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Homework 7 – 3-Sol-SAT

CS 6515: Introduction to Graduate Algorithms

NP Proof:

- We can verify the *3-Sol-SAT* problem by traversing the CNF input formula, *f*, and verifying that three distinct assignments of n-variables satisfy *f*.
    - This is done by traversing the literals within each m-clause in O(nm) time.
- Validating that three distinct assignments of *n*-variables exist takes O(n) time.
- Overall runtime is O(nm), which is also polynomial.

NP Complete Proof:

Reduction: SAT -> 3-Sol-SAT (3SolSAT)

I.T.:

- Given the CNF input formula, *f*.
- Create *f'* by adding two new variables, *x* and *y*, and two additional tautology clauses to *f*.
    - One of the new clauses will contain a true and false literal of *x*, and the other with *y*.
        - Example: *(f) ^ (x V !x) ^ (y V !y)*.
    - This takes O(1) time.
- Pass *f'* to 3SolSAT in O(1) time.
- Overall runtime is O(1), which is polynomial.

O.T.:

- Return NO, if 3SolSAT returns NO in O(1) time.
- Return the solution of 3SolSAT, dropping the newly added *x* and *y* variables, returning one of the distinct truth assignments for the original variables in *f*.
    - This takes O(n) time.
- Overall runtime is O(n), which is polynomial.

Correctness:

- If 3SolSAT returns a solution, we have a solution for SAT by dropping the newly added x and y variables, resulting in satisfiable assignments to f.
- If SAT returns a solution, we have a solution for 3SolSAT by adding two new variables, x and y, and two tautology clauses, $(x \vee !x)$ and $(y \vee !y)$, to f. Making it so 3SolSAT generates at least three distinct variable assignments from the new x and y variables alone.
- 3SolSAT has a satisfiable solution IFF the solution for SAT is satisfiable.

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