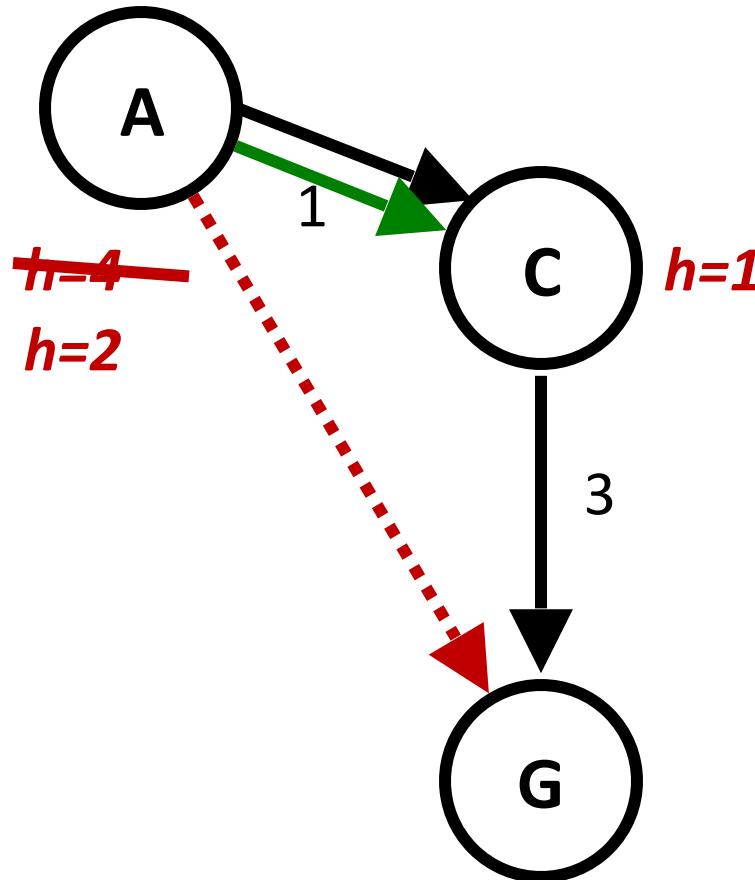
State space graph



Search tree



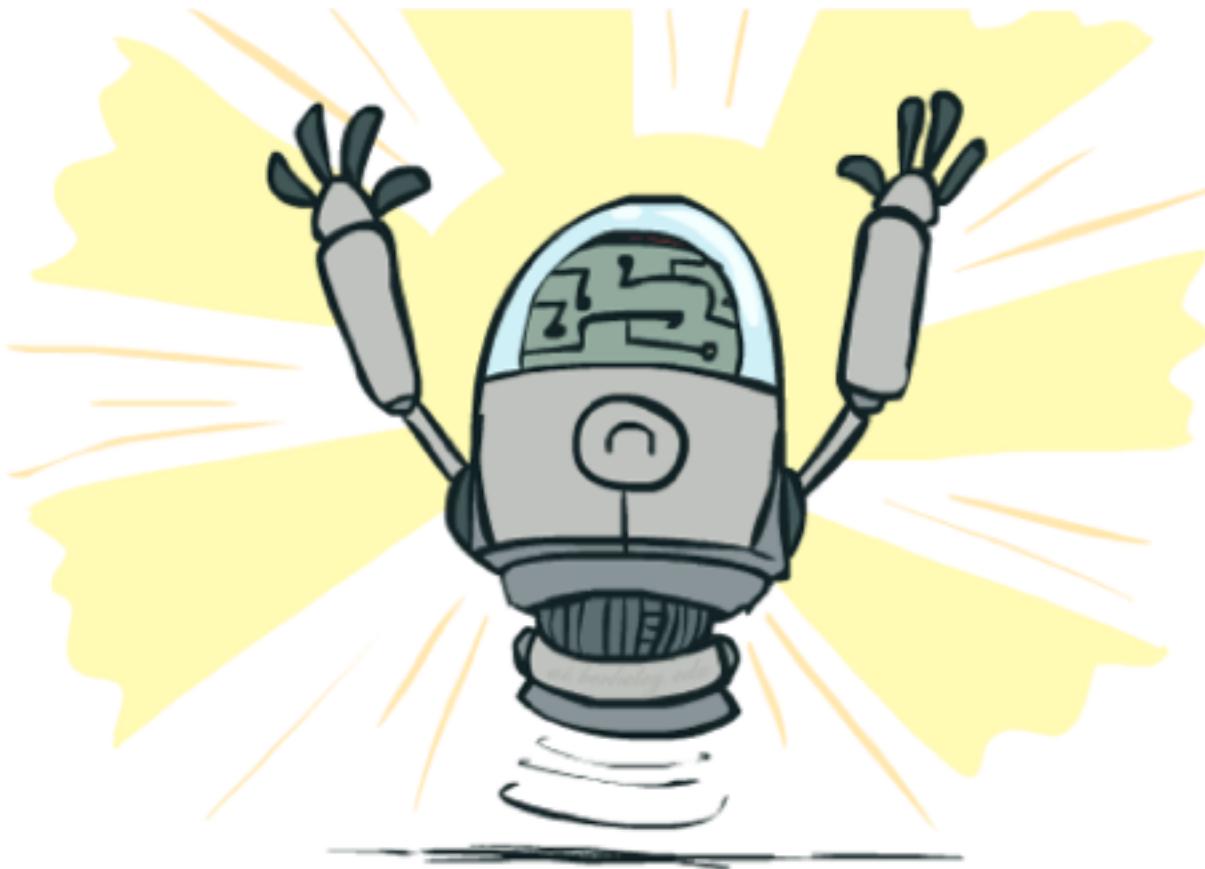
# Consistency of Heuristics



- Main idea: estimated heuristic costs  $\leq$  actual costs
  - Admissibility: heuristic cost  $\leq$  actual cost to goal
$$h(A) \leq \text{actual cost from A to G}$$
  - Consistency: heuristic “arc” cost  $\leq$  actual cost for each arc
