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```
% TTK4135 - Helicopter lab
% Hints/template for problem 2.
% Updated spring 2017, Andreas L. Fl?ten
```

## Initialization and model definition

```
code42; % NB: Change this to the init file corresponding to your
helicopter

global N mx

% Discrete time system model. x = [lambda r p p_dot e e_dot]'
A1 = A;
B1 = B;

% Number of states and inputs
mx = size(A1,2); % Number of states (number of columns in A)
mu = size(B1,2); % Number of inputs(number of columns in B)

% Initial values
x1_0 = pi; % Lambda
x2_0 = 0; % r
x3_0 = 0; % p
x4_0 = 0; % p_dot
x5_0 = 0;
x6_0 = 0;
x0 = [x1_0 x2_0 x3_0 x4_0 x5_0 x6_0]'; % Initial values

% Time horizon and initialization
N = 80; % Time horizon for states
M = N; % Time horizon for inputs
z = zeros(N*mx+M*mu,1); % Initialize z for the whole
horizon
z0 = z; % Initial value for
optimization

% Bounds
ul = -30*pi/180; % Lower bound on control -- ul
uu = 30*pi/180; % Upper bound on control -- ul
```

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```

xl      = -Inf*ones(mx,1);           % Lower bound on states (no
    bound)
xu      = Inf*ones(mx,1);           % Upper bound on states (no
    bound)
xl(3)   = ul;                       % Lower bound on state x3
xu(3)   = uu;                       % Upper bound on state x3
%%xl(4) = -0.1;                     % Lower bound on state x4
%%xu(4) = 0.1;                     % Upper bound on state x4

% Generate constraints on measurements and inputs
[vlb,vub] = hints_genbegr2(N,M,xl,xu,ul,uu); % hint:
    genbegr2
vlb(N*mx+M*mu) = 0;                 % We want the
    last input to be zero
vub(N*mx+M*mu) = 0;                 % We want the
    last input to be zero

% Generate the matrix Q and the vector c (objective function weights
    in the QP problem)
Q1 = zeros(mx,mx);
Q1(1,1) = 1;                       % Weight on state x1
Q1(2,2) = 0;                       % Weight on state x2
Q1(3,3) = 0;                       % Weight on state x3
Q1(4,4) = 0;                       % Weight on state x4
Q1(5,5) = 0;                       % Weight on state x5
Q1(6,6) = 0;                       % Weight on state x6
q1 = 1;
q2 = 1;
P1 = hints_blkdiag(q1,q2);         % Weight on input
Q = 2*hints_genq2(Q1,P1,N,M,mu);   % Generate Q
%%c = zeros(N*mx+M*mu,1);          % Generate c

```

## Generate system matrixes for linear model

```

Aeq = hints_gena2(A1,B1,N,mx,mu);   % Generate A, hint: gena2
beq = [A1*x0; zeros((N-1)*mx,1)];
%%beq = zeros(mx*N, 2);             % Generate b
%%beq(1:mx) = A1*x0;               % Initial value

%User input
%G = hints_blkdiag(Q,c);

```

## Solve QP problem with linear model

```

options = optimset('fmincon');
options.MaxFunEvals = 20000
options.Display = 'iter'
f = @(z) z'*Q*z;
tic
[z, fval] = fmincon(f,z0,[],[],Aeq,beq,vlb,vub,@func_constraint,
    options);
t1=toc;

