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Homework 4 – Well-Connected Graph

CS 6515: Introduction to Graduate Algorithms

Algorithm:

- To find if graph G is well-connected, we will use the strongly-connected component (SCC) algorithm.
- Pass graph G into the SCC algorithm, returning a metagraph DAG in topological order, with a source SCC vertex first and sink SCC vertex last. Call it G_meta.
- Next, examine G_meta:
    - If only one SCC vertex exists in the graph, return TRUE.
    - If more than one SCC vertex exists, from left to right, check if each SCC vertex has a path to the vertex adjacent to it (vertex to its immediate right). If so, return TRUE, otherwise return FALSE.

Correctness:

- By running the SCC algorithm for graph G, we can generate a metagraph DAG (G_meta) with SCCs as its vertices in topological order, with a source SCC first and sink SCC last. SCCs are only valid if there exists a path from u → v *and* v → u. So, we know the vertices that make up an SCC are already well-connected by its definition: a path from u → v *or* v → u.
- If only one SCC vertex exists in G_meta, we know graph G is an SCC and is well-connected, so we return TRUE.
- If there are multiple SCC vertices, we check if each SCC vertex in G_meta has a path to the vertex adjacent to it.
    - If so, return TRUE as there is a complete path from the source SCC vertex to the sink SCC vertex in G_meta. This means there exists a path from u → v *or* v → u for each pair of distinct u and v vertices in graph G, making it well-connected.
    - If at least one SCC vertex does not connect to the vertex adjacent to it, return FALSE as we know graph G is not well-connected since there is no complete path from the source SCC vertex to the sink SCC vertex in G_meta.

Runtime:

- Running the SCC algorithm as a blackbox takes O(n + m) time.
- Examining the SCC vertices within the metagraph takes O(n + m) time.
- Overall runtime is O(n + m).

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