DATE: April 3, 2017

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SUBJECT: Determination of Importance of Reynolds Number in Air Flow Rate Measurement Using an Orifice Flow Meter

INTRODUCTION

This memorandum conveys the findings of an experiment conducted to observe the flow of air through an orifice flow meter. The objectives of this experiment are to:

- Conduct the procedure at various high and low Reynold's numbers.
- Compute the orifice coefficient C of an orifice flow meter for both categories of Reynold's number using the data recorded during the experiment.
- For both the high and low Reynold's number datasets, calculate the mean Reynold's number as well as the mean and standard deviation of orifice coefficients.
- Observe the probability density distribution for the high and low Reynold's number datasets.
- Conduct a pooled variance analysis to determine if the orifice coefficient is dependent on the Reynold's number.

In previous work, the orifice coefficient C was found to be about $C = 0.61$ on average [1]. The equation used to determine C from the data recorded in this experiment can be seen in Equation (1). This equation does not rely on the Reynold's number; however, to analyze the relation of C to the Reynold's number Re for comparison with the recorded experimental data, Equation (2) is used [2].

$$C = \frac{\dot{m}}{YA_o\sqrt{2\rho_{mix}\Delta P_o}}\sqrt{1-\beta^4} \tag{1}$$

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{Re^{0.75}} \tag{2}$$

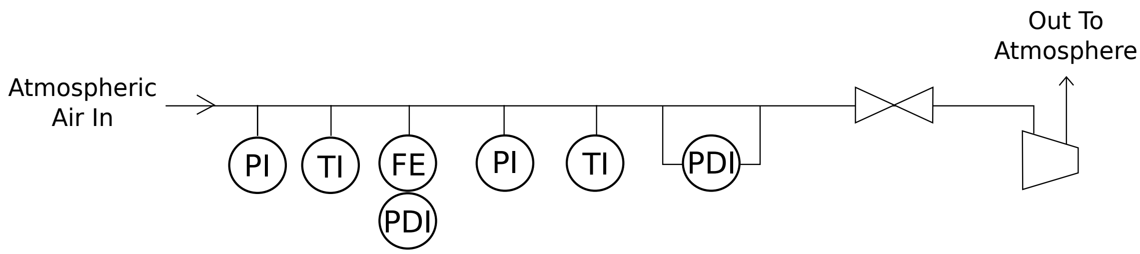

Figure 1: Experiment System Diagram

PROCEDURE

The experiment system diagram is shown in Figure 1. Prior to conduction of experiments, the air conditions in the room are measured using a sling psychrometer. An air compressor is used to suck air from the surroundings into a pipe where an orifice flow meter is housed. The flow rate is controlled using a gate valve on the pipe preceding the compressor. Temperature and pressure readings are taken at the inlet of the pitot-tube station. Two Prandtl pitot-tubes are mounted perpendicular to the air flow such that data can be recorded across two perpendicular diameters. Pressure differentials are measured across the pipe to determine the average flow velocity, and subsequently the mass flow rate of the air in the pipe. Temperature and pressure at the inlet and across the orifice are also measured.

This procedure was repeated at various high and low flow rates. Three groups ran the experiment, each group conducting three high flow rate experiments and three low flow rate experiments. The groups' data was then combined to create a larger dataset.

DATA PRESENTATION & ANALYSIS

The orifice coefficient C was calculated for each experiment conducted, yielding 9 values of C for low flow rates and 9 values of C for high flow rates. For the low flow rates, the calculated mean and standard deviation are $\overline{C_{low}} = 0.620$ and $\sigma_{low} = 0.0052$. For the high flow rates, the calculated mean and standard deviation are $\overline{C_{high}} = 0.631$ and $\sigma_{high} = 0.0069$. The mean Reynold's number for low and high flow rates were found to be $Re_{low} = 372483$ and $Re_{high} = 1179831$.

Using these values, the probability density distribution of the orifice coefficient was plotted for both high and low Reynold's numbers, as seen in Figure 2. A pooled variance analysis was conducted to determine the probability of the apparent relation between C and Re being an actual, observable relationship or just a random coincidence. The probability that the results would be repeated in further experimentation was calculated to be $C_{1-way} = 99.87\%$.

