EXERCISE SHEET (1)

Ex.1

$$C = D_{KL}(P \| Q) = \sum_j P_j \log\left(\frac{P_j}{q_j}\right)$$

WHERE $\sum_j P_j = 1$ AND $\sum_j q_j = 1$

- Show that:

$$\frac{\delta C}{\delta q_i} = -\frac{P_i}{q_i}$$

$$C = \sum_j \left(P_j \log(P_j) - P_j \log(q_j) \right)$$

$$\frac{\delta C}{\delta q_i} = -P_i \times \frac{1}{q_i} = -\frac{P_i}{q_i}$$

- For $q_i = \dfrac{e^{x_i}}{\sum_k e^{x_k}}$, Show that:

$$\frac{\delta C}{\delta x_i} = -P_i + q_i$$

$$C = \sum_j \left(P_j \log(P_j) - P_j \cdot x_j + P_j \cdot \log\left(\sum_k e^{x_k}\right) \right)$$

$$\frac{\delta C}{\delta x_i} = -P_i + \sum_j \left(P_j \cdot \frac{e^{x_i}}{\sum_k e^{x_k}} \right) =$$

$$= -P_i + \underbrace{\frac{e^{x_i}}{\sum_k e^{x_k}}}_{= q_i} \cdot \underbrace{\sum_j P_j}_{= 1} = -P_i + q_i$$

INCOMPLETE !

$\boxed{Ex. 2}$

$$P_{ij} = \frac{(1 + \|x_i - x_j\|^2)/(\sigma^2))^{-1}}{\sum_{k \neq l} (1 + \|x_k - x_l\|^2/(\sigma^2))^{-1}}$$

$$q_{ij} = \frac{e^{-\|y_i - y_j\|^2}}{\sum_{k \neq l} e^{-\|y_k - y_l\|^2}}$$

$$C = \sum_j P_j \log P_j - P_j \log q_j =$$

$$= \sum_j \sum_i P_{ij} \log P_{ij} - P_{ij} \log \left(\frac{e^{-\|y_i - y_j\|^2}}{\sum_{k \neq l} e^{-\|y_k - y_l\|^2}} \right) =$$

$$= \sum_j \sum_i P_{ij} \log P_{ij} + P_{ij} . \|y_i - y_j\|^2 + P_{ij} \log \left(\sum_{k \neq l} e^{-\|y_k - y_l\|^2} \right)$$

$$\frac{\delta C}{\delta y_i} = 2 . \sum_j P_{ij} (y_i - y_j) - 2 . \underbrace{\sum_j P_{ij}}_{=1} \underbrace{\frac{(y_i - y_j) . e^{-\|y_i - y_j\|^2}}{\sum_{k \neq l} e^{-\|y_k - y_l\|^2}}}_{= q_{ij}} =$$

$$= 2 . \sum_j P_{ij} (y_i - y_j) - 2 \sum_j q_{ij} (y_i - y_j)$$

Incomplete !