Electromagnetics 2FH4
MATLAB Set (8) – Spherical Electric Flux

Instructor: Dr. M.H Bakr

Department of Electrical and Computer Engineering
McMaster University

Matthew Jarzynowski – jarzynom – 400455803

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Problem

Exercise: Given the surface charge density $\varrho_s = 2.0\ \mu C/m^2$ existing in the region r = 1.0 m, $0 < \varphi < 2\pi$, $0 < \theta < \pi$ and is zero elsewhere (See Figure 8.2). Find analytically the energy stored in the region bounded by 2.0 m < r < 3.0 m, $0 < \varphi < 2\pi$ and $0 < \theta < \pi$.

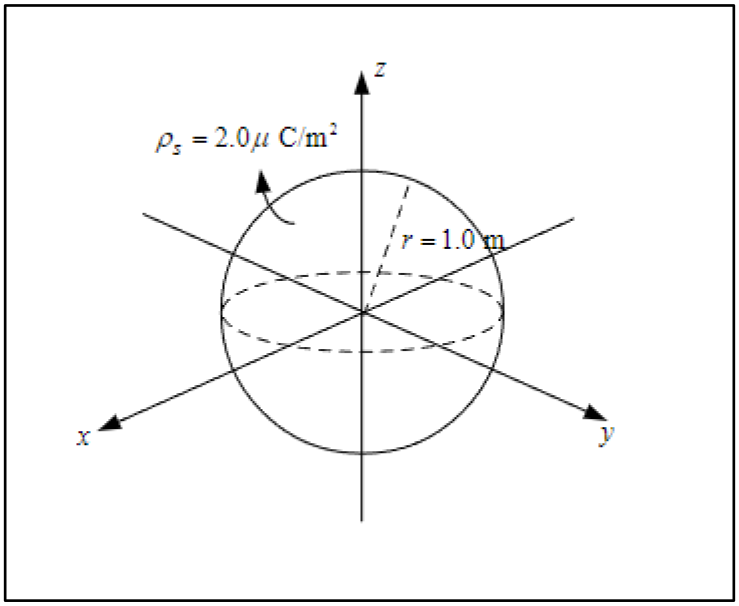


Figure 8.2 The surface charge density ϱ_s = 2 0 $\mu C/m^2$ at r = 1.0 m.

Solution

Consider the following derived from lab manual solution, in terms of MATLAB,

```
clc; % Clear the command bar
clear; % Remove all prior variables

% Bound Definitions
r_upper = 3;
r_lower = 2;
phi_upper = 2*pi;
phi_lower = 0;
theta_upper = pi;
theta_lower = 0;

% Discretization Steps
r_steps = 100;
phi_steps = 100;
theta_steps = 100;

% Differential Elements
dr = (r_upper - r_lower)/r_steps;
dphi = (phi_upper - phi_lower)/phi_steps;
dtheta = (theta_upper - theta_lower)/theta_steps;
```

Continued…

```
WE = 0; % Initial energy stored

% Constants
Eo = 8.85e-12;
D = 2.0e-6;


% Calculating the Relative Energy Stored (J)
for j=1:theta_steps
    for k=1:phi_steps
        for i=1:r_steps
            r = r_lower + dr/2+(i-1)*dr; % R, for current volume
            theta = theta_lower + dtheta/2+(i-1)*dtheta; % Theta, for current volume
            phi = phi_lower + dphi/2+(j-1)*dphi; % Phi, for current volume

            Emag = D/(Eo*(r*r)); % Relative magnitude

            dV = (r*r)*sin(theta)*dtheta*dphi*dr; % Volume of current element
            dWE = (1/2)*Eo*(Emag*Emag)*dV; % Energy stored in current element

            WE = WE +dWE; % Sum relevant contribution
        end
    end
end

fprintf('The energy stored is approx. equal to, %f J (Joules)\n', WE);
```

With the following "formatted output",

```
The energy stored is approx. equal to, 0.465022 J (Joules)
```

Where, W_E = 0.465022 J.

