Electromagnetics 2FH4
MATLAB Set (3) – Disc-Based Electric Fields

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MATLAB Set (3) – Disc-Based Electric Fields

Problem

Given the surface charge density, ρ_s =2.0 μC/m^2 , existing in the region ρ <1.0 m, z = 0, and zero elsewhere, find E at P (ρ = 0, z = 1.0) and write a MATLAB program to verify your answer.

Solution

Consider the following derived from lab manual solution, in terms of MATLAB,

```
% Matlab Set 3 - Disc-Based Electric Fields
% Matthew Jarzynowski

clc; % Clear the command line
clear; % Clear previous variables

Eo = 8.854e-12; % Premetivity constant, with respect to air
S = 2e-6; % Disc-surface charge density

P = [0 0 1]; % Observation point
E = [0 0 0]; % Initial electric field, 0 in all components

% Step sizes for relavent "integrals"
rho_steps = 5000;
phi_steps = 5000;

% Defining our bounds.
rho_L = 0;
rho_U = 1;

phi_L = 0;
phi_U = (2*pi);

% Relavent infinitesimally small dimension
d_rho = (rho_U - rho_L)/rho_steps;
d_phi = (phi_U - phi_L)/phi_steps;

ds = d_rho * d_phi; % Relative area of a single element
dQ = S * ds; % The charge on a single element
```

Continued…

```
% Double integration, using "for loops"
for j=1:rho_steps
    for i=1:phi_steps

        rho = rho_L + d_rho/2+(i-1)*d_rho; % The rho component of an element
        phi = phi_L + d_phi/2+(j-1)*d_phi; % The phi component of an element

        % Direction vector to observation point
        R = P - [rho*cos(phi) rho*sin(phi) 0];

        Rm = norm(R); % Direction vectors magnitude

        % Relative electric field contribution
        E = E + (rho*dQ/(4 * Eo * pi * Rm^3))*R;
    end
end

E
```

With the following output,

```
E =

   1.0e+04 *

   -0.0000   -0.0000    3.3080
```

Where it can be expressed as, E = [0, 0, 3.31×10^4].

Continued…

Also, consider the hand-articulated solution,

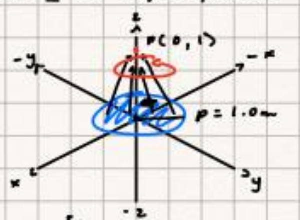
Matlab 4 - Disc-Based Electric Fields

Matthew Jaregnowski

Problem

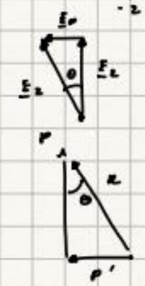
Given the surface charge density, $\rho_s = 2.0\ nC/m^2$ ($\rightarrow 2 \times 10^{-6}$), on the region, $\rho < 1.0m$, $z = 0$

Find $\underline{E}$ at $P(\rho = 0, z = 1)$



Consider, due to symmetry, the horizontal electric (x, y) components will cancel out, thus only the vertical components will sum $\underline{E}_T$.

$$dQ = \rho_s \cdot \rho\, d\rho\, d\varphi$$



$E_z = E\cos\theta$, all horizontal components will cancel out.

$$dE_z = dE\cos\theta$$

$$R = \sqrt{(\rho')^2 + (1)^2}$$

$$\cos\theta = \frac{1}{\sqrt{(\rho')^2 + (1)^2}}$$

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$$

=> For any point, on the disc, $\underline{E}$, is,

$$dE_z = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 |R|^2} \times \cos\theta = \frac{\rho_s \times \rho\, d\rho\, d\varphi}{4\pi\varepsilon_0\left(\sqrt{(\rho')^2 + (1)^2}\right)^2} \cdot \frac{1}{\sqrt{(\rho')^2 + (1)^2}},$$

$$\Rightarrow dE_z = \frac{\rho_s \rho'}{4\pi\varepsilon_0 \sqrt{(\rho')^2 + (1)^2}^{\,3}}\, d\rho'\, d\theta'$$

∴ For all the points, consider a double integral, where,

$$\underline{E}_T = \int_0^{2\pi}\int_0^1 \frac{\rho_s \rho'}{4\pi\varepsilon_0 \sqrt{(\rho')^2 + (1)^2}^{\,3}}\, d\rho'\, d\theta'$$

$$\underline{E}_T = \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \int_0^1 \frac{\rho'}{\sqrt{(\rho')^2 + (1)^2}^{\,3}}\, d\rho'$$

$$\underline{E}_T = \frac{2.0 \times 10^{-6}}{\cancel{4\pi}\left(\frac{1}{36\pi} \times 10^{-9}\right)}(\cancel{2\pi})\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \underline{E}_T = \left[0,\ 0,\ \frac{2}{\frac{1}{18\pi} \times 10^3} - \frac{1}{\frac{\sqrt{2}}{18\pi} \times 10^3}\right]$$

$$\boxed{\underline{E}_T = [0,\ 0,\ 3.3125 \times 10^4]\ V/m}$$

Which results in the same values as found in MATLAB, relatively speaking.