```

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```

% Calculate objective value
phil = 0.0;
PhiOut = zeros(N*mx+M*mu,1);
for i=1:N*mx+M*mu
    phil=phil+Q(i,i)*z(i)*z(i);
    PhiOut(i) = phil;
end

options =

        Display: 'final'
    MaxFunEvals: 20000
        MaxIter: []
        TolFun: 1.0000e-06
        TolX: []
    FunValCheck: 'off'
    OutputFcn: []
    PlotFcns: []
ActiveConstrTol: []
    Algorithm: 'interior-point'
AlwaysHonorConstraints: 'bounds'
DerivativeCheck: 'off'
    Diagnostics: 'off'
    DiffMaxChange: Inf
    DiffMinChange: 0
    FinDiffRelStep: []
    FinDiffType: 'forward'
GoalsExactAchieve: []
    GradConstr: 'off'
    GradObj: 'off'
    HessFcn: []
    Hessian: []
    HessMult: []
    HessPattern: 'sparse(ones(numberofvariables))'
    HessUpdate: []
    InitialHesType: []
    InitialHessMatrix: []
    InitBarrierParam: 0.1000
InitTrustRegionRadius: 'sqrt(numberofvariables)'
    Jacobian: []
    JacobMult: []
    JacobPattern: []
    LargeScale: []
    MaxNodes: []
    MaxPCGIter: 'max(1,floor(numberofvariables/2))'
    MaxProjCGIter: '2*(numberofvariables-numberofequalities)'
    MaxSQPIter:
'10*max(numberofvariables,numberofinequalities...)'
    MaxTime: []
    MeritFunction: []
    MinAbsMax: []
    NoStopIfFlatInfeas: []
    ObjectiveLimit: -1.0000e+20

```

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```

PhaseOneTotalScaling: []
Preconditioner: []
PrecondBandWidth: 0
RelLineSrchBnd: []
RelLineSrchBndDuration: 1
ScaleProblem: 'none'
Simplex: []
SubproblemAlgorithm: 'ldl-factorization'
TolCon: 1.0000e-06
TolConSQP: 1.0000e-06
TolGradCon: []
TolPCG: 0.1000
TolProjCG: 0.0100
TolProjCGAbs: 1.0000e-10
TypicalX: 'ones(numberofvariables,1)'
UseParallel: 0

options =

    Display: 'iter'
    MaxFunEvals: 20000
    MaxIter: []
    TolFun: 1.0000e-06
    TolX: []
    FunValCheck: 'off'
    OutputFcn: []
    PlotFcns: []
    ActiveConstrTol: []
    Algorithm: 'interior-point'
    AlwaysHonorConstraints: 'bounds'
    DerivativeCheck: 'off'
    Diagnostics: 'off'
    DiffMaxChange: Inf
    DiffMinChange: 0
    FinDiffRelStep: []
    FinDiffType: 'forward'
    GoalsExactAchieve: []
    GradConstr: 'off'
    GradObj: 'off'
    HessFcn: []
    Hessian: []
    HessMult: []
    HessPattern: 'sparse(ones(numberofvariables))'
    HessUpdate: []
    InitialHessType: []
    InitialHessMatrix: []
    InitBarrierParam: 0.1000
    InitTrustRegionRadius: 'sqrt(numberofvariables)'
    Jacobian: []
    JacobMult: []
    JacobPattern: []
    LargeScale: []
    MaxNodes: []

