Numerical results for space-time mortar.

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1 Details of experiment.

1. Master space domain: $[0,1]^2$. Master time domain: $[0,0.5]$. Space-time domain (3D): $[0,1]^2 \times [0,0.5]$.
2. Number of subdomains used: 4.
3. Sub domain finite element space: $\mathcal{RT}_0 \times DGQ_0$ in space.
4. For mortar projections:
   (a) For space-time interface for subdomains, FE space which are $\mathcal{RT}_0$ in space and DGQ_0 in time.
   (b) For space-time Mortar spaces at the interface:
      i. DGQ_1 in 3d (linear).
      ii. DGQ_2 in 3d (quadratic).
5. Simulator is written using deal.II finite element package.
6. GMRES is designed and used to solve the interface problem. Parameters used:
   (a) tolerence 1.e-6
   (b) max_iteration 500.
7. Manufactured solutions using: $p(x,y) = \sin(8t)\left(\sin(11x)\cos(11y - \frac{\pi}{4})\right)$ and $K = I_{2\times 2}$.

2 Convergence results.

2.1 Linear mortar with $H = h/2$ and $\Delta T = \Delta t/2$.

1. Subdomain space mesh structure: checkerboard with the ratio $3:2:4:3:1$ for $sub0 : sub1 : sub2 : sub3 : mortar$.
2. Time mesh(including mortar) is refined 4 times successively along with the space mesh. Final time = 0.5.

Table 1: Linear mortar: initial and final mesh parameters.

3. (a)

subdomain id	$\Delta t_i^{initial-refinement}$	$\Delta t_i^{final-refinement}$	$\Delta h_i^{initial-refinement}$	$\Delta h_i^{final-refinement}$
0	0.5/3	0.5/48	1/3	1/48
1	0.5/2	0.5/32	1/2	1/32
2	0.5/4	0.5/64	1/4	1/64
3	0.5/3	0.5/48	1/3	1/48
mortar	0.5/1	0.5/16	1/1	1/16

Table 2: Linear mortar convergence with $H = h/2$and $\Delta T = \Delta t/2$.

ref	h	Δt	# gmres		$\|z - z_h\|_{L^2(L^2)}$		$\|p - p_h\|_{DG}$		$\|p - p_h\|_{L^2(L^2)}$		$\|p - \lambda_{pH}\|_{L^2(L^2)}$	
1	$\frac{1}{2}$	$\frac{1}{4}$	11	-	6.504e-01	-	1.209e+00	-	7.906e-01	-	7.979e-01	-
2	$\frac{1}{4}$	$\frac{1}{8}$	23	-1.06	3.628e-01	0.84	7.213e-01	0.75	4.759e-01	0.73	5.111e-01	0.64
3	$\frac{1}{8}$	$\frac{1}{16}$	39	-0.76	1.744e-01	1.06	3.188e-01	1.18	2.461e-01	0.95	2.336e-01	1.13
4	$\frac{1}{16}$	$\frac{1}{32}$	59	-0.60	8.625e-02	1.02	1.460e-01	1.13	1.245e-01	0.98	1.201e-01	0.96
5	$\frac{1}{32}$	$\frac{1}{64}$	86	-0.54	4.288e-02	1.01	6.928e-02	1.08	6.248e-02	1.00	6.112e-02	0.97

2.2 Quadratic mortar with $H = C\sqrt{h}$ and $\Delta T = C\sqrt{\Delta t}$.

1. Subdomain space mesh structure: checkerboard with the ratio $3 : 2 : 4 : 3 : 1$ for $sub0 : sub1 : sub2 : sub3 : mortar$.

2. Space-time meshes are refined 4 times successively for subdomains and for mortar space, refined every other time in order to attain $H = C\sqrt{h}$ and $\Delta T = C\sqrt{\Delta t}$.

Table 3: Quadratic mortar : initial and final mesh parameters.

