

- **DFS** (outputs connected components, topological sort on a DAG. You also have access to the pre and post arrays.)
  - Types of graphs: Unweighted/Undirected graphs, directed graphs, in particular - Directional Acyclic Graph (DAG)
    - DFS ccnum is only useful if the Graph is undirected. For directed graph, always use SCC.
  - Inputs:  $G(V, E)$  in adjacency list representation
  - Access to: pre[], post[], prev[], and visited[T/F] arrays shared between explore & DFS. (visited is all True at end of run so not helpful for tracking) [#506](#)
    - pre[] & post[] are the pre and post order numbers and are created by Explore (not returning these to DFS/but we have access to pre & post)
    - prev[] is array which provides the 'parent' vertex for backtracking purposes
  - Outputs:
    - Undirect G = Vertices labelled by connected component number (ccnum)
    - Directed G = list of subgraphs (1 for each subcomponent)
  - Runtime:  $O(|V| + |E|)$  or  $O(n+m)$
  - Data structure: Stack
- **Explore** subroutine (used by DFS)
  - Types of graphs: Unweighted/undirected or directed,
    - Directional Acyclic Graph (DAG)
  - Inputs:  $G(V, E)$  and start vertex  $v$  in  $V$
  - Access to:
    - previsited (prev[]) = arrays of vertices before a given vertices (but not used by Explore, needed for DFS)
    - ccnum[]
  - Outputs:
    - visited(u) set to true for all vertices  $u$  reachable from  $v$ . Array of all the nodes in the graph, with the ones reachable from  $s$  set to True.
  - Runtime:  $O(|V| + |E|)$  or  $O(n+m)$
  - Data structure:
- **BFS**
  - Types of graphs: Unweighted/undirected or directed
  - Inputs:  $G=(V, E)$  and start  $v$  in  $V$
  - Access to: prev(u) giving vertex preceding  $u$  in shortest path from  $v$
  - Outputs: dist(u) set to shortest path between  $v$  and reachable vertex  $u$ , or infinity if not reachable
  - Runtime:  $O(|V| + |E|)$  or  $O(n+m)$
  - Data structure: Queue

- **Dijkstra's algorithm** - finds the shortest distance from a source vertex to all other vertices and a path can be recovered backtracking over the pre labels.
  - Types of graphs: Weighted/undirected or directed graphs (no negative weights)
  - Inputs:  $G=(V,E)$ , start  $v$  in  $V$
  - Access to:  $\text{prev}(u)$  giving vertex preceding  $u$  in shortest path from  $v$
  - Outputs:  $\text{dist}(u)$  set to shortest distance between  $v$  and reachable vertex  $u$ , or infinity if not reachable
  - Runtime:  $O((|V|+|E|) \log |V|)$  or  $O((n+m) \log n)$
  - Data structure: Priority queue/ minimum priority queue
- **Bellman-Ford** (compute the shortest path from  $s$  to  $t$  (weights allowed to be negative))
  - Types of graphs: Weighted/directed or undirected (can have negative weights)
  - Inputs:  $G=(V,E)$ , start vertex ( $s$ )
  - Access to: detect negative cycles by comparing  $T[n,.]$  to  $T[n-1,.]$
  - Outputs: shortest path from  $v$  to all other vertices
  - Runtime:  $O(|V|*|E|)$  or  $O(nm)$
  - Data structure:
- **Floyd-Warshall** to compute the shortest path from all nodes to all other nodes (neg weights ok)
  - Types of graphs: Weighted/directed or undirected (can have negative weights)
  - Inputs:  $G=(V,E)$
  - Access to: detect negative cycles by checking diagonals  $T[n,i,i]$
  - Outputs: shortest path from all vertices to all other vertices
  - Runtime:  $O(|V|^3)$  or  $O(n^3)$
  - Data structure:
- **SCCs** (outputs strongly connected components, and the metagraph of connected components.) (Create reverse graph =  $G^R$ , run DFS on  $G^R$ , find sink vertices of  $G$  by ordering by decreasing post #, run DFS again on this list, which returns  $\text{ccnum}$ ) finding the maximal set of SCC
  - Types of graphs: general directed graphs ([info](#))
  - Inputs:  $G(V,E)$
  - Access to: strongly connected components via  $\text{ccnum}(u)$  of first DFS run, and all other DFS outputs/structures
  - Outputs: metagraph that has to be a DAG (contains connected components from 2nd DFS run)
  - Runtime:  $O(|V|+|E|)$  or  $O(n+m)$
  - Data structure:
- **Kruskal's algorithm** to find a Minimum Spanning Tree (MST) (negative weights ok)
  - Types of graphs: connected, undirected, weighted graphs
  - Inputs: A connected undirected  $G=(V,E)$  with edge weights  $w_e$

- Access to:
  - Outputs: A minimum spanning tree defined by the edges  $X$
  - Runtime:  $O(|E| \log |V|)$  or  $O(m \log n)$
  - Data structure: disjoint-set
- **Prim's** algorithm to find a Minimum Spanning Tree (MST) [helpful site](#) (negative weights ok)
  - Types of graphs: connected, undirected, weighted graphs
  - Inputs: A connected undirected  $G=(V,E)$  with edge weights  $w_e$
  - Access to:
  - Outputs: A minimum spanning tree defined by the array `prev[]`
  - Runtime:  $O((|E| \log |V|))$  or  $O(m \log n)$  [runtime explanation](#)
  - Data structure: Binary heap
- **Ford-Fulkerson** greedy algorithm to find max flow on networks.
  - Types of graphs: directed graphs with capacity of edges
  - Inputs:  $G=(V,E)$  with flow capacity  $c$ , a source node  $s$ , and a sink node  $t$
  - Access to: Can trivially create the final residual network with  $G$ , and the outputted flow. Something along the lines of: [explanation](#)
    1. We run FF on the flow network to get the max flow.
    2. We use this to construct the residual graph.
  - Outputs: max flow
  - Runtime:  $O(C * |E|)$  or  $O(C*m)$ , where  $C$  is size of max flow
  - Data structure: linked list that stores edge capacity & queue to store augmenting paths
- **Edmonds-Karp** to find max flow on networks. (Identical to Ford-Fulkerson, except search order for finding augmenting path must be the shortest path (BFS for  $G$  with all edge weights = 1.) that has available capacity.)
  - Types of graphs: directed graphs with capacity of edges
  - Inputs:  $G=(V,E)$  with flow capacity  $c$ , a source node  $s$ , and a sink node  $t$
  - Access to: Can trivially create the final residual network with  $G$  and the outputted flow. Something along the lines of:
    1. We run EK on the flow network to get the max flow.
    2. We use this to construct the residual graph.
  - Outputs: max flow
  - Runtime:  $O(|V| |E|^2)$  or  $O(n m^2)$
  - Data structure:
- Key Difference: Ford-Fulkerson uses the DFS and Edmonds-Karp uses BFS
- **2-SAT** which takes a Conjunctive Normal Form (CNF) with all clauses of size  $\leq 2$  and returns a satisfying assignment if it exists. (uses SCC)
  - Types of graphs:

- Inputs: CNF  $f$
- Access to: truth assignment of variables
- Outputs: Boolean (T = satisfiable, F = unsatisfiable)
- Runtime:  $O(|V|+|E|)$  or  $O(n+m)$
- Data structure: