Tandem Trainer Aircraft

Sizing model for a tandem trainer plane.

Mission

Variables	Value	Units	Description
$MTOW$		$[lbf]$	max take off weight
$W_{fuel-tot}$		$[lbf]$	total fuel weight
R_{min}	400.000	$[nmi]$	minimum flight range

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & ['Mission']_{[0]} \\ & ['Mission']_{[0]} \\ & MTOW = W_{start} \\ & MTOW \geq W_{fuel-tot} + W_{zfw} \\ & W_{fuel-tot} \geq W_{fuel-fs} \\ & W_{end} \geq W_{zfw} \\ & W_{structures} \geq MTOW f_{structures} \\ & \vec{R} \geq 0.2 R_{min} \end{array}$$

Aircraft

Variables	Value	Units	Description
W_{pay}	500.000	$[lbf]$	payload
$f_{structures}$	0.700		fractional structural weight
$W_{structures}$		$[lbf]$	structural weight
W_{zfw}		$[lbf]$	zero fuel weight

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & ['Mission','Aircraft']_{[0,0]} \\ & W_{structures} = W_{structures} \\ & f_{structures} = f_{structures} \\ & W_{zfw} \geq W_{pay} + W_{structures} \end{array}$$

FlightSegment

Variables	Value	Units	Description

$W_{fuel-fs}$	$[lbf]$	flight segment fuel weight

minimize 1

subject to
$['Mission']_{[0]}$
$['Mission','FlightSegment']_{[0,0]}$
$['Mission','FlightSegment']_{[0,0]}$
$['Mission','FlightSegment']_{[0,0]}$
$['Mission','FlightSegment']_{[0,0]}$
$W_{fuel-fs} \geq W_{fuel} + W_{fuel} + W_{fuel} + W_{fuel} + W_{fuel}$
$\left[W_{end}\ \text{[lbf]} \quad W_{end}\ \text{[lbf]} \quad W_{end}\ \text{[lbf]} \quad W_{end}\ \text{[lbf]}\right] \geq \left[W_{start}\ \text{[lbf]} \quad W_{start}\ \text{[lbf]} \quad W_{start}\ \text{[lbf]}\right.$

Wing

Variables	Value	Units	Description
c_{MAC}		$[ft]$	mean aerodynamic chord
S		$[ft**2]$	planform area
AR	27.000		aspect ratio
b		$[ft]$	wing span

minimize 1

subject to
$b^2 = ARS$
$c_{MAC} = \frac{S}{b}$

FlightState

Variables	Value	Units	Description
ρ	0.771	$[kg/m**3]$	air density
V_{min}	10.000	$[m/s]$	minimum true airspeed
μ	0.000	$[N*s/m**2]$	dynamic viscosity
V		$[m/s]$	true airspeed
h	15000.000	$[ft]$	flight altitude
h_{ref}	15000.000	$[ft]$	reference altitude

minimize 1

subject to
$\vec{V} \geq \vec{V_{min}}$
$\vec{\rho} = \vec{\rho}$
$\vec{\mu} = \vec{\mu}$
$\vec{h} = \vec{h}$
$\vec{h_{ref}} = \vec{h_{ref}}$

AircraftPerf

Variables	Value	Units	Description
η_{prop}	0.700		propulsive efficiency
C_D			aircraft drag coefficient
W_{start}		$[lbf]$	vector-begin weight
CDA_0	0.005		non-wing drag coefficient
W_{end}		$[lbf]$	vector-end weight

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & \vec{C_D} \geq \begin{bmatrix} CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d \end{bmatrix} \\ & \vec{W_{start}} = \vec{W_{start}} \\ & \vec{W_{end}} = \vec{W_{end}} \\ & \vec{\eta_{prop}} = \vec{\eta_{prop}} \\ & ['Mission', 'FlightSegment', 'AircraftPerf']_{[0,0,0]} \\ & ['Mission', 'FlightSegment', 'AircraftPerf']_{[0,0,0]} \end{array}$$

