Coding algorithm.

Manu Jayadharan

October 21, 2019

1 Interface problem.

Let $u_h^{\Delta t} \in V_h^{\Delta t}$, $p_h^{\Delta t} \in W_h^{\Delta t}$ and $\lambda_h^{\Delta t} \in \Lambda_h^{\Delta t}$ the solution to the space-time multiscale problem that we want to solve.

Let $\mathcal{P}_i^t : V_{h,i}^{\Delta t} \cdot n_i|_{\Gamma_i} \to \Lambda_{h,i}^{\Delta t}$ and $\mathcal{P}_i : \Lambda_{h,i}^{\Delta t} \to V_{h,i}^{\Delta t} \cdot n_i|_{\Gamma_i}$ be the projection operators corresponding to the sub-domain index i.

1. For first two equations in the system, we can solve the system without integration over time using Backward Euler. ?? **Can we do this!!** Where do we actually use the numerical time integration formulation give in the paper. Make sure the way we solve these main equations makes sense.

2. For imposing the third equation(weak continuity of normal flux across non-matching space-time interface, we need to solve the following interface problem:

$$s(\lambda, \mu) = g(\mu),$$

   where $s(\lambda, \mu) = \int_0^T \sum_{i=1}^N \langle u_{h,i}^*(\lambda_h) \cdot n_i, \mu \rangle_{\Gamma_i}$ and $g(\mu) = \int_0^T \sum_{i=1}^N \langle \bar{u_{h,i}} \cdot n_i, \mu \rangle_{\Gamma_i}$.

3. So if $S$and G denote the matrix and vector equivalent of the interface problem, then the problem reads as follows: Solve

$$S\lambda = G, \quad \forall \lambda \in \Lambda_h^{\Delta t}$$

   where $S\lambda = \sum_{i=1}^1 \mathcal{P}_i^t(u_{h,i}^*(\lambda_h) \cdot n_i)$ and $G = \sum_{i=1}^1 \mathcal{P}_i^t(\bar{u_{h,i}} \cdot n_i)$. Note that this projection happens over the space time interface so it incorporates the integration over time as well.

2 Dealii coding framework.

1. We will use RT_0 for subdomain spaces and DGQ_k where $k \geq 1$ for space-time MORTAR spaces.

2. Each processor will have its own subdomain. We start with RT_0spaces in each $2D$ subdomain Ω_i and solve the bar problems to find $\bar{u_{h,i}}$ using zero Dirichlet boundary condition except on the boundary of full domain Ω. Note that each subdomain has its own time step

3. Then use RT_0in $3D : \Omega_i \times [0, T]$ in special DOF numbering to have DOFs only on the space-time interface. We use $\bar{u_{h,i}}$ at different time steps to form $u_{h,i}^{\Delta t}$ which now lives in the $3D$ space. Note that the normal trace(now the normal is to the space-time interface) gives us a DGQ_0 space in $3D$ which is exactly what we need. We can now project this interface values to the interface of mortar RT_k space in $3D$ to find G. G will be given by the normal trace of this mortar RT_k space at the space-time interface.

4. Start with lambda initial guess $\lambda_{h,i}^{\Delta t} = 0$.

5. Using projection $\mathcal{P}_i(\lambda_{h,i}^{\Delta t})$, solve star problem for each $2D$ subdomain Ω_i using different Δt_i to find $u_{h,i}^*$, use techniques used in previous step to extend it to the $3D$ space-time domain(with $2D$ space-time interface) and project it to the mortar space-time interface. Then add over all subdomains to get $S\lambda$.

6. Use GMRES to find an updated $\lambda_{h,i}^{\Delta t}$ project it back to the space-time interface of RT_0 in $3D$ to get $\lambda_{h,i}$ for each different level of time in Ω_i. Again smart dof_renumbering techniques could be used for this so that no extra computations and interpolations are needed. Ex: DOF ordering from FE_FaceQ could be used. FE_FaceQ is exactly the normal trace of RT space.

7. Repeat steps $5-6$ until λ converges. Solve star problems one last time using info from the final lambda and present solution as $u = \sum_{i=1}^{N} u_{h,i}^* + \bar{u_{h,i}}$. Need to do some work regarding visualization of this, since each subdomain will have different fps for images.

8. Computation of error at the end of each cycle before next refinement.