steel, or the rather abrupt loss in toughness starting at about 200 ksi TS. Basic to understanding these fractures is the concept of plastic instability in pure shear, or localization of deformation along slip lines (characteristics) as described in slip line fields in macroscopic plasticity theory. Families of slip line fields intersect orthogonally in plane strain, but not necessarily orthogonally in plane stress.

The sequence of events observed under the notch is the general expansion of the plastic zone, by the usual crystallographic modes of deformation, up to a critical root strain, at which point the material can be considered to undergo a rheological transformation to a fluidized state, simulating the ideal plastic body. Deformation can now be concentrated along the slip line directions (directions of both maximum shear stress and of pure shear). These slip lines are logarithmic spirals emanating from the notch surface. Any perturbations of deformation velocities along these directions can be 1) damped out instantaneously, giving a stable slip surface, 2) damped out in finite time, giving a weak instability, or 3) not damped out in finite time, giving a strong instability. The second event corresponds to stable crack propagation, and the third event corresponds to unstable crack propagation. In stable propagation, it was observed that the crack (always originating at the notch surface) occasionally shifts from one family of characteristics to the second family. If the state of stress corresponds to plane strain, these two segments of the crack must be orthogonal. If the material is sufficiently brittle, the stable propagation will eventually charge to unstable propagation (strong instability).

It was found that notch ductility is mainly controlled by two material parameters, namely a) the critical root plastic strain (ϵ^*) at which the material simulates the ideal plastic body, and b) the tolerance for weak instabilities which controls the extent of stable propagation along characteristics. As the strength level is increased, ϵ^* decreases and the tolerance for weak instabilities also decreases. This means that the length of stable propagation path decreases.

It is suggested that the "plastic stretch zone" corresponds to the logarithmic path of stable crack propagation prior to unstable propagation, in which fracture is effected by intensely localized deformation in these directions of pure shear. Since the notch ductility also depends on ϵ^*, it is not expected, in general, to be able to predict precisely the K_{Ic} value from a parameter related only to the tolerance for weak instabilities.

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Stress States for {123} ⟨111⟩ Multiple Slip

G. Y. CHIN AND B. C. WONSIEWICZ

PREVIOUSLY Bishop[1] has developed a list of stress states which could activate simultaneously five or more {111}⟨110⟩ slip systems required for an arbitrary shape change of a deforming crystal. By invoking Bishop and Hill's principle of maximum work[2] one can then select the appropriate stress state(s), hence the active slip systems, for a given deformation. Hosford and Chin[3] have obtained the list of stress states for {112}⟨111⟩ slip appropriate for bcc crystals and {111}⟨112⟩ twinning for fcc crystals. In this note we present the list of stress states for {123}⟨111⟩ slip, appropriate for bcc crystals.

The {123}⟨111⟩ systems are defined in Table I and illustrated in Fig. 1. For a set of stresses σ_{ij} the resolved shear stress τ_l on a given slip system l can be written[1]

Table I. The Twenty-Four {123} <111> Slip Systems

Direction	111						$\bar{1}\bar{1}1$					
Plane	$\bar{3}12$	$2\bar{3}1$	$12\bar{3}$	$\bar{3}21$	$1\bar{3}2$	$21\bar{3}$	$3\bar{1}2$	$\bar{2}31$	$\bar{1}\bar{2}\bar{3}$	$3\bar{2}1$	$\bar{1}32$	$\bar{2}\bar{1}\bar{3}$
System	*a*1	*a*2	*a*3	*a*4	*a*5	*a*6	*b*1	*b*2	*b*3	*b*4	*b*5	*b*6
Direction	$\bar{1}11$						$1\bar{1}1$					
Plane	312	$\bar{2}\bar{3}1$	$\bar{1}2\bar{3}$	321	$\bar{1}\bar{3}2$	$\bar{2}1\bar{3}$	$\bar{3}\bar{1}2$	231	$1\bar{2}\bar{3}$	$\bar{3}\bar{2}1$	132	$2\bar{1}\bar{3}$
System	*c*1	*c*2	*c*3	*c*4	*c*5	*c*6	*d*1	*d*2	*d*3	*d*4	*d*5	*d*6

