UNIVERSITY OF LOUISIANA AT LAFAYETTE

CONTROL SYSTEMS

MCHE 474

Lab 6

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G = System plant
G_c = System controller
H = System feedback
T = Transfer Function
K = Controller gain
K_{cr} = Critical gain

Introduction

This lab was conducted using MATLAB to analyze a control system. The system?s plant is given in Equation 1, the controller in Equation 2, and the negative feedback in Equation 3. The system's critical gain K at the limit of stability is found using the RH criterion. The root-locus is then plotted by hand, as well as the Bode plot. The gain margin and phase margin are found using the Bode plot and the gain margin is compared to the value calculated using the RH criterion. The system root-locus and Bode plots were then plotted in MATLAB and the system step response when $K = 0.5K_{cr}$ was plotted and performance criteria was calculated.

$$G = \frac{s+10}{s(s^2+3s+25)} \tag{1}$$

$$G_c = K \tag{2}$$

$$H = -1 \tag{3}$$

Theory

The root-locus plot of a system can reveal much about the behavior of the system. The root-locus is a plot showing the location of zeros and poles of the system. Zeros of a system are the values of s that cause the numerator of the transfer function to equal zero. Poles are the values of s that cause the denominator of the transfer function to equal zero. The zeros and poles of a system are often times complex numbers. For this reason, the root-locus plot of a system is plotted on the real-imaginary plane. The root-locus plot shows how the poles and zeros change as an applied gain K goes from 0 to ∞.

A Bode plot is a plot of the system's amplitude and phase at different input frequencies. The amplitude is typically expressed in Decibels (dB) and the frequency is typically expressed in logarithmic increments. The Bode plot contains two plots: the amplitude plot, and the phase plot.

Procedure & Analysis

The transfer function of the system was first defined, shown in Equation 4. As shown in Figure 1, the RH criterion was then used to find K_{cr} at the limit of stability and the root locus was plotted by hand. The Bode plot was also plotted by hand, as seen in Figure 2 and Figure 3.

