

Fundamentals of Machine Learning

: 기하학적 접근
데이터를 점으로 나타내고 (기하학적 접근)

⊕ Statistically
Improved 접근 Tool 소개

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Weekly Objectives

- Learn the most classical methods of machine learning
 - Rule based approach
 - Classical statistics approach
 - Information theory approach
- Rule based machine learning
 - How to find the specialized and the generalized rules
 - Why the rules are easily broken
- Decision Tree
 - How to create a decision tree given a training dataset
 - Why the tree becomes a weak learner with a new dataset
- Linear Regression
 - How to infer a parameter set from a training dataset
 - Why the feature engineering has its limit

) D.T
Forest
:

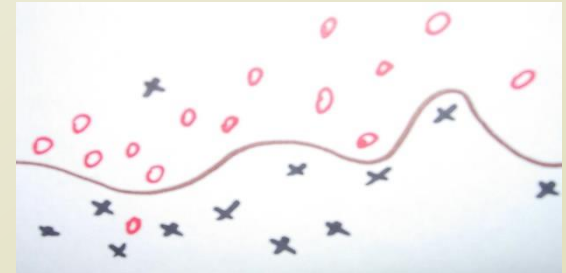
RULE BASED MACHINE LEARNING

From the Last Week

"머신러닝" = learning

- Definition of machine learning
 - A computer program is said to
 - learn from experience E
 - With respect to some class of tasks T
 - And performance measure P , if its performance at tasks in T , as measured by P , improves with experience E
- More experience \rightarrow more thumbtack toss, more prior knowledge
 - Data: We have observed the sequence data of D with a_H and a_T
 - Our hypothesis
 - The gambling result of thumbtack follows the binomial distribution of θ
- Our first trial other than thumbtack
 - Rule based learning
 - Still, about choosing a better hypothesis

You know the true answers of some of instances



A Perfect World for Rule Based Learning

- Imagine

- A perfect world with

- No observation errors, No inconsistent observations
 - No stochastic elements in the system we observe
(= random effect)
 - Full information in the observations to regenerate the system

Training data is
error-free, noise-
free

Target function is
deterministic

Target function is contained
in hypotheses set

- A perfect world of “EnjoySport”

- Observation on the people

- Sky, Temp, Humid, Wind, Water, Forecast → EnjoySport

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Function Approximation → ML의 본질

- Machine Learning?

- The effort of producing a better approximate function
- Remember PAC Learning Theory?

- In the perfect world of EnjoySport

- Instance X

- Input
- Features: O : <Sunny, Warm, Normal, Strong, Warm, Same>
 - Label: Y : <Yes>

- Training Dataset D

- A collection of observations on the instance

- Hypotheses H

- Potentially possible function to turn X into Y
- h_i : <Sunny, Warm, ?, ?, ?, Same> → Yes
- How many hypotheses exist?