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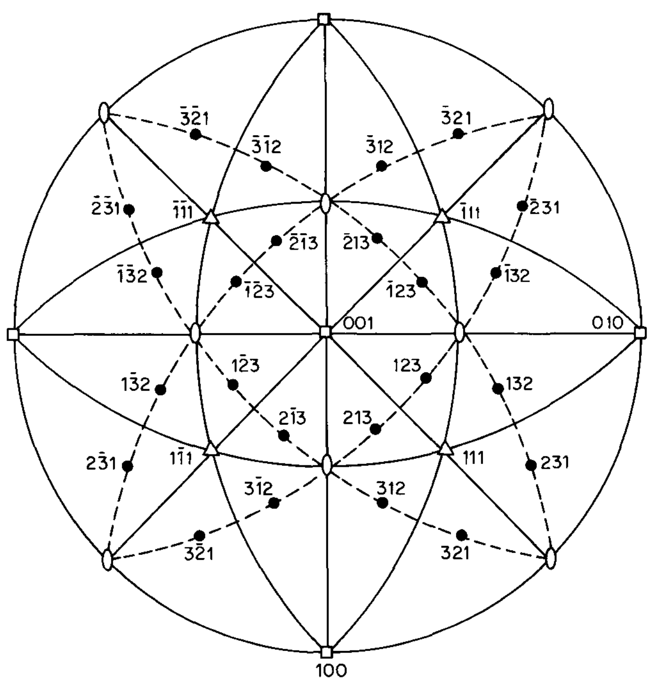

Fig. 1—(001) standard stereographic projection of cubic crystal showing {123} planes and ⟨111⟩ directions. See Table I for notation of slip systems.

Table II. Active {123} <111> Slip Systems Associated with the 106 Stress States.
Plus Sign Means Slip in the Direction Defined in Table I; Negative Sign Denotes the Reverse. Values of Stress (*A* to *H*) are in Multiples of 1/90.

