Convergence of solution for test case: Monolithic DD with mortars of different degree

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1. Physical parameters used: double c0=1;
   double alpha=1;
   int num_cycle=4;
   int max_itr=500;
   double tolerence = 1.e-12;
   BiotParameters bparam (0.001,2,c0,alpha);
2. $\Delta t = 10^{-3}$, final_step= $2 \times \Delta t$.
3. Tests were done using coupled monolithic scheme for five variable quasi-static Biot system with weakly imposed symmetry.
4. Number of subdomains = 4.
5. Let h_m, h_1, h_2 be the mesh size of the mortar space, $\{\Omega_1, \Omega_3\}$ and $\{\Omega_2, \Omega_4\}$ respectively. Then $h_m : h_1 : h_2 = 1 : 1/4 : 1/9$.
6. FE space used: $\Lambda_u \times \Lambda_p = \mathcal{RT}_m^2 \cdot n \times \mathcal{RT}_m \cdot n$, where m is the mortar degree.
7. As expected, the accuracy seems to be increasing with increase in degree of mortar space, m.

1 Convergence table

Table 1: Linear, $m = 1$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$	
0	22	-	1.349e+00	-	5.999e-02	-	5.849e-01	-	5.778e-01	-
1	40	-0.86	5.561e-01	1.28	2.970e-02	1.01	2.883e-01	1.02	2.915e-01	0.99
2	75	-0.91	1.616e-01	1.78	1.479e-02	1.01	1.425e-01	1.02	1.460e-01	1.00
3	145	-0.95	4.048e-02	2.00	7.394e-03	1.00	7.078e-02	1.01	7.306e-02	1.00

Table 2: Quadratic, $m = 2$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$	
0	32	-	6.490e-01	-	5.923e-02	-	5.508e-01	-	5.786e-01	-
1	57	-0.83	1.971e-01	1.72	2.958e-02	1.00	2.796e-01	0.98	2.916e-01	0.99
2	108	-0.92	5.302e-02	1.89	1.479e-02	1.00	1.403e-01	0.99	1.461e-01	1.00
3	201	-0.90	1.480e-02	1.84	7.395e-03	1.00	7.023e-02	1.00	7.306e-02	1.00

Table 3: Cubic, $m = 3$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$	
0	42	-	6.332e-01	-	5.919e-02	-	5.513e-01	-	5.785e-01	-
1	76	-0.86	5.289e-02	3.58	1.479e-02	2.00	1.404e-01	1.97	1.461e-01	1.99
2	149	-0.97	4.214e-03	3.65	3.697e-03	2.00	3.512e-02	2.00	3.653e-02	2.00