Biot DD with mortar: convergence tables and stability

August 14, 2019

1 Progress from previous report.

1. Fixed the code to give out correct convergence rate for solution variables: the convergence rates seems to be in match with the case of Biot DD without mortars.
2. Added the multi-scale basis functionality for all mortar orders.
3. Added a function to the code to calculate the interface error in a special norm: this is similar to what eldar has done for Elasticity case in his paper, but three different norms are added now. Please check the last three columns of the convergence tables. Still not sure whether the $\|p - \lambda\|$ error makes sense, since in the initial paper about mortars, it seems to be 2.
4. Debugged the code to fix some multiple threads overwriting memory bug. As a result the code will run on all machines with dealii installed without crashing.
5. Changed the dealii compilation mode to "Release" from debug, so it now runs about 5-10x times faster than before.
6. Added the Biot DD as a repository on git. Now different kind of modifications to the repository is done on different branches on git. Version control is taken care of.

2 Example 1: Testing convergence rate

1. Physical parameters used: double c0=1;

   double alpha=1;

   int num_cycle=5;

   int max_itr=500;

double tolerence = 1.e-12;

BiotParameters bparam (0.001,5,c0,alpha);

2. $\Delta t = 10^{-3}$, final_step= $5 \times \Delta t$.
3. Tests were done using coupled monolithic scheme for five variable quasi-static Biot system with weakly imposed symmetry.
4. Number of subdomains = 4.
5. Let h_m, h_1, h_2 be the mesh size of the mortar space, $\{\Omega_1, \Omega_3\}$ and $\{\Omega_2, \Omega_4\}$ respectively. Then $h_m : h_1 : h_2 = 1 : 1/4 : 1/9$.
6. FE space used: $\Lambda_u \times \Lambda_p = \mathcal{RT}_m^2 \cdot n \times \mathcal{RT}_m \cdot n$, where m is the mortar degree.
7. As expected, the accuracy seems to be increasing with increase in degree of mortar space, m.

3 Example 2: Testing stability.

1. In this section, we have run the test case with the same physical parameters as in example 1, but with final time = $50 \times \Delta t$.

Table 1: Convergence rate for Example 1.

Table 2: Quadratic, $m = 2$

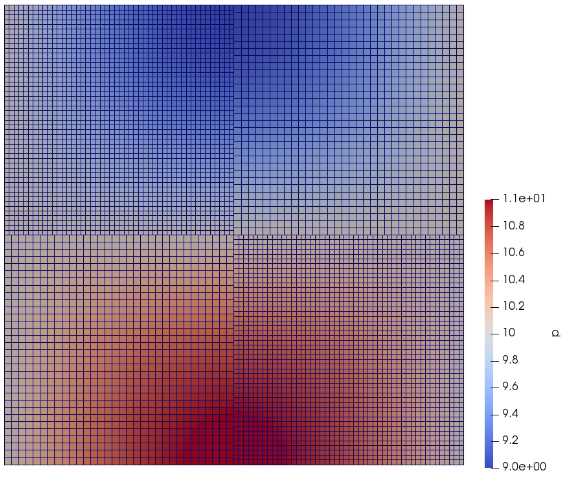
cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
0	32	-	2.123e+00	-	5.923e-02	-	5.783e-01	-	5.786e-01	-	1.436e-02	-	1.204e-02	-	1.300e-02	-
1	57	-0.83	1.059e+00	1.00	2.958e-02	1.00	2.942e-01	0.97	2.916e-01	0.99	3.583e-03	2.00	6.497e-03	0.89	5.685e-03	1.19
2	108	-0.92	4.634e-01	1.19	1.479e-02	1.00	1.478e-01	0.99	1.461e-01	1.00	9.277e-04	1.95	3.339e-03	0.96	2.801e-03	1.02
3	201	-0.90	1.931e-01	1.26	7.395e-03	1.00	7.396e-02	1.00	7.306e-02	1.00	2.573e-04	1.85	1.703e-03	0.97	1.404e-03	1.00
4	363	-0.85	8.605e-02	1.17	3.697e-03	1.00	3.699e-02	1.00	3.653e-02	1.00	9.167e-05	1.49	9.120e-04	0.90	7.408e-04	0.92

Table 3: Quadratic, $m = 2$

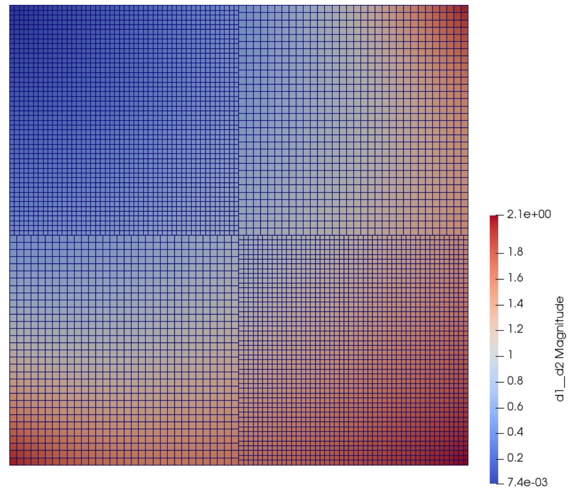
cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
0	32	-	2.123e+00	-	5.923e-02	-	5.783e-01	-	5.786e-01	-	1.436e-02	-	1.204e-02	-	1.300e-02	-
1	57	-0.83	1.059e+00	1.00	2.958e-02	1.00	2.942e-01	0.97	2.916e-01	0.99	3.583e-03	2.00	6.497e-03	0.89	5.685e-03	1.19
2	108	-0.92	4.634e-01	1.19	1.479e-02	1.00	1.478e-01	0.99	1.461e-01	1.00	9.277e-04	1.95	3.339e-03	0.96	2.801e-03	1.02
3	201	-0.90	1.931e-01	1.26	7.395e-03	1.00	7.396e-02	1.00	7.306e-02	1.00	2.573e-04	1.85	1.703e-03	0.97	1.404e-03	1.00
4	363	-0.85	8.605e-02	1.17	3.697e-03	1.00	3.699e-02	1.00	3.653e-02	1.00	9.167e-05	1.49	9.120e-04	0.90	7.408e-04	0.92