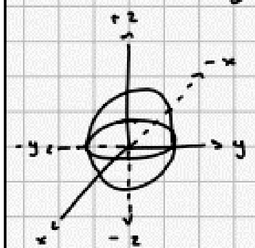
Continued…

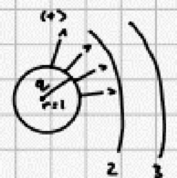
Also, consider the hand-articulated solution,

MATLAB (5) – Spherical "Flux" Inquiry

Consider, $\rho_s = 2.0\ nC/m^2$, $1\ nC = 1\times10^{-6}\ C$, with the bounds of $0 < \theta < \pi$, $0 < \phi\ 2\pi$, $2.0 < r < 3.0$

Note, that the existing sphere has a radius of 1.0 m



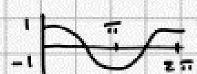


First, we need the "interior" charge

$Q_{enc} = \iiint \rho_s\, dS$, consider dS as $r^2 \sin\theta\, d\theta\, d\phi$, for a system in spherical coordinates

$$Q_{enc} = \int_0^{2\pi}\int_0^{\pi} (2.0\times10^{-6})(r^2 \sin\theta\, d\theta\, d\phi)\Big|_{r=1.0}$$

$$Q_{enc} = 2.0\times10^{-6}\int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi$$



$$Q_{enc} = 2.0\times10^{-6}\left(\int_0^{2\pi} d\phi + \int_0^{\pi} \sin\theta\, d\theta\right)$$

$$Q_{enc} = 2.0\times10^{-6}(2\pi)\left(-\cos\theta\Big|_0^{\pi}\right)$$

$$Q_{enc} = 2.0\times10^{-6}(2\pi)\big(1 - (-1)\big)$$

$$Q_{enc} = 4\pi\,(2.0\times10^{-6})$$

$$Q_{enc} = 8\pi\times10^{-6}\ C$$

Continued…

Continued…

Now, consider the "electric flux density", D

$$\oiint D \cdot dS = Q_{enc} = \iint \rho_s \cdot dS \quad , \quad dS = 4\pi r^2$$

$$D_r(4\pi r^2) = (8\pi \times 10^{-6})$$

$$D_r = \frac{8\cancel{\pi} \times 10^{-6}}{\cancel{4\pi} r^2} \text{ C/m}^2 \quad , \quad D_r = \frac{2 \times 10^{-6}}{r^2} \text{ C/m}^2 \quad , \quad D_r = \rho_s \left(\frac{1}{r^2}\right) \text{ (C/m)}$$

"In general"

Also, consider the field, $\bar{E}$

$$D = \epsilon_0 \bar{E} \quad , \quad \bar{E} = \frac{D}{\epsilon_0} \quad , \quad \epsilon_0 = \frac{1}{36\pi \times 10^{-m}}$$

$$\Rightarrow \bar{E} = \frac{2 \times 10^{-6}}{\epsilon r^2}$$

Finally, we can calculate the energy stored in the system, just the work, W_E, J

$$W_E = \frac{1}{2} \iiint_V \epsilon |E|^2 \, dV = \frac{1}{2} \iiint_V D \cdot \bar{E} \, dV \quad , \quad dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$W_E = \frac{1}{2} \int_0^{2\pi} \int_0^{\pi} \int_2^3 \left(\frac{2 \times 10^{-6}}{r^2}\right) \left(\frac{2 \times 10^{-6}}{\epsilon r^2}\right) (r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

$$W_E = \frac{1}{2} \int_0^{2\pi} \int_0^{\pi} \int_2^3 \left(\frac{4 \times 10^{-12}}{\cancel{\epsilon r^4}}\right) (\cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi)$$

$$W_E = \frac{1}{2} \int_0^{2\pi} \int_0^{\pi} \int_2^3 \left(\frac{4 \times 10^{-12}}{r^2}\right) (\sin\theta \, dr \, d\theta \, d\phi)$$

$$W_E = \frac{4 \times 10^{-12}}{2\epsilon_0} (2\pi)(2) \left(\int_2^3 \frac{1}{r^2} \, dr \right) \quad r^{-2} = r^{-1} = \frac{-1}{r} \Big|_2^3$$

$$W_E = \frac{4 \times 10^{-12}}{2\epsilon_0} (4\pi) \left(\frac{1}{6}\right) \qquad \frac{-1}{3} - \left(\frac{-1}{2}\right) \qquad \frac{-1}{3} + \frac{1}{2} = \frac{1}{6}.$$

$$W_E = \frac{16\pi \times 10^{-12}}{12\epsilon_0} = \frac{4\pi \times \cancel{10^{-12}}}{3\epsilon_0} \quad , \quad \epsilon_0 = 8.85 \times \cancel{10^{-12}}$$

$$\doteq \frac{4\pi}{3 \times 8.85}$$

$$\boxed{\Rightarrow \text{The energy stored in the "system" is} \approx 0.47 \text{ J}}$$

Which results in the same values as found in MATLAB, relatively speaking.