Universal Portfolios

Jamal Bajwa

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Modern Portfolio Theory

The Kelly Criterion

Shannon's Demon

Cover's Universal Portfolio

Speculations

Modern Portfolio Theory

Mean-Variance Portfolio Optimization

- Sharpe and Markowitz (1960s)
- Maximize Expected Arithmetic Returns subject to a variance constraint
- Modern portfolio theory tries to estimate expected return mean and variance of various stocks – Implicit in this assumption is that the return distribution is *Gaussian*, and can be estimated from empirical history of a stock
- These estimates are inputs into an optimization model that allocates portfolio fractions subject to overall portfolio risk – Variance of the returns is thought of as "risk"

Mathematical Formulation

$$\textit{Let } b_i = \textit{ the portfolio fraction invested in the } i^{th} \textit{ stock}, \sum_{i=1}^{N} b_i = 1$$

$$\textit{Assume each stock is Gaussian and Independent : } N(\mu_i, \sigma_i)$$

$$\mu_i = \textit{mean expected return from } i^{th} \textit{ stock}$$

$$\sigma_i = \textit{variance of } i^{th} \textit{ stock}$$

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$$\textit{maximize} \sum_{i=1}^{N} b_i \mu_i$$

$$s.t.\ \sigma_{portfolio} < \sigma_{max}$$

Efficient Frontier

- As you tweek desired portfolio variance, the expected return of the new portfolio will go up (high risk, high return)

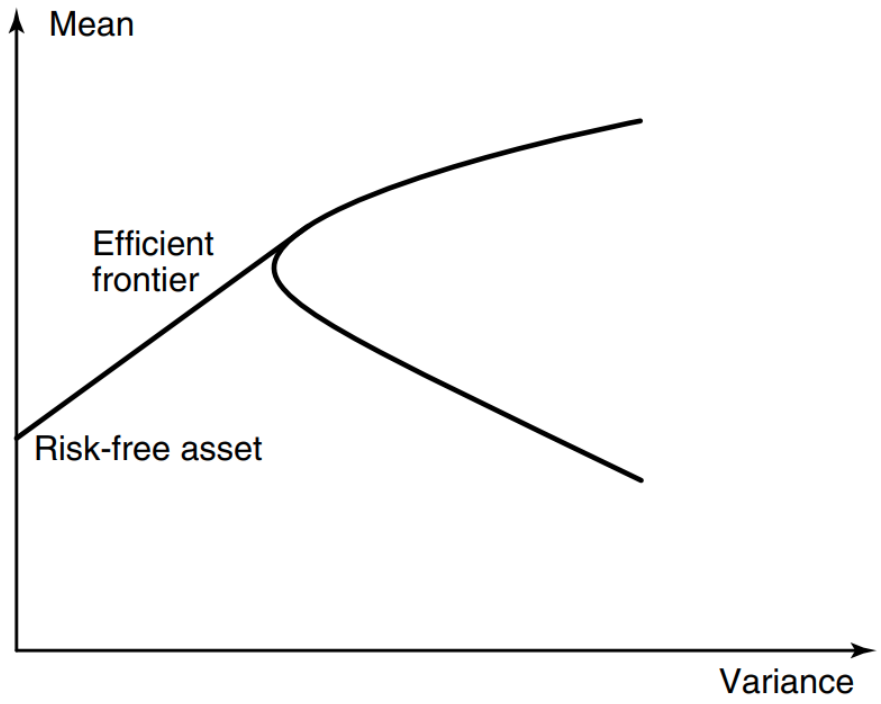

Figure 1: Sample Efficient Frontier

Shortcomings

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The Kelly Criterion

Derivation

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Why was this so different?

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Samuelson's Criticisms

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Shannon's Demon

Cover's Universal Portfolio

Some primitives from Information Theory and Data Compression

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Basic Idea of Universal Portfolio

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Why does this work?

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Some examples

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Speculations