3. (a)

subdomain id	$\Delta t_i^{initial-refinement}$	$\Delta t_i^{final-refinement}$	$\Delta h_i^{initial-refinement}$	$\Delta h_i^{final-refinement}$
0	0.5/3	0.5/48	1/3	1/48
1	0.5/2	0.5/32	1/2	1/32
2	0.5/4	0.5/64	1/4	1/64
3	0.5/3	0.5/48	1/3	1/48
mortar	0.5/1	0.5/4	1/1	1/4

Table 4: Quadratic mortar with $H = C\sqrt{h}$ and $\Delta T = C\sqrt{\Delta t}$

ref	h	Δt	# gmres		$\|z - z_h\|_{L^2(L^2)}$		$\|p - p_h\|_{DG}$		$\|p - p_h\|_{L^2(L^2)}$		$\|p - \lambda_{pH}\|_{L^2(L^2)}$	
1	$\frac{1}{2}$	$\frac{1}{4}$	18	-	6.810e-01	-	1.347e+00	-	8.390e-01	-	2.133e+00	-
3	$\frac{1}{8}$	$\frac{1}{16}$	34	-0.46	1.697e-01	1.00	3.511e-01	0.97	2.508e-01	0.87	2.820e-01	1.46
5	$\frac{1}{32}$	$\frac{1}{64}$	57	-0.37	4.477e-02	0.96	8.589e-02	1.02	6.591e-02	0.96	9.201e-02	0.81