- Target Function c (= 일기아무리 / 불필요하다 함수) ∴ h 은 c 로 대체 가능하다

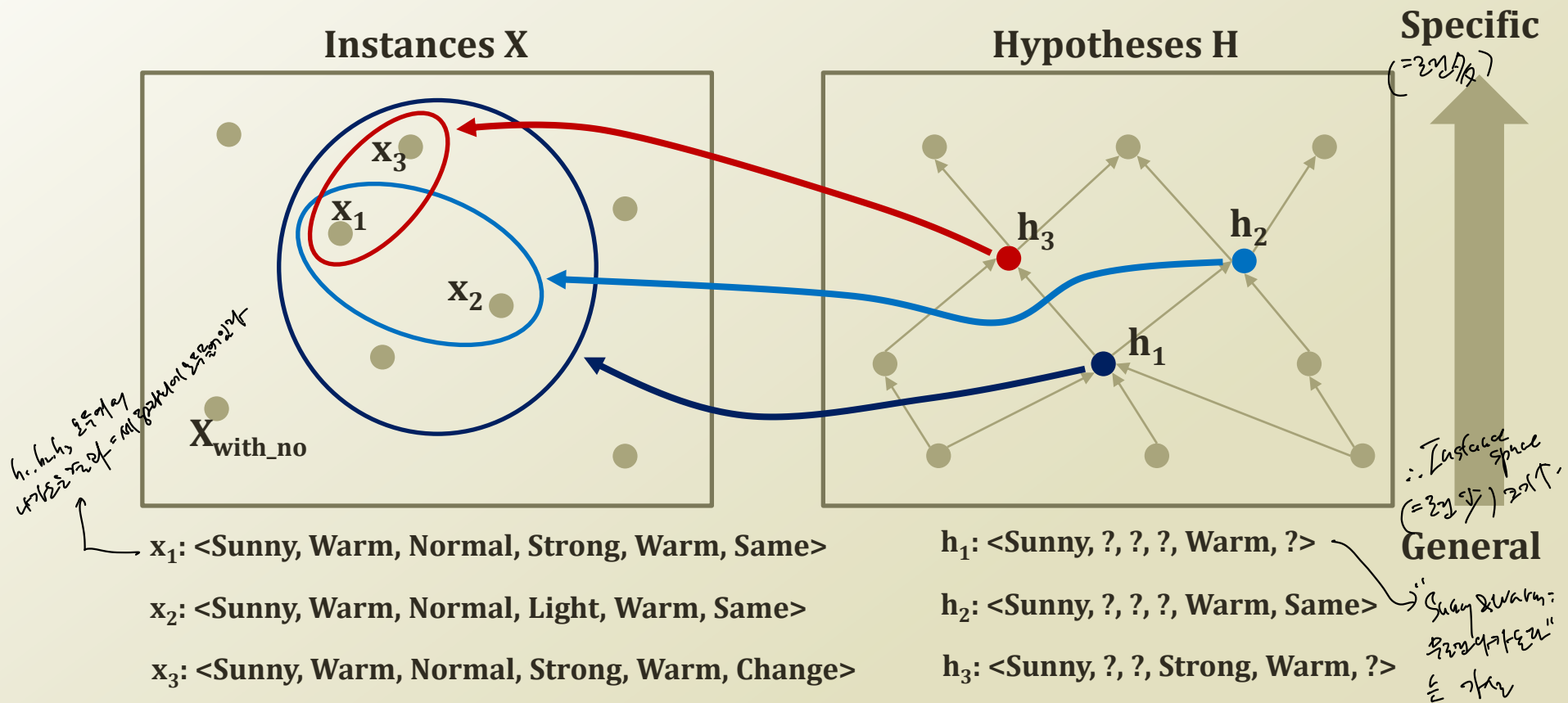
- Unknown target function between the features and the label

Determine
A hypothesis h in H such
that $h(x)=c(x)$ for all x in X



Determine
A hypothesis h in H such
that $h(x)=c(x)$ for all x in D

Graphical Representation of Function Approximation



- What would be the better function approximation?
 - Generalization vs. Specialization

Find-S Algorithm

- Find-S Algorithm

- Initialize h to the most specific in H
- For instance x in D

- if x is positive
 - For feature f in O
 - If f_i in $h == f_i$ in x
 - Do nothing
 - Else
 - f_i in $h = f_i$ in $h \cup f_i$ in x

- Return h

- Instances

- x_1 : <Sunny, Warm, Normal, Strong, Warm, Same>
- x_2 : <Sunny, Warm, Normal, Light, Warm, Same>
- x_4 : <Sunny, Warm, Normal, Strong, Warm, Change>

