Exercises for the course
Machine Learning 1
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Exercise Sheet 9

Exercise 1: The Dual SVM (10+20+10 P)

The primal program for the linear hard margin SVM is

$$\begin{aligned} &\min_{\boldsymbol{w},\theta} \ \|\boldsymbol{w}\|^2 \\ &\text{subject to } y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + \theta) \geq 1, \qquad \text{for } 1 \leq i \leq n, \end{aligned}$$

where $\|.\|$ denotes the Euclidean norm, and the minimization is performed in $\boldsymbol{w} \in \mathbb{R}^d, \theta \in \mathbb{R}$, while the data $\boldsymbol{x}_i \in \mathbb{R}^d, y_i \in \{-1, 1\}$ are regarded as fixed constants.

(a) *Write down* the Lagrange function $\Lambda(\boldsymbol{w}, \theta, \boldsymbol{\alpha})$ for the hard margin SVM, where $\boldsymbol{\alpha} \in \mathbb{R}^n$ is the vector of Lagrange multipliers.

(b) *Derive* the Lagrange dual of the program above. *Describe* how the solution for the primal program can be obtained from a solution of the dual program. (Hint: The Lagrange dual is a maximization problem in $\boldsymbol{\alpha}$.)

(c) *Write down* the kernelized versions of the primal program and of the dual program.

Exercise 2: SVMs and Quadratic Programming (10 P)

We consider the `CVXOPT` Python software for convex optimization. The method `cvxopt.solvers.qp` solves the quadratic optimization problem given in matrix form as:

$$\begin{aligned} \min_x \quad & \frac{1}{2}\boldsymbol{x}^\top P \boldsymbol{x} + \boldsymbol{q}^\top \boldsymbol{x} \\ \text{subject to} \quad & G\boldsymbol{x} \preceq \boldsymbol{h} \\ \text{and} \quad & A\boldsymbol{x} = \boldsymbol{b}. \end{aligned}$$

Here, $\preceq$ denotes the element-wise inequality: $(\boldsymbol{h} \preceq \boldsymbol{h}') \Leftrightarrow (\forall_i : h_i \leq h'_i)$. Note that the meaning of the variable $\boldsymbol{x}$ is different from that of the same variable in the previous exercise.

(a) *Express* the matrices and vectors $P, \boldsymbol{q}, G, \boldsymbol{h}, A, \boldsymbol{b}$ in terms of the variables of Exercise 1, such that the quadratic minimization problem above corresponds to the kernel dual SVM.

Exercise 3: Programming (50 P)

Download the programming files on ISIS and follow the instructions.