$$T = \frac{K}{s(s^2+3s+25)+K} \tag{4}$$

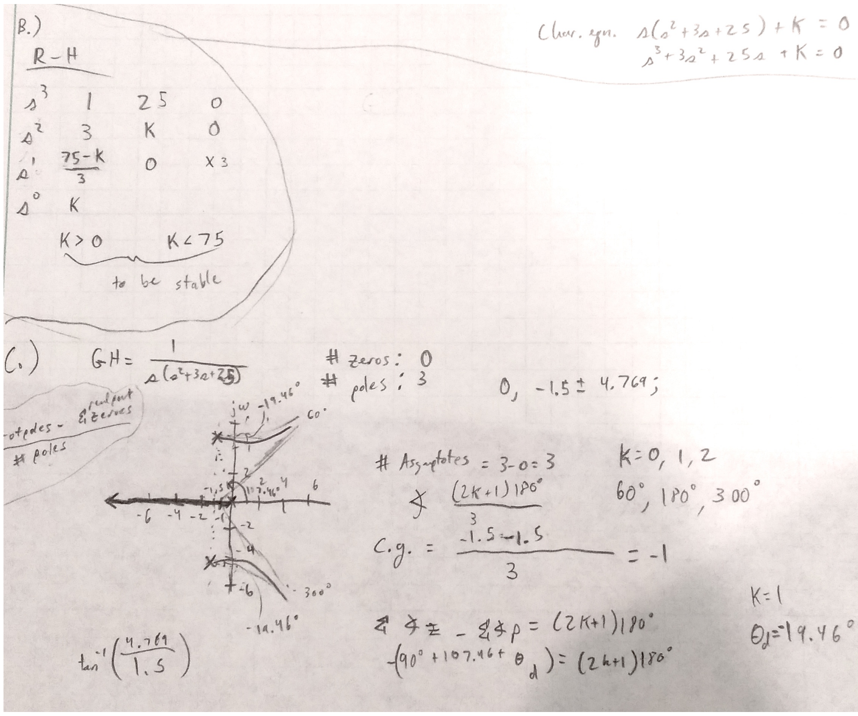

Figure 1: Hand Performed RH Criterion and Root Locus Plot

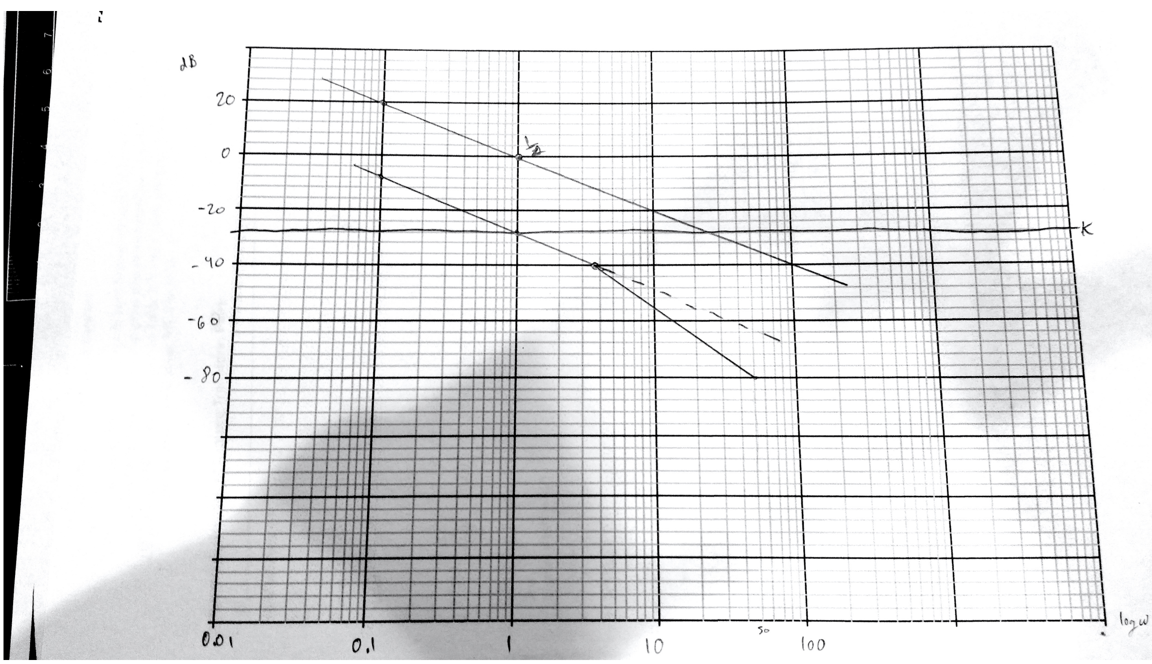

Figure 2: Hand Performed Bode Plot Magnitude

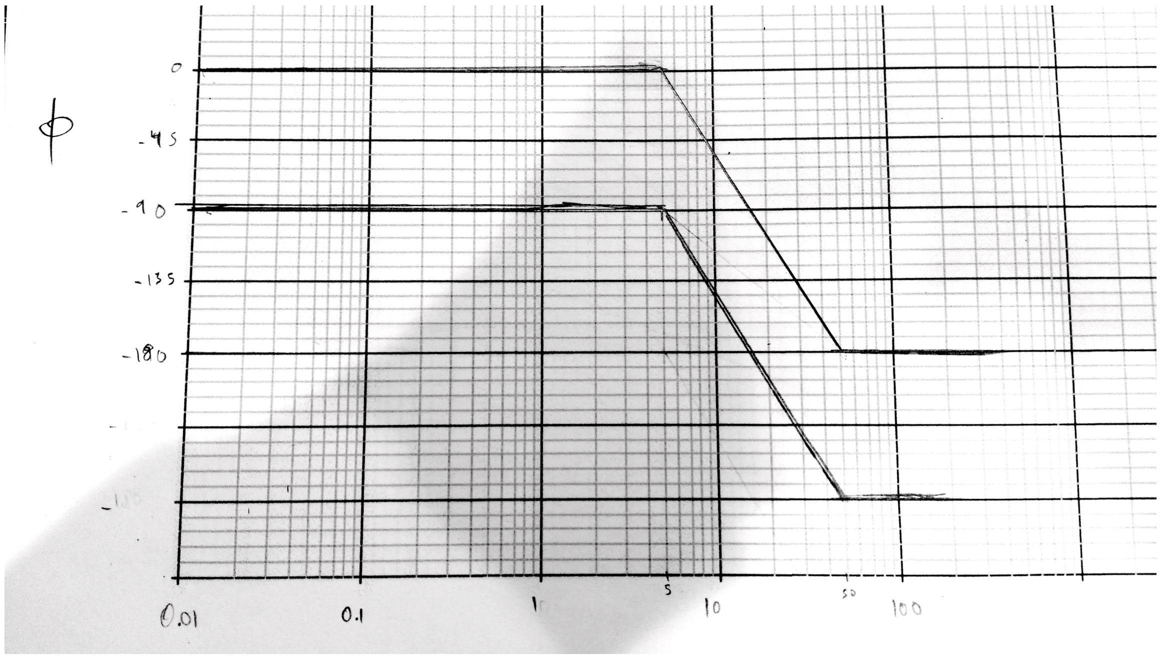

Figure 3: Hand Performed Bode Plot Phase

The root-locus was then plotted using MATLAB, shown in Figure 4. Note that the hand-drawn plot closely matches the computer generated plot. The Bode plots were also done in MATLAB and are shown in Figure 5. The phase margin and gain margin were found to be 89.725 and 75.0, respectively, which agrees with the values calculated by hand using the RH criterion.

The system was then subjected to a design criteria at which the gain $K = 0.5K_{cr} = 32.5$ and the system's transfer function was determined, as shown in Equation 5. The system's step response at this K was also plotted and the performance criteria calculated, as shown in Figure 6 and Figure 7. It can be seen that the system is stable, however if the gain $K = K_{cr} = 75$ then the system is marginally stable for a step response. Any gain $K > 75$ results in the system being unstable.