- Hypotheses

시작 = null $h_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$: 아무것도 모르겠을 때

- $h_1 = \langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle$

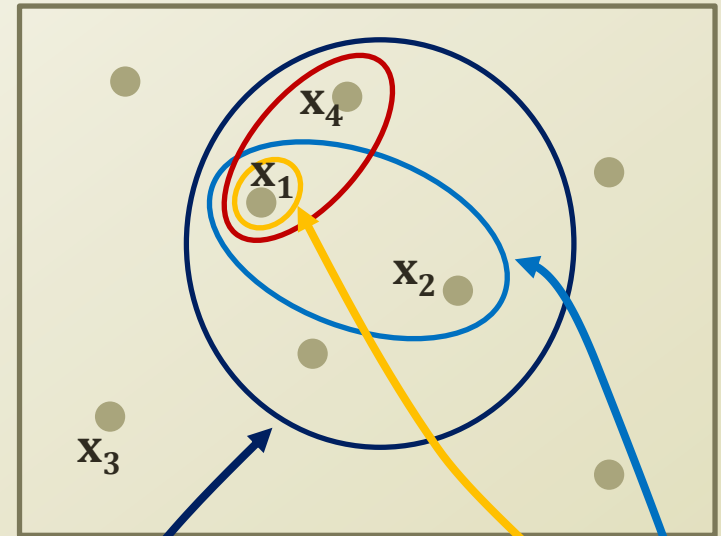
- $h_{1,2,3} = \langle \text{Sunny, Warm, Normal, ?, Warm, Same} \rangle$

- $h_{1,2,3,4} = \langle \text{Sunny, Warm, Normal, ?, Warm, ?} \rangle$

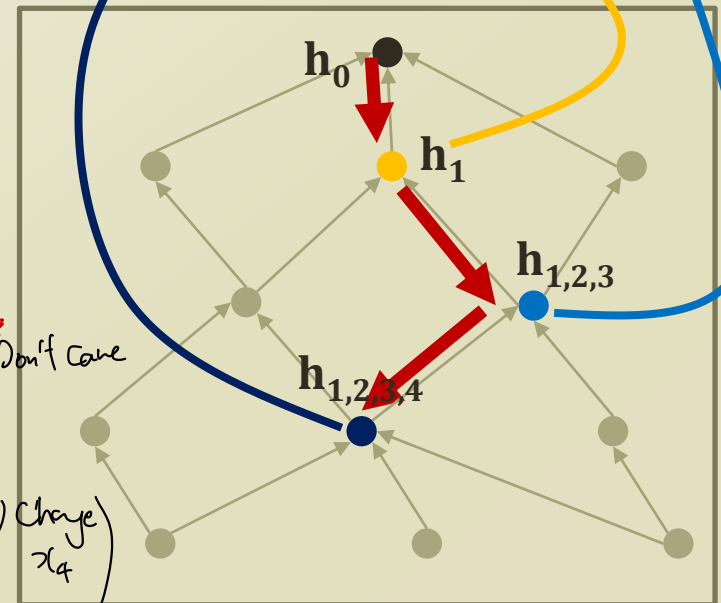
- Any problems?

- Many possible h s, and can't determine the converge : 아직 정답이 나오지 않을 수 있음.

Instances X



Hypotheses H



Version Space

영역 = boundary를 정하는 것.

- Many hypotheses possible, and No way to find the convergence
- Need to setup the perimeter of the possible hypothesis
- The set of the possible hypotheses ^(= boundary) == Version Space, **VS**

- General Boundary, **G**

- Is the set of the maximally general hypotheses of the version space

- Specific Boundary, **S**

- Is the set of the maximally specific hypotheses of the version space

- Every hypothesis, **h**, satisfies

- $VS_{H,D} = \{h \in H \mid \exists s \in S, \exists g \in G, g \geq h \geq s\}$

where $x \geq y$ means x is more general or equal to y

S
↓
S = Version
↓
G
↓
G = Maximally General

S: {<Sunny, Warm, ?, Strong, ?, ?>}

<Sunny, ?, ?, Strong, ?, ?>

<Sunny, Warm, ?, ?, ?, ?>

<?, Warm, ?, Strong, ?, ?>

G: {<Sunny, ?, ?, ?, ?, ?>, <?, Warm, ?, ?, ?, ?>}

① Maximally General = G
② Maximally Specific = S
③ Version Space = V-S

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
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Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Candidate Elimination Algorithm

$$= \begin{pmatrix} \text{가장 } S \\ \text{가장 } G \end{pmatrix} \text{의 } V.S \text{의 } \text{가장 } S \text{의 } \text{가장 } G \text{의 } \text{가장 } S \text{의 } \text{가장 } G$$