```

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```

MaxPCGIter: 'max(1,floor(numberofvariables/2))'
MaxProjCGIter: '2*(numberofvariables-numberofequalities)'
MaxSQPIter:
'10*max(numberofvariables,numberofinequalities...)'
MaxTime: []
MeritFunction: []
MinAbsMax: []
NoStopIfFlatInfeas: []
ObjectiveLimit: -1.0000e+20
PhaseOneTotalScaling: []
Preconditioner: []
PrecondBandWidth: 0
RelLineSrchBnd: []
RelLineSrchBndDuration: 1
ScaleProblem: 'none'
Simplex: []
SubproblemAlgorithm: 'ldl-factorization'
TolCon: 1.0000e-06
TolConSQP: 1.0000e-06
TolGradCon: []
TolPCG: 0.1000
TolProjCG: 0.0100
TolProjCGAbs: 1.0000e-10
TypicalX: 'ones(numberofvariables,1)'
UseParallel: 0

```

Iter	F-count	$f(x)$	Feasibility	First-order optimality	Norm of step
0	641	0.000000e+00	3.142e+00	4.130e-08	
1	1290	6.094652e-01	2.592e+00	2.229e-02	5.845e-01
2	1935	5.259258e+00	1.552e+00	5.086e-02	1.173e+00
3	2579	6.137941e+00	1.494e+00	3.567e-01	4.432e-01
4	3220	1.458992e+01	1.277e+00	6.493e-01	1.797e+00
5	3861	2.022316e+01	1.189e+00	9.959e-01	7.250e-01
6	4503	2.023725e+01	1.189e+00	9.964e-01	1.683e-03
7	5144	2.025145e+01	1.189e+00	9.968e-01	1.706e-03
8	5785	2.597526e+01	1.083e+00	5.154e-01	1.069e+00
9	6426	4.598311e+01	8.437e-01	5.919e+00	3.155e+00
10	7067	4.436232e+01	7.832e-01	5.905e+00	2.490e+00
11	7708	4.324773e+01	7.661e-01	5.899e+00	1.145e+00
12	8349	4.307538e+01	7.487e-01	5.890e+00	9.732e-01
13	8990	4.658664e+01	7.063e-01	5.887e+00	1.504e+00
14	9631	4.662406e+01	7.060e-01	5.887e+00	1.588e-02
15	10272	4.743276e+01	7.029e-01	5.884e+00	3.403e-01
16	10913	4.743697e+01	7.029e-01	5.884e+00	1.739e-03
17	11554	5.237544e+01	6.207e-01	3.388e+00	1.095e+00
18	12195	5.084459e+01	6.178e-01	3.338e+00	8.373e-01
19	12836	4.926780e+01	6.152e-01	3.251e+00	1.252e+00
20	13477	5.019938e+01	6.139e-01	2.887e+00	1.343e+00
21	14118	4.860875e+01	6.132e-01	1.993e+00	8.129e-01
22	14759	4.754577e+01	6.128e-01	6.739e-01	4.657e-01
23	15400	4.704155e+01	6.126e-01	3.366e-01	2.683e-01
24	16041	4.632699e+01	6.125e-01	1.653e-01	3.133e-01
25	16682	4.576749e+01	6.124e-01	1.253e-01	2.962e-01

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26	17323	4.570847e+01	6.124e-01	1.097e-01	1.039e-01
27	17964	4.567254e+01	6.124e-01	1.086e-01	6.265e-02
28	18605	4.565835e+01	6.124e-01	1.046e-01	5.393e-02
29	19246	4.565295e+01	6.124e-01	1.019e-01	1.425e-02
30	19887	4.564947e+01	6.124e-01	1.008e-01	6.231e-03

Iter	F-count	$f(x)$	Feasibility	First-order optimality	Norm of step
31	20528	4.564811e+01	6.124e-01	1.004e-01	3.148e-03

*Solver stopped prematurely.*

*fmincon stopped because it exceeded the function evaluation limit, options.MaxFunctionEvaluations = 20000 (the selected value).*

## Extract control inputs and states

```

u1 = [z(N*mx+1:mu:N*mx+M*mu); z(N*mx+M*mu)-1]; % Control input from
solution
u2 = [z(N*mx+2:mu:N*mx+M*mu); z(N*mx+M*mu)];

x1 = [x0(1); z(1:mx:N*mx)]; % State x1 from solution
x2 = [x0(2); z(2:mx:N*mx)]; % State x2 from solution
x3 = [x0(3); z(3:mx:N*mx)]; % State x3 from solution
x4 = [x0(4); z(4:mx:N*mx)]; % State x4 from solution
x5 = [x0(5); z(5:mx:N*mx)]; % State x5 from solution
x6 = [x0(6); z(6:mx:N*mx)]; % State x6 from solution

num_variables = 10/Theta_t;
zero_padding = zeros(num_variables,1);
unit_padding = ones(num_variables,1);

u1 = [zero_padding; u1; zero_padding];
u2 = [zero_padding; u2; zero_padding];
x1 = [pi*unit_padding; x1; zero_padding];
x2 = [zero_padding; x2; zero_padding];
x3 = [zero_padding; x3; zero_padding];
x4 = [zero_padding; x4; zero_padding];
x5 = [zero_padding; x5; zero_padding];
x6 = [zero_padding; x6; zero_padding];

```

## Plotting