$$h(A) - h(C) \leq \text{cost}(A \text{ to } C)$$
- Consequences of consistency:
  - The f value along a path never decreases
$$h(A) \leq \text{cost}(A \text{ to } C) + h(C)$$
  - A\* graph search is optimal

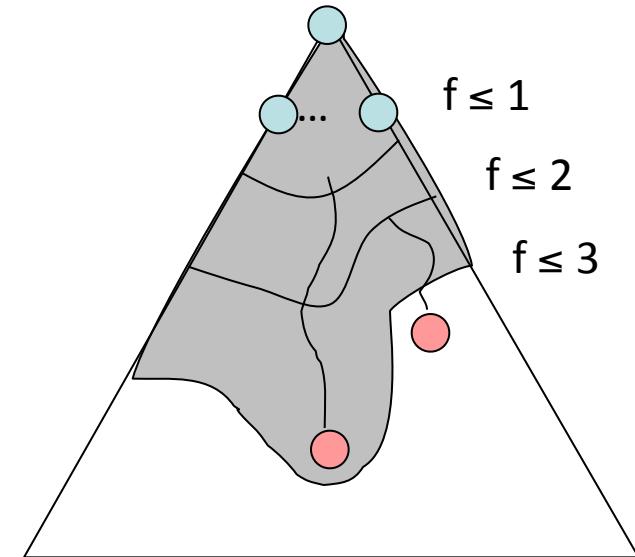
# Optimality of A\* Graph Search

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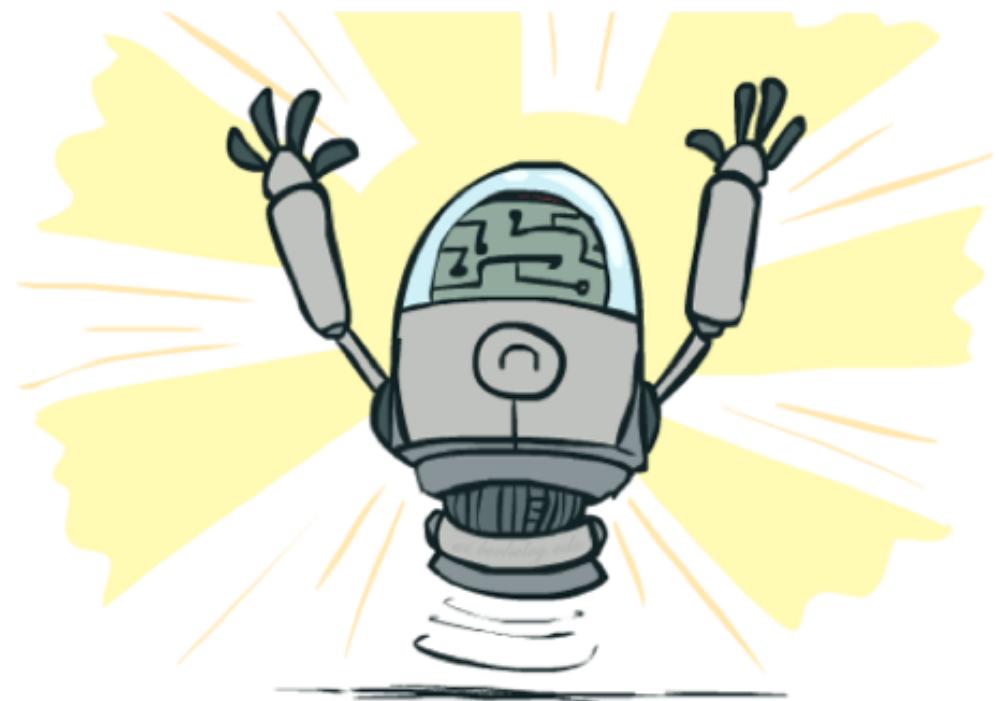
# Optimality of A\* Graph Search