- Candidate Elimination Algorithm

- Initialize S to maximally specific h in H

- Initialize G to maximally general h in H

$S_0: \{ \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle \}$: 가장 안 좋은

- For instance x in D \rightarrow Data 이 (예제)를 주면 G or S 를 수정한다

- If y of x is positive

- Generalize S as much as needed to cover o in x

- Remove any h in G , for which $h(o) \neq y$

- If y of x is negative

- Specialize G as much as needed to exclude o in x

- Remove any h in S , for which $h(o) = y$

- Generate h that satisfies $\exists s \in S, \exists g \in G, g \geq h \geq s$

most S 를 generalize
of instance feature 가비 항목이 있으면 가장 gen.

Error 발생 시 (예제)를
specialize.

$G_0: \{ \langle ?, ?, ?, ?, ?, ? \rangle \}$: 가장 좋은

Progress of Candidate Elimination Algorithm

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Handwritten: 2: Don't Care

$S_0: \{ \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle \}$

Handwritten: 타사라이지. x_1, x_2 만일 1이면 relax 시키지

$\downarrow \tau_1$

$S_1: \{ \langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle \}$

$\downarrow \tau_2$

$S_2: \{ \langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle \}$

Handwritten: gen

$G_0, G_1, G_2: \{ \langle ?, ?, ?, ?, ?, ? \rangle \}$

Handwritten: 타't.

Handwritten: 0 $\rightarrow x_1, x_2$ 만일 3.

Progress of Candidate Elimination Algorithm

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

S0: {<∅, ∅, ∅, ∅, ∅, ∅>}

S1: {<Sunny, Warm, Normal, Strong, Warm, Same>}

S2, S3: {<Sunny, Warm, ?, Strong, Warm, Same>}

가장 S = 가장 S
∴ 가장 S x.

(가장 S (가장 S) 22222222)

{<... Light ...>
S 222222222222
onk(222)

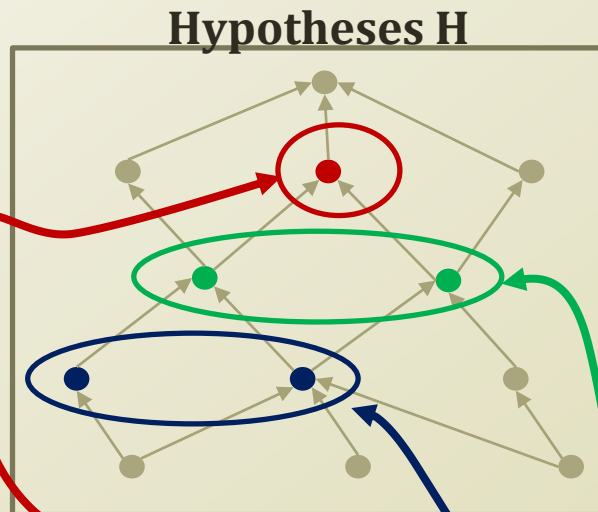
G3: {<Sunny,?,?,?,?,?>, <?,Warm,?,?,?,?,?>, <?,?,?,?,?,Same>}

G0, G1, G2: {<?,?,?,?,?,?>}

2 case 222222222222

Progress of Candidate Elimination Algorithm

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes



$S_0: \{ \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle \}$

$S_1: \{ \langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle \}$

$S_2, S_3: \{ \langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle \}$

$S_4: \{ \langle \text{Sunny, Warm, ?, Strong, ?, ?} \rangle \}$

Still many *hs* : *이건 True for Color? (= Rule Based)*

$G_4: \{ \langle \text{Sunny, ?, ?, ?, ?, ?} \rangle, \langle \text{?, Warm, ?, ?, ?, ?} \rangle \}$

