Fubini rankings with n competitors are n-tuples $\alpha = (a_1, a_2, \ldots, a_n) \in [n]^n$ that record valid rankings of n competitors with ties allowed. The set is denoted by FR_n. I enumerated Fubini Rankings through their lucky sets, I, which are the competitors which placed first in their rankings. I also enumerated Fubini rankings through their outcomes, denoted $\mathcal{O}(\alpha) = \pi$.

Imagine $\alpha \in \mathrm{FR}_n$ where $k = 1$, $I = \{1\}$. Then $\alpha_i = 1$ for all $i \in [n]$.

$$\boxed{1} \times 1 \ldots \times 1.$$

Append a second lucky car at index i_2, $I = \{1, i_2\}$. Then $\alpha_1 \in \{1, i_2\}$ and $\alpha_j = \alpha_1$ for all $j \in [2, i_2 - 1]$ and $\alpha_k \in \{1, i_2\}$ for all $k \in [i_2 + 1, n]$.

$$\boxed{2} \times 1 \ldots \times 1 \times \boxed{1} \times 2 \ldots \times 2.$$

Append a third lucky car at index i_3, $I = \{1, i_2, i_3\}$. Then $\alpha_1 \in \{1, i_2, i_3\}$ and $\alpha_j = \alpha_1$ for all $j \in [2, i_2 - 1]$ and $\alpha_k \in \{1, i_2\}$ for all $k \in [i_2 + 1, n]$ and $\alpha_m \in \{1, i_2, i_3\}$ for all $m \in [i_3 + 1, n]$.

$$\boxed{3} \times 1 \ldots \times 1 \times \boxed{2} \times 2 \ldots \times 2 \times \boxed{1} \times 3 \ldots \times 3.$$

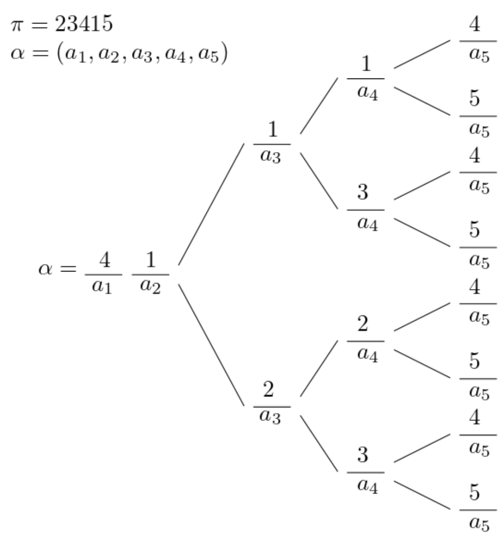

FIGURE 1. Counting through outcomes.