Electromagnetics 2FH4
MATLAB Set (3) – Electric Fields in Relation to Uniform Linear Charges

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MATLAB Set (2) – Electric Fields in Relation to Uniform Linear Charges

Problem

An infinite uniform linear charge ρ_L = 2.0 nC/m lies along the x axis in free space, while point charges of 8.0 nC each are located at (0, 0, 1) and (0, 0, -1). Find E at (2, 3, 4). Write a MATLAB program to verify your answer

Solution

Consider this MATLAB code "snippet",

```
% Matlab Set 3 - Field-Based Charge Density
% Matthew Jarzynowski

clc % Clear command bar
clear % Remove prior variables

% Consider this, "variable definition"

% Charges, Q1, Q2, in nC
Q1 = 8e-9;
Q2 = 8e-9;

pL = 4e-9; % Linear charge density
Eo = 8.8419e-12; % Permittivity of free space

P = [0 0 0]; % Observation point
A = [0 1 1]; % Q1 coordinates
B = [0 -1 1]; % Q2 coordinates

C = [3.5 3.5 0]; % Coordinates of the line charges centre, midpoint

stepL = 100000; % Step size of L

% Vector Manipulation

R1 = (P - A); % Vector from Q1 to observation point
R2 = (P - B); % Vector from Q2 to observation point

R1m = norm(R1); % R1 vector magnitude
R2m = norm(R2); % R2 vector magnitude

% Electric Field Calculation

E1 = Q1/(4*pi*Eo*R1m^3)*R1; % Field by Q1
E2 = Q2/(4*pi*Eo*R2m^3)*R2; % Field by Q2

D = norm(P - C); % Distance from observation to midpoint on line
L = sqrt(98)*D; % Length of the line, (m)
```

Continued…

```
length = sqrt(98); % Relative length
dir_vec = [-7/sqrt(98) 7/sqrt(98) 0]; % Direction vector

dL = length/stepL; % Length of a segment
dL_Vector = dL*dir_vec; % A vector of a segment

% Consider changing to a vector pointing in the direction of the line

EL = [0 0 0]; % Initialize the field of the line segment

% Perpendicular centre of a segment
C_Start = C - length/2 * dir_vec;
C_Segment = C_Start;

for i=1: stepL

    % A vector from the observation point, P, to the centre
    % of the linearly charged line
    R = P - C_Segment;
    Rm = norm(R); % R vector magnitude

    EL = EL +dL * pL/(4*pi*Eo*(Rm)^3)*R; % Each segments contribution

    % Relative centre of the "i-th" segment
    C_Segment = C_Segment + dL_Vector;
end

ET = E1 + E2 + EL; % Sum each electric field contribution

ET
```

With the following output,

```
ET =

   -7.2732   -7.2731  -50.9119
```

Continued…

Also, consider this hand-calculated solution,

Matlab 3 - Electric Fields in Relation to Uniform Linear Charges

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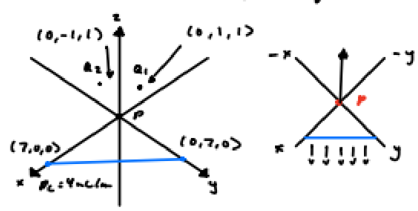
Problem

A "finite" uniform linear charge, $\rho_L = 4\,nC/m$, is on the xy-plane. 2 charges, $Q_{1,2}$, each "charged" with 8 nC, are placed at (0, 1, 1) and (0, -1, 1) respectively.

Find $\underline{E}$ at (0, 0, 0), the origin.



Consider the "analytical" solution,

$$\underline{E}_P = \frac{Q}{4\pi\varepsilon_0|\Delta r|^3}\Delta r \quad , \quad \underline{E}_T = \underline{E}_1 + \underline{E}_2 + \underline{E}_L$$

$$1\,nC = 1 \times 10^{-9}\,C$$

$Q_1\,(0, 1, 1)$

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$$

$$\Delta r_1 = \underline{P} - \underline{Q}_1$$
$$\Delta r_1 = (0, 0, 0) - (0, 1, 1)$$
$$\Delta r_1 = (0, -1, -1)$$

$$|\Delta r_1|^3 = \left(\sqrt{(0)^2 + (-1)^2 + (-1)^2}\right)^3$$
$$|\Delta r_1|^3 = (\sqrt{2})^3 \quad , \quad \sqrt{x} = x^{\frac{1}{2}} \quad (\sqrt{x})^3 = x^{\frac{3}{2}}$$

$$\underline{E}_1 = \frac{8.0 \times \cancel{10^{-9}}}{4\cancel{\pi}\left(\frac{1}{36\cancel{\pi}} \times \cancel{10^{-9}}\right)(\sqrt{2})^3}[0, -1, -1]$$

$$\underline{E}_1 = \frac{8}{\frac{(\sqrt{2})^3}{9}}[0, -1, -1]$$

$$\underline{E}_1 = (25.46)[0, -1, -1]$$
$$\underline{E}_1 = [0, -25.46, -25.46]$$

Continued…

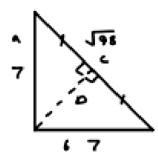
$Q_2(0, -1, 1)$

$\underline{a}_{r_2} = \underline{P} - \underline{Q}_2$
$\underline{a}_{r_2} = (0, 0, 0) - (0, -1, 1)$
$\underline{a}_{r_2} = (0, 1, -1)$

$$\underline{E}_2 = \frac{1.0 \times 10^{-9}}{4\pi\left(\frac{1}{36\pi} \times 10^{-9}\right)(\sqrt{2})^3}[0, 1, -1]$$

$\underline{E}_2 = [0, 25.46, -25.46]$

E_L (The line itself)



$c^2 = a^2 + b^2$
$c = \sqrt{(7)^2 + (7)^2}$
$c = \sqrt{98}$

$h = \frac{\sqrt{98}}{2} = \rho$

$$|E_L| = \frac{\rho_L}{4\pi\varepsilon_0\rho}$$

$$\underline{E}_L = \frac{4.0 \times \cancel{10^{-9}}}{4\pi\left(\frac{1}{36\pi} \times \cancel{10^{-9}}\right)\left(\frac{\sqrt{98}}{2}\right)}[-1, -1, 0]$$

$$\underline{E}_L = \frac{4}{\frac{1}{9}\left(\frac{\sqrt{98}}{2}\right)}[-1, -1, 0]$$

$$\underline{E}_L = \frac{4}{\frac{\sqrt{98}}{18}}[-1, -1, 0]$$

$\underline{E}_L = (7.273)[-1, -1, 0]$

$\underline{E}_T = \underline{E}_1 + \underline{E}_2 + \underline{E}_L$, $\underline{E}_1 = [0, \cancel{-25}.46, -25.46]$
$\underline{E}_2 = [0, \cancel{25}.46, -25.46]$
$\underline{E}_L = [-7.273, -7.273, 0]$

$\boxed{\underline{E}_T = [-7.273, -7.273, -50.92]}$
$\underline{a}_x = -7.273$, $\underline{a}_y = -7.273$, $\underline{a}_z = -50.92$
relatively close to matlab output.

Which results in the same values as found in MATLAB, relatively speaking.