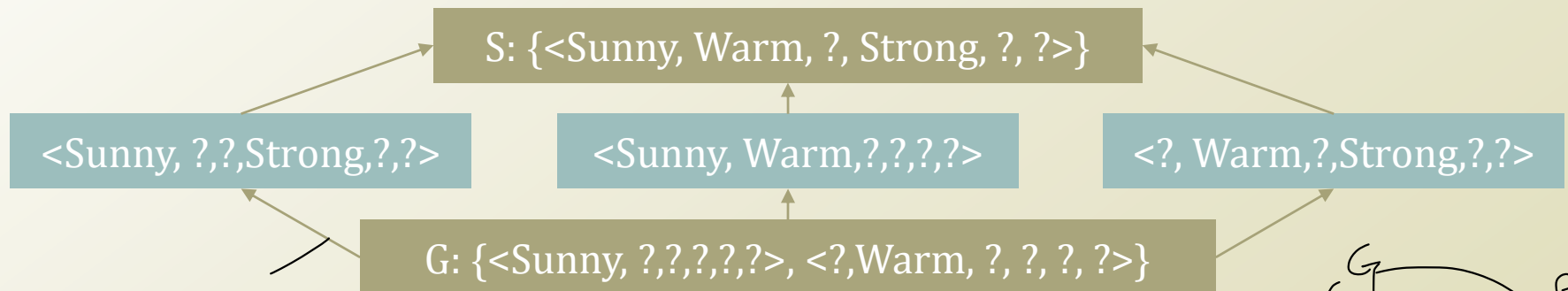
$G_3: \{ \langle \text{Sunny, ?, ?, ?, ?, ?} \rangle, \langle \text{?, Warm, ?, ?, ?, ?} \rangle, \langle \text{?, ?, ?, ?, ?, Same} \rangle \}$: *이건 넣으면 pos-쪽이*

$G_0, G_1, G_2: \{ \langle \text{?, ?, ?, ?, ?, ?} \rangle \}$

→ 222222 chye. yes 맞으면 제거.

How to classify the next instance?

Sky	Temp	Humid	Wind	Water	Forecast	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes



- Somehow, we come up with the version space
 - A subset of H that satisfies the training data, D

Imagine a new instance kicks in

Teacher

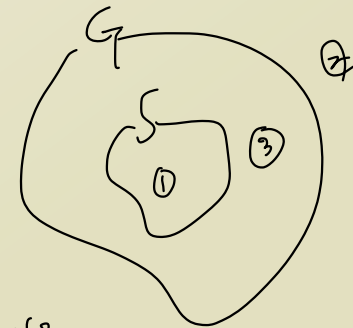
① $\langle \text{Sunny, Warm, Normal, Strong, Cool, Change} \rangle \rightarrow S$ 만족 \therefore 다 만족.

② $\langle \text{Rainy, Cold, Normal, Light, Warm, Same} \rangle \rightarrow G$ 만족 \therefore 다 만족.

③ $\langle \text{Sunny, Warm, Normal, Light, Warm, Same} \rangle \rightarrow G$ 만족, S 만족. \rightarrow Data evidence X

- How to classify these?

- Which h to apply from the subset?
- Or, a classification by all of h s in the subset
- How many are h s satisfied?



(= 여러 Case 각각)
만족

\therefore 각각 X Case! 각각
② 지역 U.S 만족
바탕 3개 100% 만족

Is this working? : Ideal or not?

- Will the candidate-elimination algorithm converge to the correct hypothesis?

- Converge? \rightarrow Able to select a hypothesis
- Correct? \rightarrow The hypothesis is true in the observed system

- Given the assumption, yes and yes

Training data is error-free, noise-free

- No observation errors, No inconsistent observations
- No stochastic elements in the system we observe
- Full information in the observations to regenerate the system

Target function is deterministic

- However, we don't live in the perfect world

- Any noise in \mathbf{o} of \mathbf{x} in D
- Decision factor other than \mathbf{o} of \mathbf{x} (noise factor)

Target function is contained in hypotheses set

\rightarrow a correct h can be removed by the noise : \rightarrow noise가 elimination을 하게 됨

\rightarrow Cannot say yes and no : noise가 임의의 h 를 제거할 수 있으므로 알고리즘이 PT