- Sketch: consider what A\* does with a consistent heuristic:
  - Fact 1: In tree search, A\* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state  $s$ , nodes that reach  $s$  optimally are expanded before nodes that reach  $s$  suboptimally
- Result: A\* graph search is optimal



# Optimality

- Tree search:
  - A\* is optimal if heuristic is admissible
  - UCS is a special case ( $h = 0$ )
- Graph search:
  - A\* optimal if heuristic is consistent
  - UCS optimal ( $h = 0$  is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems



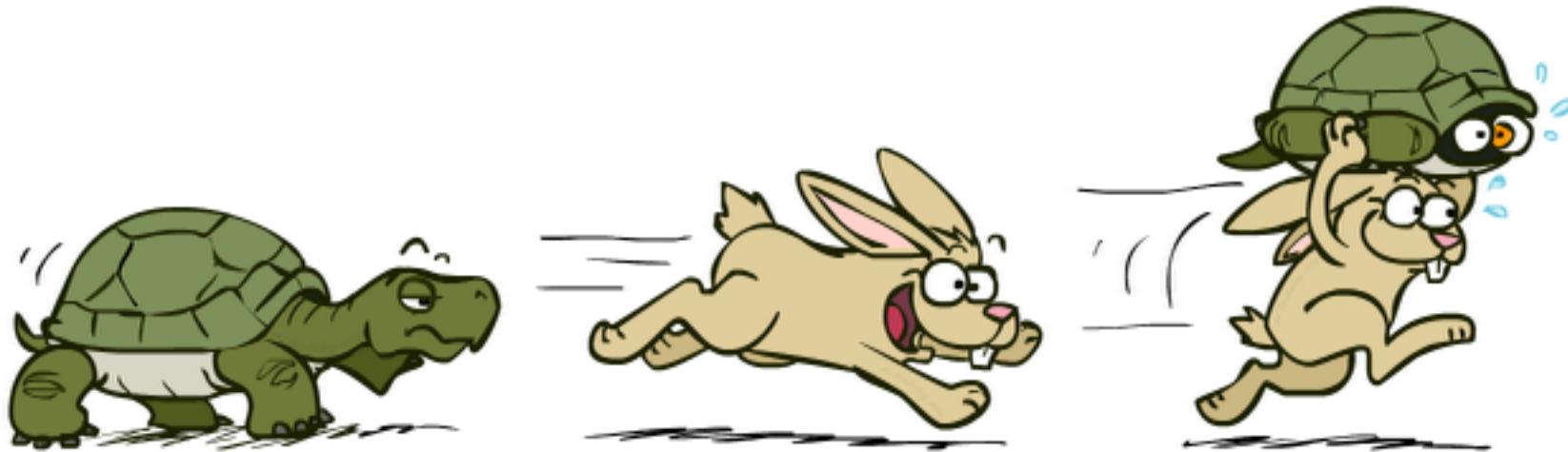
# A\*: Summary

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# A\*: Summary

- A\* uses both backward costs and (estimates of) forward costs
- A\* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



# Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
  fringe  $\leftarrow$  INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node  $\leftarrow$  REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    for child-node in EXPAND(STATE[node], problem) do
      fringe  $\leftarrow$  INSERT(child-node, fringe)
    end
  end
```

# Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
      add STATE[node] to closed
      for child-node in EXPAND(STATE[node], problem) do
        fringe ← INSERT(child-node, fringe)
    end
  end
```