Table 4: Cubic, $m = 3$

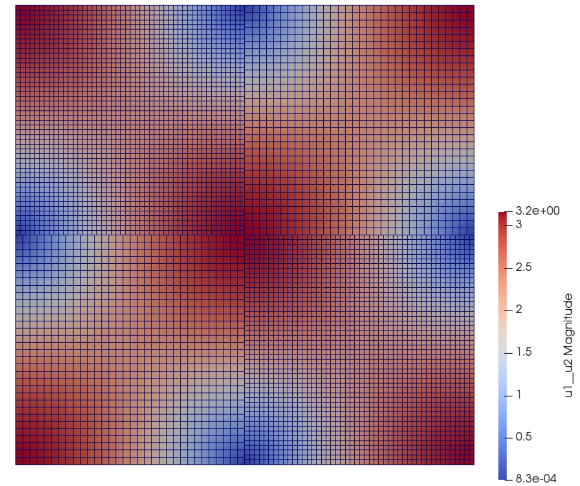
cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
0	42	-	2.102e+00	-	5.919e-02	-	5.784e-01	-	5.785e-01	-	1.491e-02	-	1.198e-02	-	1.318e-02	-
1	73	-0.80	1.060e+00	0.99	2.958e-02	1.00	2.943e-01	0.98	2.916e-01	0.99	3.729e-03	2.00	6.423e-03	0.90	5.669e-03	1.22
2	116	-0.67	4.568e-01	1.21	1.479e-02	1.00	1.478e-01	0.99	1.461e-01	1.00	9.701e-04	1.94	3.260e-03	0.98	2.745e-03	1.05
3	207	-0.84	1.881e-01	1.28	7.394e-03	1.00	7.396e-02	1.00	7.306e-02	1.00	2.629e-04	1.88	1.661e-03	0.97	1.371e-03	1.00
4	371	-0.84	8.338e-02	1.17	3.697e-03	1.00	3.699e-02	1.00	3.653e-02	1.00	9.007e-05	1.55	8.768e-04	0.92	7.131e-04	0.94



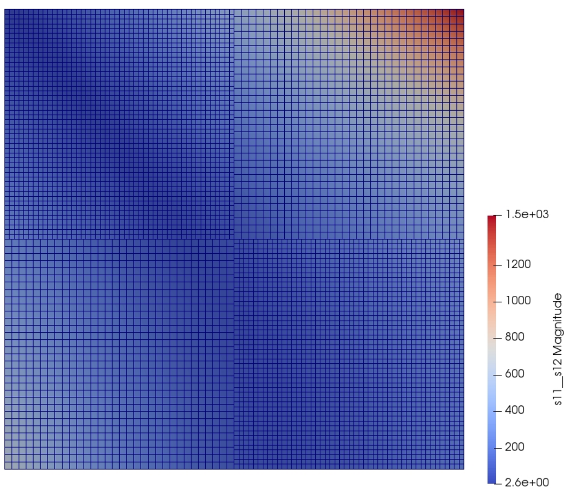
(a) pressure



(b) displacement



(c) Velocity



(d) Stress X magnitude

Figure 1: Example 1: Solution with cubic mortars at final time step and refinement

Table 5: Example 2, Stability check $50 \times \Delta t$: Error table.

(a) Linear, $m = 1$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
0	40	-	3.325e+00	-	2.970e-02	-	2.981e-01	-	2.915e-01	-	4.780e-03	-	1.037e-02	-	8.806e-03	-
1	75	-0.91	1.831e+00	0.86	1.479e-02	1.01	1.487e-01	1.00	1.460e-01	1.00	1.235e-03	1.95	5.355e-03	0.95	4.418e-03	0.99

(b) Quadratic, $m = 2$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
1	57	-	1.059e+00	-	2.958e-02	-	2.942e-01	-	2.916e-01	-	3.586e-03	-	7.404e-03	-	6.378e-03	-
2	108	-0.92	4.634e-01	1.19	1.479e-02	1.00	1.478e-01	0.99	1.461e-01	1.00	9.283e-04	1.95	3.865e-03	0.94	3.227e-03	0.98

(c) Cubic, $m = 3$

cycle	# gmres		$\|z - z_h\|_{L^\infty(H_{div})}$		$\|p - p_h\|_{L^\infty(L^2)}$		$\|\sigma - \sigma_h\|_{L^\infty(H_{div})}$		$\|u - u_h\|_{L^\infty(L^2)}$		$\|u - \lambda_{uH}\|_{d_H}$		$\|p - \lambda_{pH}\|_{d_H}$		$\|(u,p) - \lambda_H\|_{d_H}$	
0	73	-	1.060e+00	-	2.958e-02	-	2.943e-01	-	2.916e-01	-	3.732e-03	-	7.139e-03	-	6.214e-03	-
1	116	-0.67	4.568e-01	1.21	1.479e-02	1.00	1.478e-01	0.99	1.461e-01	1.00	9.707e-04	1.94	3.716e-03	0.94	3.114e-03	1.00