NO.	A	B	C	F	G	H	1 A1	2 A2	3 A3	4 A4	5 A5	6 A6	7 B1	8 B2	9 B3	10 B4	11 B5	12 B6	13 C1	14 C2	15 C3	16 C4	17 C5	18 C6	19 D1	20 D2	21 D3	22 D4	23 D5	24 D6
1	30	-30	0	0	0	0			+			+			+			+			+			+			+			+
2	0	30	-30	0	0	0	+			+			+			+			+			+			+			+		
3	-30	0	30	0	0	0		+			+			+			+			+			+			+			+	
4	0	0	0	30	0	0	+			+			-			-			+			+			-			-		
5	0	0	0	0	30	0		+			+			-			-			-			-			+			+	
6	0	0	0	0	0	30			+			+			+			+			-			-			-			-
7	18	-9	-9	27	0	0		-		+			-					+		-		+			-					+
8	18	-9	-9	-27	0	0	-					+		-		+			-					+		-		+		
9	-9	18	-9	0	27	0			-		+			-		+				-		+					-		+	
10	-9	18	-9	0	-27	0		-		+					-		+				-		+			-		+		
11	-9	-9	18	0	0	27	-					+	-					+			-		+				-		+	
12	-9	-9	18	0	0	-27			-		+				-		+		-					+	-					+
13	30	-15	-15	15	0	0		-			-				+			+		-			-				+			+
14	30	-15	-15	-15	0	0			+			+		-			-				+			+		-			-	
15	-15	30	-15	0	15	0			-			-	+			+			+			+					-			-
16	-15	30	-15	0	-15	0	+			+					-			-			-			-	+			+		
17	-15	-15	30	0	0	15	-			-			-			-				+			+			+			+	
18	-15	-15	30	0	0	-15		+			+			+			+		-			-			-			-		
19	0	-24	24	3	9	9	-			-			-					+								+		-		
20	0	-24	24	-3	-9	9	-					+	-			-				+		-								
21	0	-24	24	3	-9	-9								+		-			-			-			-					+
22	0	-24	24	-3	9	-9		+		-									-					+	-			-		
23	24	0	-24	9	3	9		-			-				+		-			-		+								
24	24	0	-24	-9	-3	9			+		-			-			-									-		+		
25	24	0	-24	9	-3	-9		-		+										-			-				+		-	
26	24	0	-24	-9	3	-9								-		+					+		-			-			-	
27	-24	24	0	9	9	3			-			-							+					-			-		+	
28	-24	24	0	-9	-9	3									-			-			-		+		+					-
29	-24	24	0	9	-9	-3	+					-			-		+				-			-						
30	-24	24	0	-9	9	-3			-		+		+					-									-			-
31	0	-15	15	0	15	15	-			-					+			+								+			+	
32	0	-15	15	0	-15	15			+			+	-			-				+			+							
33	0	-15	15	0	15	-15		+			+										+			+	-			-		
34	0	-15	15	0	-15	-15								+			+		-			-					+			+
35	15	0	-15	15	0	15		-			-				+			+	+			+								
36	15	0	-15	-15	0	15			+			+		-			-								+			+		
37	15	0	-15	15	0	-15	+			+										-			-				+			+
38	15	0	-15	-15	0	-15							+			+					+			+		-			-	
39	-15	15	0	15	15	0			-			-							+			+				+			+	
40	-15	15	0	-15	15	0		+			+		+			+											-			-
41	-15	15	0	15	-15	0	+			+				+			+				-			-						
42	-15	15	0	-15	-15	0									-			-		+			+		+			+		
43	0	0	0	15	15	15									+			+	+			+				+			+	
44	0	0	0	-15	-15	15			+			+								+			+		+			+		
45	0	0	0	15	-15	-15	+			+				+			+										+			+
46	0	0	0	-15	15	-15		+			+		+			+					+			+						
47	0	-3	3	24	9	9							-					+	+			+				+		-		
48	0	-3	3	-24	-9	9	-					+								+		-			+			+		
49	0	-3	3	24	-9	-9	+			+				+		-									-					+
50	0	-3	3	-24	9	-9		+		-			+			+			-					+						
51	3	0	-3	9	24	9									+		-			-		+				+			+	
52	3	0	-3	-9	-24	9			+		-									+			+			-		+		
53	3	0	-3	9	-24	-9		-		+				+			+										+		-	
54	3	0	-3	-9	24	-9		+			+			-		+					+		-							
55	-3	3	0	9	9	24									+			+	+					-			-		+	
56	-3	3	0	-9	-9	24			+			+									-		+		+					-
57	-3	3	0	9	-9	-24	+					-			-		+										+			+
58	-3	3	0	-9	9	-24			-		+		+					-			+			+						