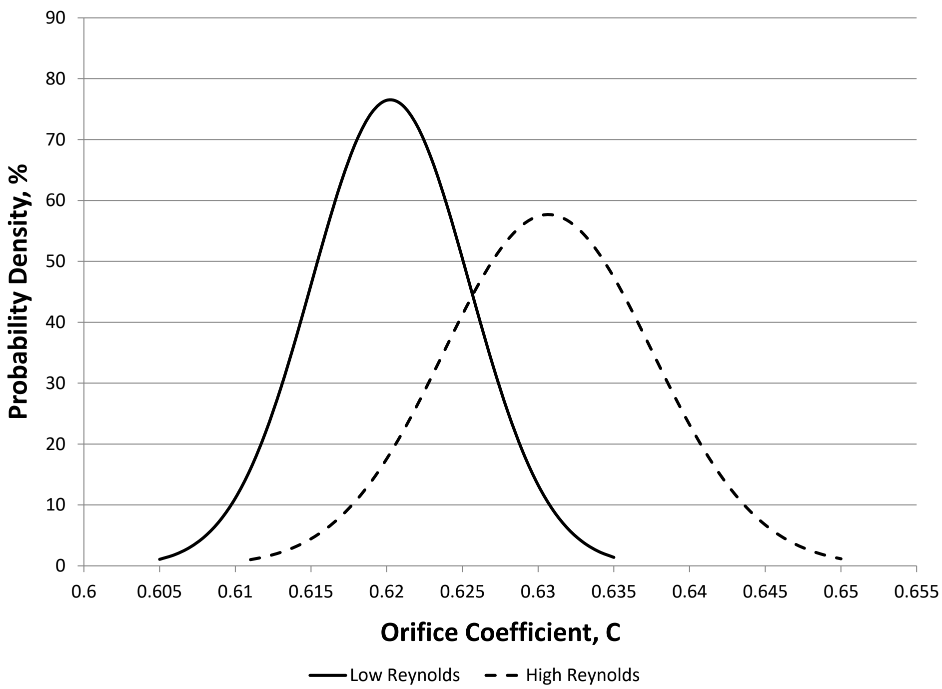

Figure 2: Probability Density Distribution

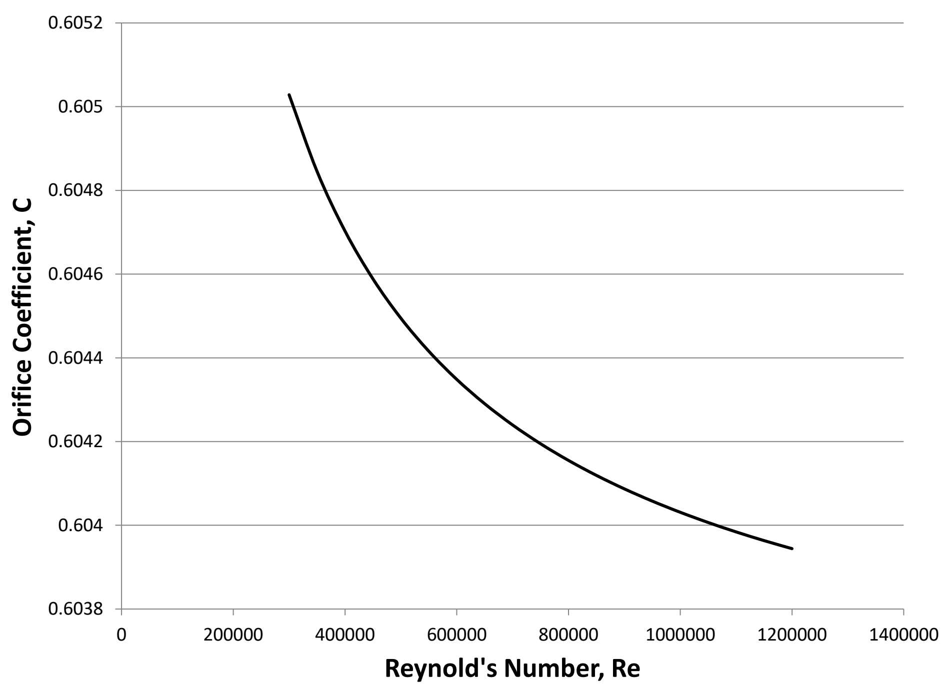

Figure 3: Theoretical Model of C as Function of Re

The model referenced in Equation (2) can be seen plotted in Figure 3 for a range of Reynold's numbers encompassing the range of Reynold's numbers calculated from the experiment data. To compare the model to the experiment data, the mean C of the data was calculated to be $\overline{C_{exp}} =$

0.625, and the mean C of the theoretical model was calculated to be $\overline{C_{th}} = 0.604$. The percent error between the theoretical model and the experimental data is calculated to be $E = 3.50\%$.

DISCUSSION

As seen in Figure 2, it is apparent that the average C value increases slightly with an increase in Reynold's number. As the increase in C is small and there are only two distributions to observe, a pooled variance analysis is used to determine if this trend is actually a dependent relationship or random coincidence.

As indicated by the two way confidence determined from the pooled variance analysis, $C_{1-way} = 99.87\%$, the relationship observed between C and Re is most likely true. In other words, as observed in the experiment it is likely that an increase in Reynold's number will increase the orifice coefficient. However, this result seems to disagree with the referenced theoretical model shown in Equation (2). By observing Figure 3, it is shown that C should be inversely proportional to Re. It is also seen that the theoretical model average $\overline{C_{th}} = 0.604$ is relatively much less than the experiment average $\overline{C_{exp}} = 0.625$, with a percent error of $E = 3.50\%$.

CONCLUSION

- The experimental procedure was conducted at high and low Reynold's numbers.
- The orifice coefficient C was calculated for each conducted test and the mean C and standard deviation σ are calculated for the low Reynold's dataset and high Reynold's datasets.
- The mean Reynold's numbers for the low and high Reynold's number datasets have been calculated.
- The probability density distribution of the orifice coefficient is plotted in Figure 2 for both high and low Reynold's number datasets.
- Pooled variance analysis indicates that the relation of C and Re observed in Figure 2 should hold for any further experiments conducted; meaning an increase in Re should result in a slight increase in C.
- Considering a theoretical model shown in Equation (2) and Figure 3, the relation observed in the experimental data disagrees with the model. The data indicates that an increase in Re should result in an increase in C, but the model shows that C is inversely proportional to Re.

REFERENCES

[1] 2015, "Discharge Coefficients for Nozzles and Orfices." from https://neutrium.net/fluid_flow/discharge-coefficient-for-nozzles-and-orifices/

[2] Wheeler, A., and Ganji, A., 2008, *Introduction to Engineering Experimentation*, 3^{rd} ed., Pearson, Upper Saddle River, NJ, Chap. 10.