3 Plots for linear mortar with refinement level =4.

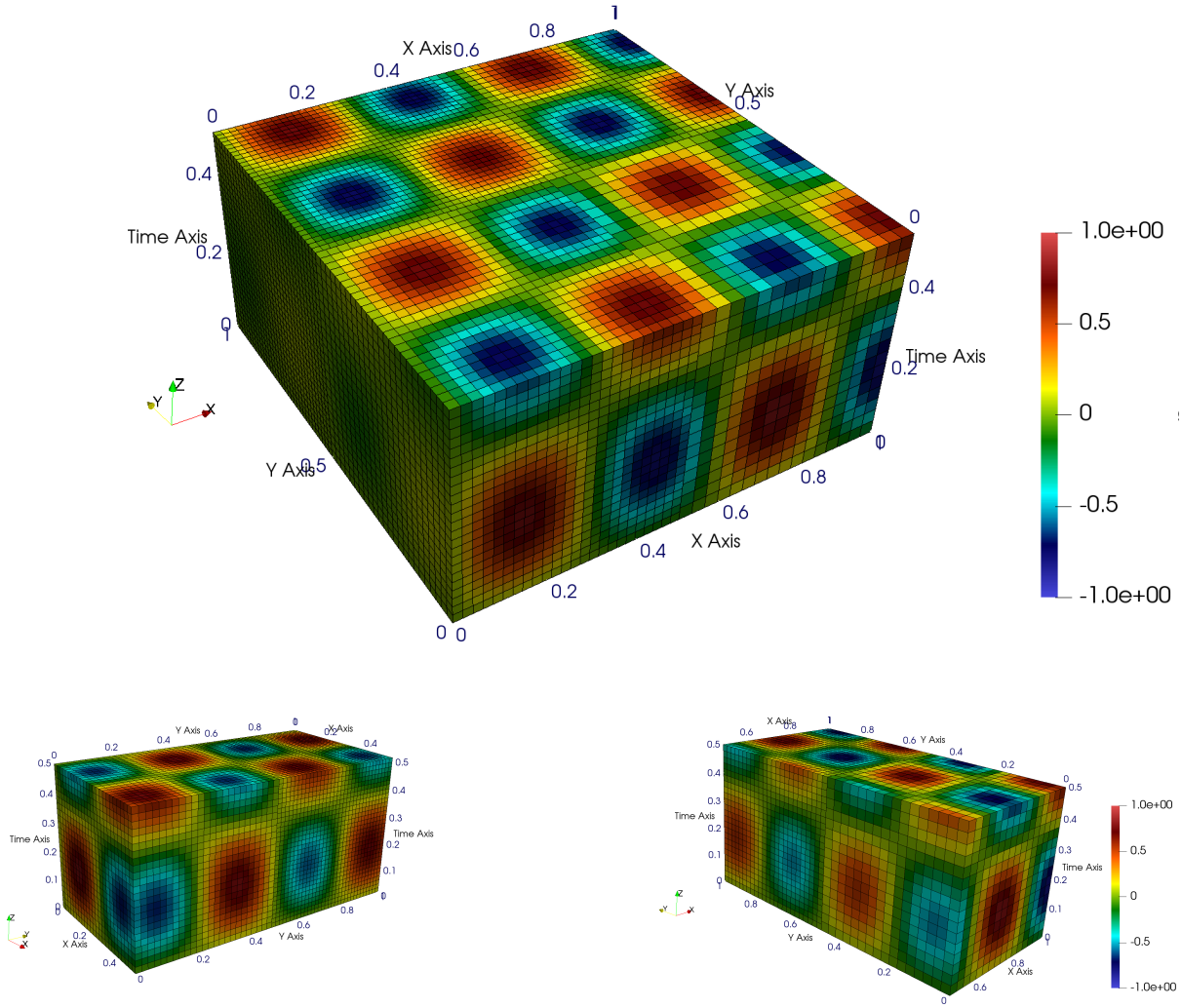

Figure 3.1: linear mortar: Pressure

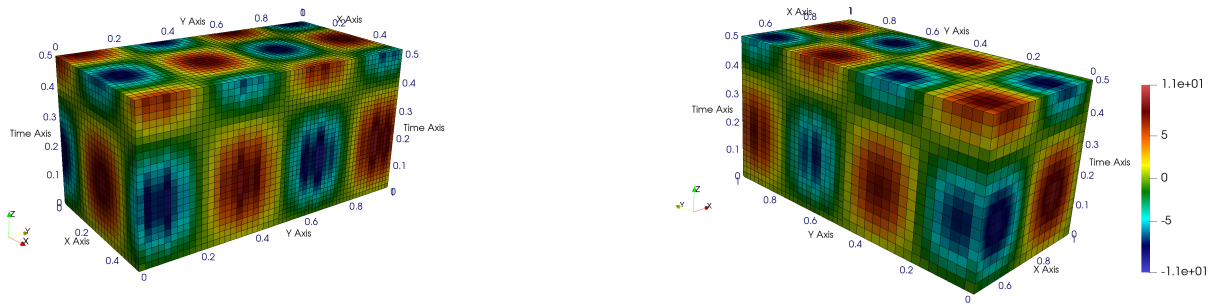

Figure 3.2: linear mortar: u_1

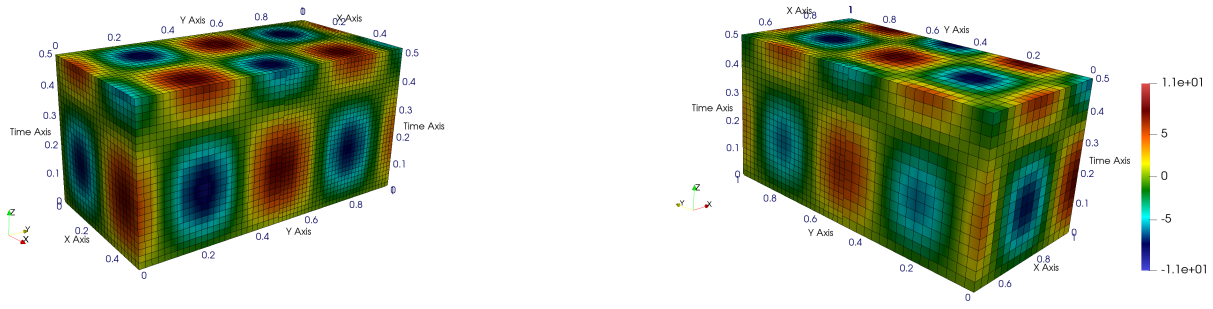

Figure 3.3: linear mortar: u_2