Biot MMMFE.

Manu Jayadharan Ivan Yotov

August 24, 2019

1 Introduction

2 Formulation of the methods

2.1 Mathematical formulation of model Problem.

2.2 Formulation of monolithic MMMFE method.

Let $\Omega = \cup_{i=1}^{n} \Omega_i$ be a union of non-overlapping shape regular polygonal subdomains. Let $\Gamma_{i,j} = \partial\Omega_i \cap \partial\Omega_j$, $\Gamma = \cup_{i,j=1}^{n} \Gamma_{i,j}$, and $\Gamma_i = \partial\Omega_i \cap \Gamma = \partial\Omega_i \setminus \partial\Omega$ denote the interior subdomain interfaces. The domain descretization technique we develop here is generalization of the techniques derived in [give reference to the other paper] to a general case where the sub-domains have non-matching grids with different scales of refinement. We make use of relatively coarser mortar finite elements of our choice satisfying certain conditions at the interface of sub-domains. Let h_i be the diameter of maximal element in the mesh on Ω_i and $h = \max_i h_i$. Talk about the computationally cheap and non-matching grid advantages this method has over the previous paper in the introduction.

For $1 \leq i \leq n$, let $X_{h,i} \times V_{h,i} \times W_{h,i} \times Z_{h,i} \times W_{h,i}$ be a collection of MFE families defined on the subdomain Ω_i. We assume that $X_{h,i}, V_{h,i}, W_{h,i}, Z_{h,i}$ and W contain polynomials of degree less than or equal to $k \geq 1, l \geq 0, q \geq 0, r \geq 1$ and $s \geq 0$ respectively. Let us define

$$X_h = \bigoplus_{1\leq i\leq n} X_{h,i}, \quad V_h = \bigoplus_{1\leq i\leq n} V_{h,i}, \quad W_h = \bigoplus_{1\leq i\leq n} W_{h,i} \quad Z_h = \bigoplus_{1\leq i\leq n} Z_{h,i}, \quad W_h = \bigoplus_{1\leq i\leq n} W_{h,i}.$$

Note that the definition of the global spaces X_h and Z_h do not impose continuity of normal component of stress or velocity across the sub-domain interfaces. This issue is taken care of using Lagrange multipliers defined on compatible mortar spaces on the interface.

References