Table II Continued

NO	A	B	C	F	G	H	1 A1	2 A2	3 A3	4 A4	5 A5	6 A6	7 B1	8 B2	9 B3	10 B4	11 B5	12 B6	13 C1	14 C2	15 C3	16 C4	17 C5	18 C6	19 D1	20 D2	21 D3	22 D4	23 D5	24 D6
59	-6	-16	22	2	6	18	-						-					+								+			+	
60	-6	-16	22	-2	-6	18	-					+	-							+			+							
61	-6	-16	22	2	-6	-18								+			+		-						-					+
62	-6	-16	22	-2	6	-18		+			+								-					+	-					
63	22	-6	-16	18	2	6		-							+			+		-		+								
64	22	-6	-16	-18	-2	6			+			+		-												-		+		
65	22	-6	-16	18	-2	-6		-		+										-							+			+
66	22	-6	-16	-18	2	-6								-		+					+			+		-				
67	-16	22	-6	6	18	2			-										+			+					-		+	
68	-16	22	-6	-6	-18	2									-						-		+		+			+		
69	-16	22	-6	6	-18	-2	+			+					-		+				-									
70	-16	22	-6	-6	18	-2			-		+		+			+											-			
71	6	-22	16	2	18	6				-					+			+								+		-		
72	6	-22	16	-2	-18	6			+			+				-				+		-								
73	6	-22	16	2	-18	-6								+		-						-					+			+
74	6	-22	16	-2	18	-6		+		-											+			+				-		
75	16	6	-22	6	2	18					-				+		-		+			+								
76	16	6	-22	-6	-2	18			+		-						-								+			+		
77	16	6	-22	6	-2	-18	+			+													-				+		-	
78	16	6	-22	-6	2	-18							+			+					+		-						-	
79	-22	16	6	18	6	2						-							+					-		+			+	
80	-22	16	6	-18	-6	2												-		+			+		+					-
81	-22	16	6	18	-6	-2	+					-		+			+							-						
82	-22	16	6	-18	6	-2		+			+		+					-												-
83	-3	-8	11	-2	-9	-24									-		+		-								+			+
84	-3	-8	11	-2	9	24	-								+			+									-		+	
85	-3	-8	11	2	-9	24			+			+	-								-		+							
86	-3	-8	11	2	9	-24			-		+										+			+	-					
87	11	-3	-8	-24	-2	-9							+			+			-					+		-				
88	11	-3	-8	-24	2	9	-					+		-											+			+		
89	11	-3	-8	24	-2	9		-					-					+	+			+								
90	11	-3	-8	24	2	-9	+			+										-					-					+
91	-8	11	-3	-9	-24	-2									-					+			+			-		+		
92	-8	11	-3	-9	24	2		+			+			-		+											-			
93	-8	11	-3	9	-24	2		-		+				+			+				-									
94	-8	11	-3	9	24	-2			-											-		+				+			+	
95	3	-11	8	-2	-24	-9								+			+					-					+		-	
96	3	-11	8	-2	24	9				-					+		-									+			+	
97	3	-11	8	2	-24	9			+		-					-				+			+							
98	3	-11	8	2	24	-9		+			+										+		-					-		
99	8	3	-11	-9	-2	-24							+					-			+			+					-	
100	8	3	-11	-9	2	24			+			+					-								+					-
101	8	3	-11	9	-2	24					-				+			+	+					-						
102	8	3	-11	9	2	-24	+					-											-				+			+
103	-11	8	3	-24	-9	-2												-		+		-			+			+		
104	-11	8	3	-24	9	2		+		-			+			+														-
105	-11	8	3	24	-9	2	+			+				+		-								-						
106	-11	8	3	24	9	-2						-							+			+				+		-		

$$\tau_l = \sum_{ij} \tfrac{1}{2}\sigma_{ij}(d_{il}n_{jl} + d_{jl}n_{il}) \qquad [1]$$

where d and n are, respectively, the direction cosines of the slip direction and slip plane normal, all referring to the same set of coordinate axes. For convenience, the cubic axes (1, 2, 3) of the crystal have been used. The resolved shear stresses on the twenty-four $\{123\}\langle 111\rangle$ systems, calculated according to Eq. [1], are as follows:

$$\tau_{a1}/\tau = 2B - C + 3F - G - 2H$$
$$\tau_{a2}/\tau = 2C - A - 2F + 3G - H$$
$$\tau_{a3}/\tau = 2A - B - F - 2G + 3H$$
$$\tau_{b1}/\tau = 2B - C - 3F + G - 2H$$
$$\tau_{b2}/\tau = 2C - A + 2F - 3G - H$$
$$\tau_{b3}/\tau = 2A - B + F + 2G + 3H$$
$$\tau_{c1}/\tau = 2B - C + 3F + G + 2H$$
$$\tau_{c2}/\tau = 2C - A - 2F - 3G + H$$
$$\tau_{c3}/\tau = 2A - B - F + 2G - 3H$$
$$\tau_{d1}/\tau = 2B - C - 3F - G + 2H$$
$$\tau_{d2}/\tau = 2C - A + 2F + 3G + H$$
$$\tau_{d3}/\tau = 2A - B + F - 2G - 3H \qquad [2]$$

$$\tau_{a4}/\tau = B - 2C + 3F - 2G - H$$

$$\tau_{a5}/\tau = C - 2A - F + 3G - 2H$$

$$\tau_{a6}/\tau = A - 2B - 2F - G + 3H$$

$$\tau_{b4}/\tau = B - 2C - 3F + 2G - H$$

$$\tau_{b5}/\tau = C - 2A + F - 3G - 2H$$

$$\tau_{b6}/\tau = A - 2B + 2F + G + 3H$$

$$\tau_{c4}/\tau = B - 2C + 3F + 2G + H$$

$$\tau_{c5}/\tau = C - 2A - F - 3G + 2H$$

$$\tau_{c6}/\tau = A - 2B - 2F + G - 3H$$

$$\tau_{d4}/\tau = B - 2C - 3F - 2G + H$$

$$\tau_{d5}/\tau = C - 2A + F + 3G + 2H$$

$$\tau_{d6}/\tau = A - 2B + 2F - G - 3H$$

where

$$A = \frac{\sigma_{22} - \sigma_{33}}{\tau\sqrt{42}},\ B = \frac{\sigma_{33} - \sigma_{11}}{\tau\sqrt{42}},\ C = \frac{\sigma_{11} - \sigma_{22}}{\tau\sqrt{42}}$$