$$T = \frac{32.5}{s^3 + 3s^2 + 25s + 32.5} \tag{5}$$

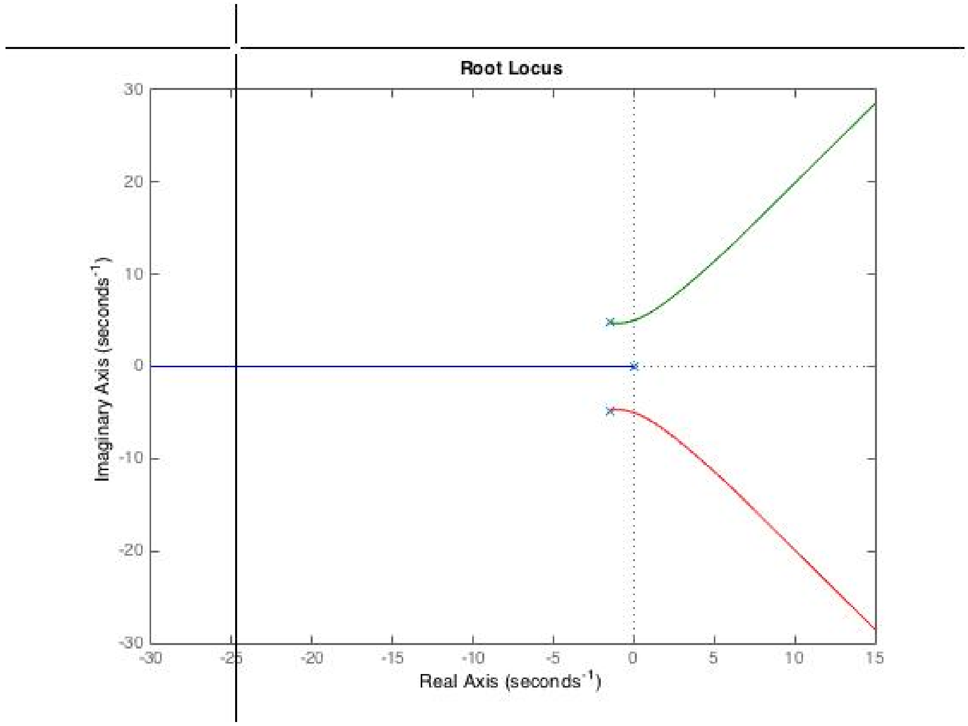

Figure 4: System Root Locus

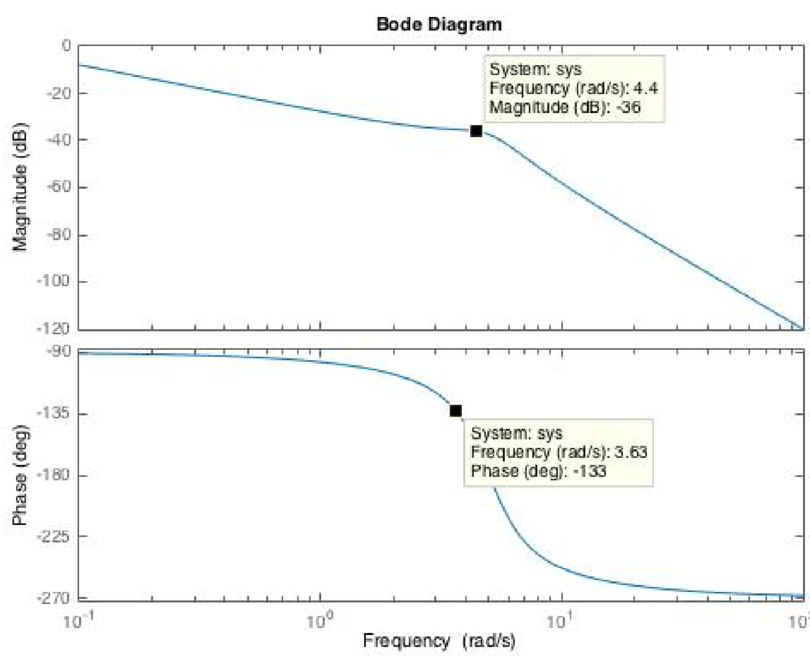

Figure 5: System Bode Plots

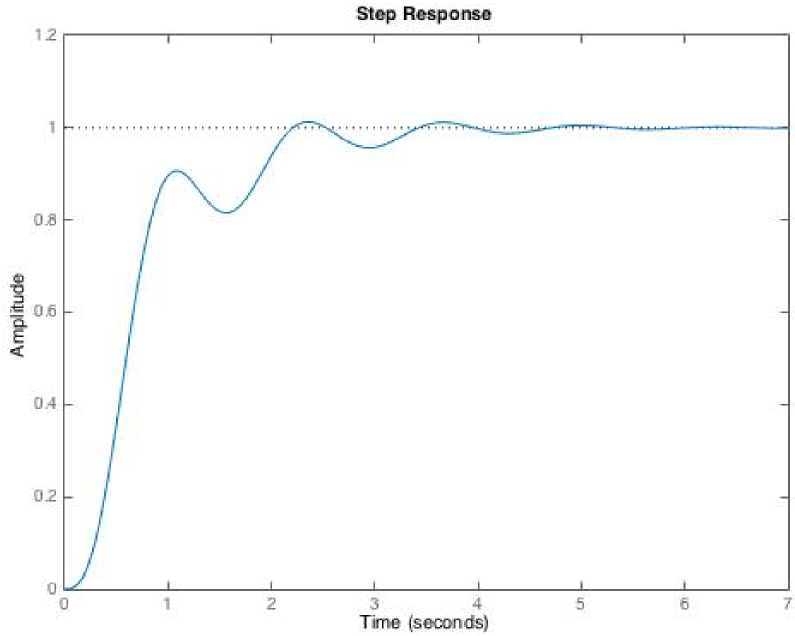

Figure 6: Step Response ($K = 0.5K_{cr}$)

```
     RiseTime: 0.7261
 SettlingTime: 3.2496
  SettlingMin: 0.8151
  SettlingMax: 1.0128
    Overshoot: 1.2846
   Undershoot: 0
         Peak: 1.0128
     PeakTime: 2.3440
```

Figure 7: Step Response Information ($K = 0.5K_{cr}$)

Conclusion

The exercises conducted in this lab reinforce the theory learned in the classroom. It is shown that the root-locus plot of a system can show much information about the system. It is also shown that the Bode plots for a system can provide information as to how the system responds at different frequencies, for amplitude and phase. With the information obtained from the root-locus and the Bode plots, a system's response behavior can be estimated quite well.