WIKIPEDIA

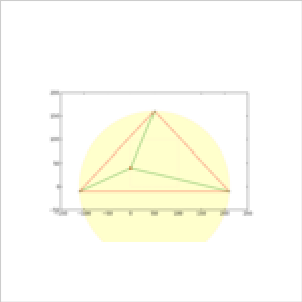
Bowyer–Watson algorithm

In computational geometry, the **Bowyer–Watson algorithm** is a method for computing the Delaunay triangulation of a finite set of points in any number of dimensions. The algorithm can be also used to obtain a Voronoi diagram of the points, which is the dual graph of the Delaunay triangulation.

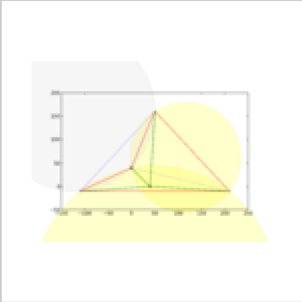
Contents

Description

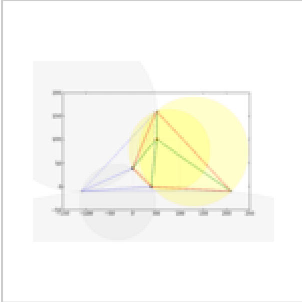
The Bowyer–Watson algorithm is an incremental algorithm. It works by adding points, one at a time, to a valid Delaunay triangulation of a subset of the desired points. After every insertion, any triangles whose circumcircles contain the new point are deleted, leaving a star-shaped polygonal hole which is then re-triangulated using the new point. By using the connectivity of the triangulation to efficiently locate triangles to remove, the algorithm can take $O(N \log N)$ operations to triangulate N points, although special degenerate cases exist where this goes up to $O(N^2)$.[1]



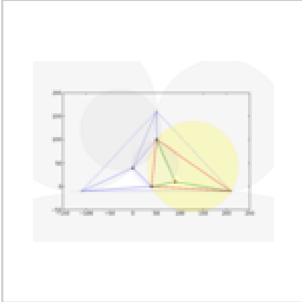
First step: insert a node in an enclosing "super"-triangle



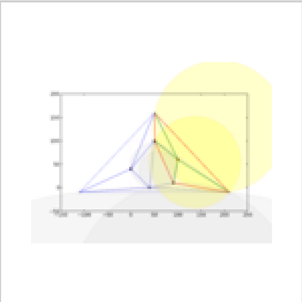
Insert second node



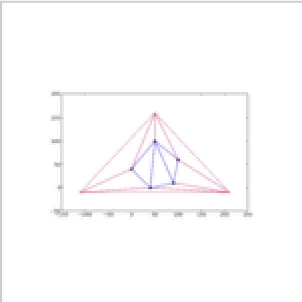
Insert third node



Insert fourth node



Insert fifth (and last) node



Remove edges with extremes in the super-triangle

History

The algorithm is sometimes known just as the **Bowyer Algorithm** or the **Watson Algorithm**. Adrian Bowyer and David Watson devised it independently of each other at the same time, and each published a paper on it in the same issue of *The Computer Journal* (see below).

Pseudocode

The following pseudocode describes a basic implementation of the Bowyer-Watson algorithm. Its time complexity is $O(n^2)$. Efficiency can be improved in a number of ways. For example, the triangle connectivity can be used to locate the triangles which contain the new point in their circumcircle, without having to check all of the triangles - by doing so we can decrease time complexity to $O(n \log n)$. Pre-computing the circumcircles can save time at the expense of additional memory usage. And if the points are uniformly distributed, sorting them along a space filling Hilbert curve prior to insertion can also speed point location.[2]

```
function BowyerWatson (pointList)
   // pointList is a set of coordinates defining the points to be triangulated
   triangulation := empty triangle mesh data structure
   add super-triangle to triangulation // must be large enough to completely contain all the
points in pointList
   for each point in pointList do // add all the points one at a time to the triangulation
      badTriangles := empty set
      for each triangle in triangulation do // first find all the triangles that are no longer
valid due to the insertion
```

```
        if point is inside circumcircle of triangle
           add triangle to badTriangles
     polygon := empty set
     for each triangle in badTriangles do // find the boundary of the polygonal hole
        for each edge in triangle do
           if edge is not shared by any other triangles in badTriangles
              add edge to polygon
     for each triangle in badTriangles do // remove them from the data structure
        remove triangle from triangulation
     for each edge in polygon do // re-triangulate the polygonal hole
        newTri := form a triangle from edge to point
        add newTri to triangulation
  for each triangle in triangulation // done inserting points, now clean up
     if triangle contains a vertex from original super-triangle
        remove triangle from triangulation
  return triangulation
```

References

1. Rebay, S. *Efficient Unstructured Mesh Generation by Means of Delaunay Triangulation and Bowyer-Watson Algorithm*. Journal of Computational Physics Volume 106 Issue 1, May 1993, p. 127.
2. Liu, Yuanxin, and Jack Snoeyink. "A comparison of five implementations of 3D Delaunay tessellation." Combinatorial and Computational Geometry 52 (2005): 439-458.

Further reading

- Bowyer, Adrian (1981). "Computing Dirichlet tessellations" (https://doi.org/10.1093%2Fcomjnl%2F24.2.162). *Comput. J.* **24** (2): 162–166. doi:10.1093/comjnl/24.2.162 (https://doi.org/10.1093%2Fcomjnl%2F24.2.162).
- Watson, David F. (1981). "Computing the *n*-dimensional Delaunay tessellation with application to Voronoi polytopes" (https://doi.org/10.1093%2Fcomjnl%2F24.2.167). *Comput. J.* **24** (2): 167–172. doi:10.1093/comjnl/24.2.167 (https://doi.org/10.1093%2Fcomjnl%2F24.2.167).
- Efficient Triangulation Algorithm Suitable for Terrain Modelling (http://paulbourke.net/papers/triangulate/) generic explanations with source code examples in several languages.

External links

- pyDelaunay2D (https://github.com/jmespadero/pyDelaunay2D) : A didactic Python implementation of Bowyer–Watson algorithm.
- Bl4ckb0ne/delaunay-triangulation (https://github.com/Bl4ckb0ne/delaunay-triangulation) : C++ implementation of Bowyer–Watson algorithm.