$$F = \frac{\sigma_{23}}{\tau\sqrt{42}},\ G = \frac{\sigma_{31}}{\tau\sqrt{42}},\ H = \frac{\sigma_{12}}{\tau\sqrt{42}}$$

with τ as the critical resolved shear stress for slip.

For five or more of the systems l to be activated simultaneously, it is necessary that $\tau_l/\tau = \pm 1$ in at least five of the expressions [2], while $-1 < \tau_l/\tau < 1$ on the remaining systems. The discrete values of A through H which can satisfy these constraints are listed in Table II together with the active slip systems identified. (The complete list is actually double that of Table II, since reversing the sense of stress activates shear in the negative direction). The values of τ_l/τ for all systems are given in the Appendix. As was with the case of $\{112\}\langle 111\rangle$ slip and $\{111\}\langle 112\rangle$ twinning,[3] τ_l/τ for $\{123\}\langle 111\rangle$ slip is not necessarily zero on all nonactive systems, see Appendix, which was true for $\{111\}\langle 110\rangle$ slip.[1]

It may be noted that the stress states fall into ten groups; the first four groups activate eight systems, the next four activate six systems, and the last two activate five systems. The particular stress states that activate the required slip for axisymmetric flow for axial orientations in the ten regions of the [110]-[110]-[111] triangle as found by Chin and Mammel,[4] are as follows: region 1, stress state –2; 2, 19; 3, 17; 4, 59; 5, 31; 6, 84; 7, 11; 8, 43; 9, 55; 10, 6.

The list of stress states in Table II were found by first taking the active slip systems for axisymmetric flow, found by Chin and Mammel for each of the ten regions of the [100]-[111]-[110] triangle, and then invoking symmetry and obtaining the equivalent active systems for the other twenty-three triangles of the hemisphere. The discrete values of A through H were then found for each stress state (region) by solving the set of appropriate simultaneous equations from Eqs. [2]. The stress on all other systems was then determined by inserting the newly-found values of A through H into the remaining equations of Eqs. [2]. The calculations were made with the help of a computer.

This method hypothesizes that a list of stress states necessary and sufficient to enforce axisymmetric flow for all axial orientations is necessary and sufficient to enforce any shape change for all orientations. While the hypothesis has not been proven theoretically, a systematic derivation of a complete list of stress states for enforcing arbitrary shape change has been made for $\{111\}\langle 110\rangle$[1,2] and $\{112\}\langle 111\rangle$[5] slips and $\{111\}\langle 112\rangle$ twinning.[5] Thus in these cases, at least, the results agree with those obtained for axisymmetric flow alone.

It should be pointed out that if the list of stress states is small, the maximum work method of Bishop and Hill is a reasonably convenient way to obtain active slip systems, particularly for persons inaccessible to an electronic computer. As the list grows long, such as the case of 106 stress states for the present $\{123\}\langle 111\rangle$ slip, it becomes extremely difficult to apply without the help of a computer. On the other hand, if a computer is available, a much more efficient method of determining the active slip system is the application of linear programming techniques to Taylor's minimum work method,[6] such as done by Chin and Mammel.[4] The equivalence of the maximum work and minimum work methods has now been established.[7] A salient advantage of the latter method is that a knowledge of the stress states is not necessary. It has been found extremely useful in problems involving mixed slip modes, *e.g.*, $\{110\}$, $\{112\}$, $\{123\}\langle 111\rangle$,[4] and mixed slip and twinning, *e.g.*, $\{111\}\langle 110\rangle$ slip plus $\{111\}\langle 112\rangle$ twinning,[8] where the stress states presumably number in the hundreds.