```

t = 0:Theta_t:Theta_t*(length(u1)-1);

figure(43)
subplot(811)
stairs(t,u1),grid
ylabel('u1')
subplot(812)
stairs(t,u2),grid

```

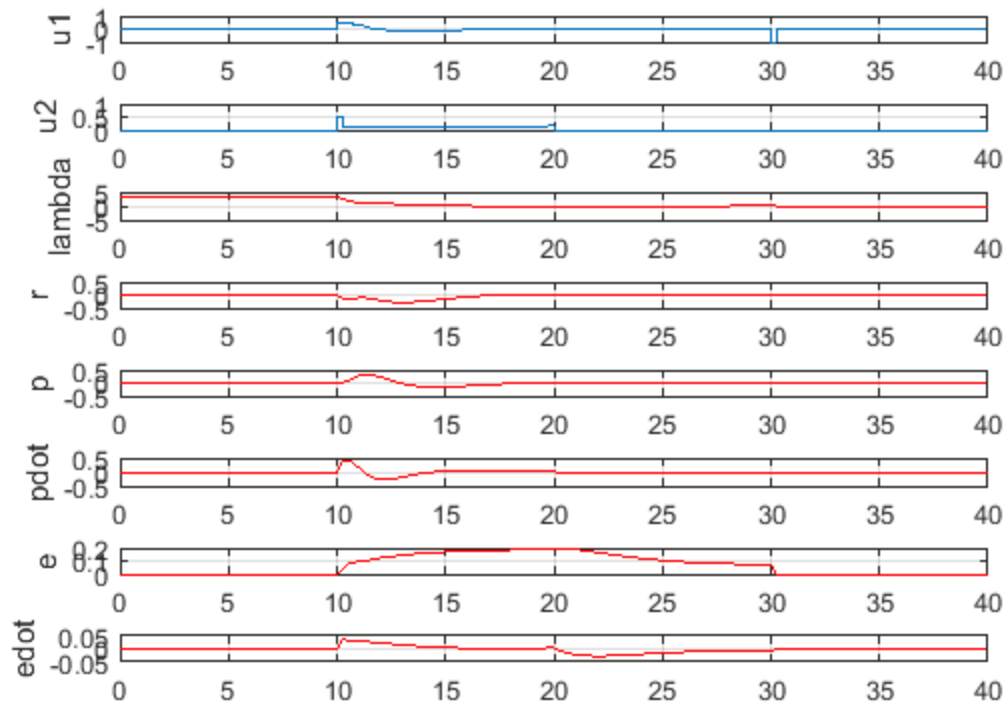
```

ylabel('u2')
subplot(813)
plot(t,x1,'m',t,x1,'r'),grid
ylabel('lambda')
subplot(814)
plot(t,x2,'m',t,x2', 'r'),grid
ylabel('r')
subplot(815)
plot(t,x3,'m',t,x3,'r'),grid
ylabel('p')
subplot(816)
plot(t,x4,'m',t,x4', 'r'),grid
ylabel('pdot')
subplot(817)
plot(t,x5,'m',t,x5', 'r'),grid
ylabel('e')
subplot(818)
plot(t,x6,'m',t,x6', 'r'),grid
ylabel('edot')
hold off

input = [t' u1 u2];
x_star = [t' x1 x2 x3 x4 x5 x6];

ut1 = [t' u1];
ut2 = [t' u2];

```



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# LQR

```
Q_LQR = [1 0 0 0 0 0; 0 1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 1 0 0; 0 0 0 0 1  
0; 0 0 0 0 0 1];  
R_LQR = [0.1 0; 0 0.1];  
K = dlqr(A, B, Q_LQR, R_LQR);
```

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