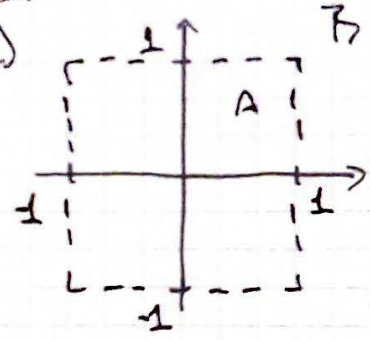
EXERCISE 10

Ex 1

a)



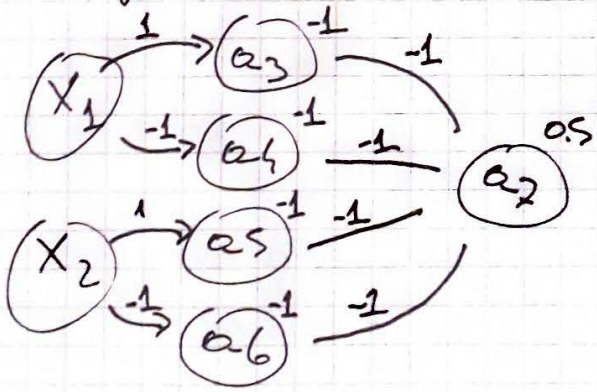
$$A = \{\mathbb{R}^2 : |x_1| < 1 \wedge |x_2| < 1\}$$

$$B = \{\mathbb{R}^2 : |x_1| > 1 \vee |x_2| > 1\}$$

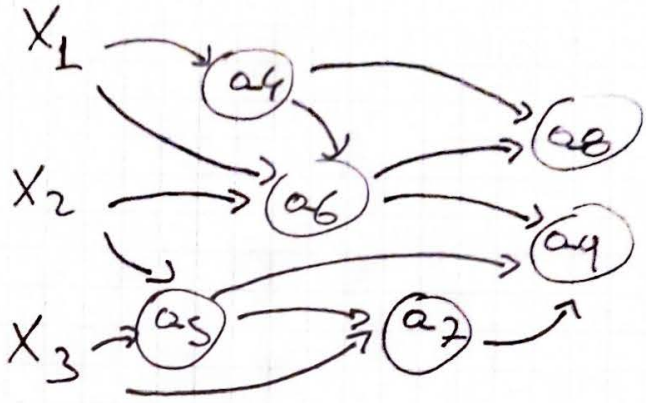
$$z_j = \sum_i a_i w_{ij} + b_j$$

$$a_j = 1_{z_j > 0} \quad \forall a_j$$

$$a_3 = g(x_1 \cdot 1 + x_2 \cdot 0 - 1)$$
$$a_4 = g(x_1 \cdot (-1) + x_2 \cdot 0 - 1)$$
$$a_5 = g(x_1 \cdot 0 + x_2 \cdot 1 - 1)$$
$$a_6 = g(x_1 \cdot 0 + x_2 \cdot (-1) - 1)$$
$$a_2 = g((-1) \cdot a_3 + (-1) \cdot a_4 + (-1) \cdot a_5 + (-1) \cdot a_6 + 0.5)$$



Ex. 2



$t = \begin{pmatrix} t_8 \\ t_9 \end{pmatrix}$, $\delta_i = \frac{\delta E}{\delta z_i}$, $z_j = \sum_i a_i w_{ij}$

$$a_j = \frac{e^{z_j}}{1 + e^{z_j}}$$

$$\frac{\delta a_j}{\delta z_j} = a_j \cdot (1 - a_j)$$

$$E(w) = \| y(x,w) - t \|^2 =$$
$$= \sum_i (a_i - t_i)^2 =$$
$$= (a_8 - t_8)^2 + (a_9 - t_9)^2$$
$$= (g(z_8) - t_8)^2 + (g(z_9) - t_9)^2$$

$z_8 = a_4 w_{48} + a_5 w_{58}$
$z_9 = (\dots)$

$$\delta_8 = \frac{\delta E}{\delta z_8} = 2 \cdot (a_8 - t_8) \cdot a_8 (1 - a_8)$$

$$\delta_9 = \frac{\delta E}{\delta z_9} = 2 (a_9 - t_9) \cdot a_9 (1 - a_9)$$

$$\delta_6 = \frac{\delta E}{\delta z_6} = \frac{\delta E}{\delta z_9} \cdot \frac{\delta z_9}{\delta z_6} + \frac{\delta E}{\delta z_8} \cdot \frac{\delta z_8}{\delta z_6} =$$
$$= \delta_9 (a_6 (1 - a_6) w_{68}) + \delta_9 (a_6 (1 - a_6) w_{69})$$

$$\frac{\delta E}{\delta w_{16}} = \frac{\delta E}{\delta z_6} \cdot \frac{\delta z_6}{\delta w_{16}} = \delta_6 \cdot x_1$$

$$\delta_4 = \frac{\delta E}{\delta z_4} = \frac{\delta E}{\delta z_6} \cdot \frac{\delta z_6}{\delta z_4} + \frac{\delta E}{\delta z_8} \cdot \frac{\delta z_8}{\delta z_4} =$$
$$= \delta_6 \times (a_4 (1 - a_4) w_{46}) + \delta_8 \times (a_4 (1 - a_4) w_{48})$$

$$\frac{\delta E}{\delta w_{14}} = \frac{\delta E}{\delta z_4} \cdot \frac{\delta z_4}{\delta w_{14}} = \delta_4 \cdot x_1$$

Ex. 3

a) $X_1 \xrightarrow{w_{12}} (z_2 | a_2) \xrightarrow{w_{23}} (z_3 | a_3)$

$z_2 = x_1 \cdot w_{12} = a_2$

$z_3 = a_2 \cdot w_{23} = a_3$

$E(w_{12}, w_{23}) = \sum_{n=1}^{3} (f(x^{(n)}) - t^{(n)})^2$

x	t	f(x)
-1	-1	0
0	0	0
1	1	0

$$\frac{\delta E}{\delta w_{23}}\Big|_{\vec{w}=0} = \sum 2 \cdot (f(x^{(n)}) - t^{(n)}) \cdot x_1 \cdot \underbrace{w_{12}}_{=0} = 0$$

$$\frac{\delta E}{\delta w_{12}}\Big|_{\vec{w}=0} = \sum 2 \cdot (f(x^{(n)}) \cdot t^{(n)} \cdot x_1 \cdot \underbrace{w_{23}}_{=0} = 0$$

b) $f(x) = w_{12} \cdot w_{23} \cdot x_1 \Rightarrow f(x) = x$

$w_{12} = v$

$w_{23} = 1/v$, if $v \neq 0$

$$\begin{pmatrix} w_{12} \\ w_{23} \end{pmatrix} \in \left\{ \begin{pmatrix} v \\ 1/v \end{pmatrix}, v \in \mathbb{R} \setminus \{0\} \right\}$$