Finally, it hardly needs pointing out that the perennial question of whether $\{123\}\langle 111\rangle$ slip does occur in bcc metals remains unsettled. In a recent theoretical paper, for example, Van der Walt examined the slip geometry of bcc crystals and concluded that planes other than $\{110\}$ and $\{112\}$ must be excluded as true slip planes.[9] Experimentally, however, Jordan and Stoloff have obtained evidence by transmission electron microscopy of $\{123\}$ slip in ordered FeCo.[10]

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Table III (Appendix). Values of Resolved Shear Stress in Multiples of 1/90, on all $\{123\}\langle 111\rangle$ Slip Systems Associated with the 106 Stress States.

NO	A	B	C	F	G	H	1 A1	2 A2	3 A3	4 A4	5 A5	6 A6	7 B1	8 B2	9 B3	10 B4	11 B5	12 B6	13 C1	14 C2	15 C3	16 C4	17 C5	18 C6	19 D1	20 D2	21 D3	22 D4	23 D5	24 D6
1	30	-30	0	0	0	0	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90
2	0	30	-30	0	0	0	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60
3	-30	0	30	0	0	0	-30	90	-60	-60	90	-30	-30	90	-60	-60	90	-30	-30	90	-60	-60	90	-30	-30	90	-60	-60	90	-30
4	0	0	0	30	0	0	90	-60	-30	90	-30	-60	-90	60	30	-90	30	60	90	-60	-30	90	-30	-60	-90	60	30	-90	30	60
5	0	0	0	0	30	0	-30	90	-60	-60	90	-30	30	-90	60	60	-90	30	30	-90	60	60	-90	30	-30	90	-60	-60	90	-30
6	0	0	0	0	0	30	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	90	60	30	-90	30	60	-90	60	30	-90	30	60	-90

Table III Continued

NO	A	B	C	F	G	H	1 A1	2 A2	3 A3	4 A4	5 A5	6 A6	7 B1	8 B2	9 B3	10 B4	11 B5	12 B6	13 C1	14 C2	15 C3	16 C4	17 C5	18 C6	19 D1	20 D2	21 D3	22 D4	23 D5	24 D6
7	18	-9	-9	27	0	0	72	-90	18	90	-72	-18	-90	18	72	-72	-18	90	72	-90	18	90	-72	-18	-90	18	72	-72	-18	90
8	18	-9	-9	-27	0	0	-90	18	72	-72	-18	90	72	-90	18	90	-72	-18	-90	18	72	-72	-18	90	72	-90	18	90	-72	-18
9	-9	18	-9	0	27	0	18	72	-90	-18	90	-72	72	-90	18	90	-72	-18	72	-90	18	90	-72	-18	18	72	-90	-18	90	-72
10	-9	18	-9	0	-27	0	72	-90	18	90	-72	-18	18	72	-90	-18	90	-72	18	72	-90	-18	90	-72	72	-90	18	90	-72	-18
11	-9	-9	18	0	0	27	-90	18	72	-72	-18	90	-90	18	72	-72	-18	90	18	72	-90	-18	90	-72	18	72	-90	-18	90	-72
12	-9	-9	18	0	0	-27	18	72	-90	-18	90	-72	18	72	-90	-18	90	-72	-90	18	72	-72	-18	90	-90	18	72	-72	-18	90
13	30	-15	-15	15	0	0	30	-90	60	60	-90	30	-60	-30	90	-30	-60	90	30	-90	60	60	-90	30	-60	-30	90	-30	-60	90
14	30	-15	-15	-15	0	0	-60	-30	90	-30	-60	90	30	-90	60	60	-90	30	-60	-30	90	-30	-60	90	30	-90	60	60	-90	30
15	-15	30	-15	0	15	0	60	30	-90	30	60	-90	90	-60	-30	90	-30	-60	90	-60	-30	90	-30	-60	60	30	-90	30	60	-90
16	-15	30	-15	0	-15	0	90	-60	-30	90	-30	-60	60	30	-90	30	60	-90	60	30	-90	30	60	-90	90	-60	-30	90	-30	-60
17	-15	-15	30	0	0	15	-90	60	30	-90	30	60	-90	60	30	-90	30	60	-30	90	-60	-60	90	-30	-30	90	-60	-60	90	-30
18	-15	-15	30	0	0	-15	-30	90	-60	-60	90	-30	-30	90	-60	-60	90	-30	-90	60	30	-90	30	60	-90	60	30	-90	30	60
19	0	-24	24	3	9	9	-90	60	30	-90	30	60	-90	18	72	-72	-18	90	-36	24	12	-36	12	24	-72	90	-18	-90	72	18
20	0	-24	24	-3	-9	9	-90	18	72	-72	-18	90	-90	60	30	-90	30	60	-72	90	-18	-90	72	18	-36	24	12	-36	12	24
21	0	-24	24	3	-9	-9	-36	24	12	-36	12	24	-72	90	-18	-90	72	18	-90	60	30	-90	30	60	-90	18	72	-72	-18	90
22	0	-24	24	-3	9	-9	-72	90	-18	-90	72	18	-36	24	12	-36	12	24	-90	18	72	-72	-18	90	-90	60	30	-90	30	60
23	24	0	-24	9	3	9	30	-90	60	60	-90	30	-18	-72	90	18	-90	72	72	-90	18	90	-72	-18	12	-36	24	24	-36	12
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66	22	-6	-16	-18	2	-6	-40	-6	46	-26	-24	50	72	-90	18	90	-72	-18	-60	-30	90	-30	-60	90	44	-90	46	70	-84	14
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71	6	-22	16	2	18	6	-84	70	14	-90	44	46	-60	-30	90	-30	-60	90	-24	-26	50	-6	-40	46	-72	90	-18	-90	72	18
72	6	-22	16	-2	-18	6	-60	-30	90	-30	-60	90	-84	70	14	-90	44	46	-72	90	-18	-90	72	18	-24	-26	50	-6	-40	46
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74	6	-22	16	-2	18	-6	-72	90	-18	-90	72	18	-24	-26	50	-6	-40	46	-60	-30	90	-30	-60	90	-84	70	14	-90	44	46
75	16	6	-22	6	2	18	14	-84	70	46	-90	44	-18	-72	90	18	-90	72	90	-60	-30	90	-30	-60	50	-24	-26	46	-6	-40
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81	-22	16	6	18	-6	-2	90	-18	-72	72	18	-90	-30	90	-60	-60	90	-30	70	14	-84	44	46	-90	-26	50	-24	-40	46	-6
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Table III Continued

