# Electromagnetics 2FH4
# MATLAB Set (2) – Surface and Volume Integrals


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MATLAB Set (2) – Surface and Volume Integrals

Problem

The surfaces r = 0 and r = 2, $\phi = 45^o$ , $\phi = 90^o$, $\theta = 45^o$ and $\theta = 90^o$ define a closed surface. Find the enclosed volume and the area of the closed surface S. Write a MATLAB program to verify your answer.

Solution

Consider the following MATLAB code "snippet",

```
clc;
clear;

% Initial volume, set to 0
volume = 0;

% Lower bounds of each integrand
rho_l = 0;
phi_l = (pi/4);
theta_l = (pi/4);

% Defining the accuracy of the relative integral
% 1000, is a good approx, although 1000 should be
% used due to floating point arithmetic

rho_steps = 1000;
phi_steps = 1000;
theta_steps = 1000;

% Relative increments, for each integrand
d_rho = ((2-0)/rho_steps);
d_phi = ((pi/2 - pi/4)/phi_steps);
d_theta = ((pi/2 - pi/4)/phi_steps);

% For loop to calculate the volume of the enclosed
% surface. Order of the for loops doesnt matter
% since each calculation is independent.
```

*Continued…*

```
for i=1:rho_steps
    for j=1:theta_steps
       for k=1:phi_steps

            % Adds volume contributions
            volume = volume + rho_l^2 * sin(theta_l) * d_rho * d_phi * d_theta;
        end

        % Increase theta each time phi is "traversed"
        theta_l = theta_l + d_theta;
    end

    %Reset theta to its lower bound, and increment rho
    % each time theta and phi have be "traversed"
    theta_l = (pi/4);
    rho_l = rho_l + d_rho;
end

% Reset rho to its lower bound
rho_l = 0;

volume

% Surface Area Calculation

% The only surfaces, out of the 5 that we care about is S1
% the surface that is on the curved face, since surfaces S(2 - 5),
% are tangential to the "fluxial field array lines"

% Since we are calculating the volume of a sphere
% from its origin, there are 5 surfaces.

% Surface Area, each surface set to 0
surf1 = 0;
surf2 = 0;
surf3 = 0;
surf4 = 0;
surf5 = 0;

rho_u = 2; % Upper rho bound

% Calculating the surface area for surfaces (1 - 2), same
for i=1:rho_steps
    for j=1:theta_steps
        surf1 = surf1 + (rho_l * d_theta * d_rho);
    end

    % Increase rho each "traversal"
    rho_l = rho_l + d_rho;
end

rho_l = 0; % Reset to 0, lower bound
surf2 = surf1; % Consider them equal
```

*Continued…*

```
rho_u = 2; % Define the upper bound of rho

% Calculating the surface area for surface 3, unique
for i=1:theta_steps
    for j=1:phi_steps
        surf3 = surf3 + (((rho_u)^2) * sin(theta_l) * d_theta * d_phi);
    end
    theta_l = theta_l + d_theta;
end

theta_l = (pi/4); % Redefine thetas lower bound
surf3;

theta_l = (pi/4);
theta_u = (pi/2);

% Calculating the surface area for surfaces (4 - 5), same
for i=1:rho_steps
    for j=1:phi_steps
        surf4 = surf4 + (sin(theta_u) * d_rho * d_phi);
        surf5 = surf5 + (sin(theta_l) * d_rho * d_phi);
    end
    rho_l = rho_l + d_rho;
end

surfaceArea = surf1 + surf2 + surf3 + surf4 +surf5; % Final sum

surfaceArea
```

With the following output,

```
volume =

    1.4785


surfaceArea =

    8.0410
```

Also, consider this hand-calculated solution,

**MATLAB Set 2**

Consider an enclosed sphere, where,

**"Relativistic volume"**

$r = [0,2]$ , $dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\phi = [\frac{\pi}{4}, \frac{\pi}{2}]$

$\theta = [\frac{\pi}{4}, \frac{\pi}{2}]$

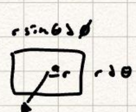
$$V = \int_0^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$V = \int_0^2 r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta\, d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi \rightarrow V = \left(\frac{r^3}{3}\Big|_0^2\right)\left(-\cos(\theta)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right)\left(\theta\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right)$$

$\boxed{V \doteq 1.48096\ m^3 \text{, relatively close to MATLAB}}$

**"Relativistic surface-area"**

$S_1$




$dS_1 = r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r$

Consider the differential element a "wedge" with 5 surfaces

$S_1, S_2, S_3, S_4, S_5$

$$S_1 = \iint_S dS_1$$

$$S_1 = \iint_S r^2 \sin\theta\, d\phi$$

$$S_1 = r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta\, d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi$$

$$S_1 = r^2\Big|_0^2 \left(-\cos(\theta)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right)\left(\theta\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right)$$

$S_1 \doteq 2.2214\ m^2$

$S_2 = r\, d\theta\, dr$

$$= \int_0^2 r\, dr \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta$$

$$= \frac{r^2}{2}\Big|_0^2 \left(\frac{\pi}{4}\right) = \frac{2\pi}{4}$$

$S_{2,3} \doteq \pi\ m^2$ , for S2 and S3

$S_4 = dr\, r\sin\theta\, d\phi$

$$S_4 = \sin\left(\frac{\pi}{2}\right)\Big|_0^2 r\, dr \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi$$

$$S_4 = \frac{r^2}{2}\Big|_0^2 \left(\frac{\pi}{4}\right)$$

$S_4 \doteq 1.57$

$S_5$, similar to $S_4$, $\left(\frac{r^2}{2}\right)\Big|_0^2\left(\frac{\pi}{2}\right)$

$S_5 = 1.107\ m^2$

$\Rightarrow S_A = S_1 + S_2 + S_3 + S_4 + S_5$

$S_A \doteq 8.045\ m^2$, relatively close to MATLAB

Which results in the same values as found in MATLAB, relatively speaking.