

- **SAT**
 - Input: boolean formula f in CNF with n -variables and m -clauses
 - Output: Satisfying assignment S where each clause evaluates to True. Returns 'NO' when no such S exists.
- **SAT Variations**
 - k -SAT is SAT with an additional input constraint k so that each clause has at most k literals. To be NP-C, $k > 2$
- **Independent-Set (Search)**
 - Input: Graph $G = (V, E)$ and goal g
 - Output: Subset S in V if there are **no edges** between every vertex in S and $|S| \geq g$. Returns 'NO' when no such S exists.
- **Clique (Search)**
 - Input: Undirected Graph $G = (V, E)$ and goal g
 - Output: subset S in V where there **are edges** between every pair of vertices in S and $|S| \geq g$. Returns 'NO' when no such S exists.
- **Vertex Cover (Search)**
 - Input: Undirected Graph $G = (V, E)$ and budget, b
 - Output: Subset S in V , if every $e=(x,y)$ has either x in S or y in S and $|S| \leq b$. Returns 'NO' when no such S exists.
- **K-Colorings**
 - Input: Undirected Graph $G=(V,E)$ & integer $k > 0$
 - Output: assign each vertex a color in $\{1, 2, \dots, k\}$ so that adjacent vertices get different colors and NO if no such k -coloring exists
 - *Note:* for $k=2$, K-Colorings $\in P$ because it can be solved with modified BFS/DFS to find a bipartite graph
- **Knapsack (Search)**
 - Input: n -items with integer weights: w_1, \dots, w_n ; integer values v_1, \dots, v_n ; capacity B ; and goal g
 - Output: subset S items where the total value V is $\geq g$ and total weight W is $\leq B$. Returns 'NO' when no such S exists.
- **Subset Sum (Search)**
 - Input: n positive integers a_1, \dots, a_n and a target, t
 - Output: Subset S of $[1, \dots, n]$ where the sum of a_i for all i in $S = t$, if such a subset exists. Returns 'NO' when no such S exists.

From Book:

Based on recommendations from course staff

(<https://edstem.org/us/courses/42665/discussion/3720834?answer=8572355>), we should focus on the problems presented in lecture; however, the examples from the book may still prove helpful.

- **Traveling Salesman**
 - Input:

- n vertices $\{1, 2, \dots, n\}$
 - $n(n-1)/2$ pairwise distances between each vertex
 - A budget b for total distance traveled
- Output: a path that visits every vertex with a total distance $\leq b$. Returns 'NO' when no such path exists.
- **Rudrata Cycle**
 - Input: $G = (V, E)$ (directed or undirected)
 - Output: a cycle c that visits every vertex exactly once. Returns 'NO' when no such S exists.
- **Balanced Cut**
 - Input: $G = (V, E)$ and a budget b ,
 - Output: partition the vertices into two sets S and T such that $|S|, |T| \geq n/3$ and such that there are at most b edges between S and T . Returns 'NO' when no such S exists.

Solution Verification Runtimes

Note: additional runtimes can be added to:

<https://edstem.org/us/courses/42665/discussion/3844271>

- **SAT**: $O(nm)$ to evaluate all literals within each clause
 - k -SAT: $O(m)$ assuming k is a constant
- **Independent Set** (Search): $O(n^2)$ to verify there are no edges between any vertices within S and $O(n)$ to check $|S| \geq g$
- **Clique** (Search): $O(n^2)$ to verify that there are edges between every vertex in S and $O(n)$ to check $|S| \geq g$
- **Vertex Cover** (Search): $O(n+m)$ to *TBD* and $O(n)$ to check $|S| \leq b$
- **K-Colorings**: For every edge, check that color assignment for each paired vertex is different $O(m)$ (or $O(n+m)$ to [be safe](#))
- **Knapsack** (Search): *TBD*
- **Subset Sum** (Search): *TBD*