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Homework 6

CS 6515: Introduction to Graduate Algorithms

1.) This problem is within an NP.

For the solution to problem B, we need to verify if the solution set of $k+1$ vertices in our induced subgraph is a star. This can be done by checking the adjacency list of graph G and confirming that every vertex except one has a size of 1, and only one vertex has a size of k. Going through the adjacency list is done in O(n) time, which is also in polynomial time. Thus, problem B is within an NP.

2.) Independent Set - > Star-Search

We can show a reduction from the independent set problem to the star-search problem. Given $G = (V, E)$ and k for the independent set problem, we first modify graph G by adding in a new vertex v' that connects to all other vertices within the graph. This new graph will be G' and adding vertex v' with edges connecting to every other vertex in the graph takes O(n + m) time. Next, we pass G' and k, into the star-search algorithm. If a star is returned (S), we can then confirm if its central vertex with a degree of k is the same as our new v' vertex. Once confirmed, we transform S by removing v' and should result in an independent set of vertices. This takes O(n) time to remove v' and O(1) time to return NO or the independent set. Finally, this means that there is a star within the induced graph IFF v' connects to every vertex in an independent set of at least size k.

References:

- https://en.wikipedia.org/wiki/Star_(graph_theory)

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