NO	A	B	C	F	G	H	1 A1	2 A2	3 A3	4 A4	5 A5	6 A6	7 B1	8 B2	9 B3	10 B4	11 B5	12 B6	13 C1	14 C2	15 C3	16 C4	17 C5	18 C6	19 D1	20 D2	21 D3	22 D4	23 D5	24 D6
83	-3	-8	11	-2	-9	-24	24	26	-50	6	40	-46	18	72	-90	-18	90	-72	-90	32	58	-78	-2	80	-60	-30	90	-30	-60	90
84	-3	-8	11	-2	9	24	-90	32	58	-78	-2	80	-60	-30	90	-30	-60	90	24	26	-50	6	40	-46	18	72	-90	-18	90	-72
85	-3	-8	11	2	-9	24	-60	-30	90	-30	-60	90	-90	32	58	-78	-2	80	18	72	-90	-18	90	-72	24	26	-50	6	40	-46
86	-3	-8	11	2	9	-24	18	72	-90	-18	90	-72	24	26	-50	6	40	-46	-60	-30	90	-30	-60	90	-90	32	58	-78	-2	80
87	11	-3	-8	-24	-2	-9	-50	24	26	-46	6	40	90	-60	-30	90	-30	-60	-90	18	72	-72	-18	90	58	-90	32	80	-78	-2
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95	3	-11	8	-2	-24	-9	6	-46	40	24	-50	26	-30	90	-60	-60	90	-30	-78	80	-2	-90	58	32	-18	-72	90	18	-90	72
96	3	-11	8	-2	24	9	-78	80	-2	-90	58	32	-18	-72	90	18	-90	72	6	-46	40	24	-50	26	-30	90	-60	-60	90	-30
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106	-11	8	3	24	9	-2	80	-2	-78	58	32	-90	-46	40	6	-50	26	24	90	-60	-30	90	-30	-60	-72	90	-18	-90	72	18

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