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Homework 6 – Flower Search

CS 6515: Introduction to Graduate Algorithms

NP Proof:

- Verify the solution, S, to the *Flower-Search* problem by first inducing the subgraph of S. This takes O(n + m) time.
- Validate that *N* vertices make up a clique by finding all vertices with a degree >= *N-1* and are connected to one another by an edge. This takes O(n^2) time.
- Validate that the remaining vertices make up a star by verifying that three vertices have a degree of one and are connected to a common central vertex. This takes O(n + m) time.
- Verify that there is a path from the central vertex in the star to a vertex in the clique. This takes O(n + m) time.
- Finally, verify $|S| = N + 4$ in O(n) time.
- Overall runtime is O(n^2), which is polynomial time.

NP Complete Proof:

Reduction: Clique -> Flower-Search (F.S.)

I.T.:

- Given *G=(V, E)* and *N*.
- A star consists of four vertices, featuring a central vertex, v', which connects to three other vertices that do not share an edge with each other.
- Create *G'* by adding a star to every vertex in *G*.
    - Each star will be added by an edge from v'.
    - This takes O(n^2) time.
- Pass *G'* and *N* into F.S. in O(1) time.
- Overall transformation time is O(n^2), which is polynomial.

O.T.:

- Return NO, if F.S. returns NO in O(1) time.

- Return solution of F.S., removing the four vertices that make up the star, leaving a clique.
    - To find the star, find the three vertices with a degree of one that are connected to a common vertex (the fourth vertex). These four vertices will be removed.
    - This takes O(n + m) time.
- Overall transformation time is O(n + m), which is polynomial.

Correctness:

- If Clique returns a solution, S, we have a solution for F.S. by adding a star, from its central vertex, to each vertex in S, resulting in N + 4 vertices that form a flower.
- If F.S. returns a solution, S, we have a solution for Clique by removing the star from S, resulting in an N-sized clique.
- F.S. has a solution IFF there is a star added, from its central vertex, to the Clique solution.

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