SteadyLevelFlight

Variables	Value	Units	Description
T		$[N]$	thrust

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & \begin{bmatrix} {W_{end}}^{0.5}{W_{start}}^{0.5}\ [\text{lbf}] & {W_{end}}^{0.5}{W_{start}}^{0.5}\ [\text{lbf}] & {W_{end}}^{0.5}{W_{start}}^{0.5}\ [\text{lbf}] & {W_{end}}^{0.5}{W_{start}}^{0.5}\ [\text{lbf}] \end{bmatrix} \\ & \vec{T} \geq \begin{bmatrix} 0.5SC_DV^2\rho\ \left[\frac{\text{ft}^2\cdot\text{kg}}{(\text{m}\cdot\text{s}^2)}\right] & 0.5SC_DV^2\rho\ \left[\frac{\text{ft}^2\cdot\text{kg}}{(\text{m}\cdot\text{s}^2)}\right] & 0.5SC_DV^2\rho\ \left[\frac{\text{ft}^2\cdot\text{kg}}{(\text{m}\cdot\text{s}^2)}\right] & 0.5SC_DV^2\rho\ \left[\frac{\text{ft}^2\cdot\text{kg}}{(\text{m}\cdot\text{s}^2)}\right] \end{bmatrix} \\ & \vec{P_{shaft}} \geq \begin{bmatrix} \frac{TV}{\eta_{prop}}\ \left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right] & \frac{TV}{\eta_{prop}}\ \left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right] & \frac{TV}{\eta_{prop}}\ \left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right] & \frac{TV}{\eta_{prop}}\ \left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right] & \frac{TV}{\eta_{prop}}\ \left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right] \end{bmatrix} \end{array}$$

BreguetRange

Variables	Value	Units	Description
g	9.810	$[m/s**2]$	gravitational acceleration
R		$[nmi]$	range
W_{fuel}		$[lbf]$	segment-fuel weight
z_{bre}			Breguet coefficient
ρ_{JetA}	6.750	$[lb/gal]$	Jet A fuel density

minimize 1

subject to

$$\vec{z_{bre}} \geq \left[0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{}{VW}\right.$$

$$\left[\frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}}\right] \geq \left[0.1667z_{bre}{}^{3} + 0.5z_{bre}{}^{2} + z_{bre} \quad 0.1667z_{bre}{}^{3} + 0.5z_{bre}{}^{2} +\right.$$

$$\vec{W_{start}} \geq \left[W_{end} + W_{fuel} \ [\mathrm{lbf}] \quad W_{end} + W_{fuel} \ [\mathrm{lbf}] \quad W_{end} + W_{fuel} \ [\mathrm{lbf}] \quad W_{end} + W_{fuel} \ [\mathrm{lbf}] \quad W\right.$$

NACA652Aero

Variables	Value	Units	Description
Re			Reynold's number
c_{dp}			wing profile drag coeff
C_L			lift coefficient
e	0.900		Oswald efficiency
C_d			wing drag coefficient

minimize 1

subject to

$$\vec{C_d} \geq \left[0.3183\frac{C_L{}^{2}}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^{2}}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^{2}}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^{2}}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^{2}}{ARe} + c_{d}\right.$$

$$\left[c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18}\right] \geq \left[1.15 \times 10^{56}\frac{C_L{}^{93}}{Re^{14}} + 1.563 \times 10^{-10}\frac{C_L{}^{0.062}}{Re^{5.2}} + 2.443 \times 10^{-49}C\right.$$

$$\vec{Re} = \left[0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu}\right]$$

EnginePerf

Variables	Value	Units	Description
$\eta_{alternator}$	0.800		alternator efficiency
P_{shaft}		$[hp]$	Shaft power
P_{total}		$[hp]$	Total power, avionics included
RPM		$[rpm]$	Engine operating RPM
RPM_{max}	5800.000	$[rpm]$	Maximum RPM
P_{avn}	40.000	$[W]$	Avionics power
$\dot{m}$		$[l/hr]$	fuel burn rate
L_{eng}	1.000		shaft power loss factor
$P_{shaft-max}$		$[hp]$	Max shaft power at altitude
$\dot{m}_{f-min}$	7.000	$[l/hr]$	minimum fuel burn rate

$$\begin{aligned}
\text{minimize} \quad & 1 \\
\text{subject to} \quad & \begin{bmatrix} \frac{P_{total}}{P_{shaft-max}} & \frac{P_{total}}{P_{shaft-max}} & \frac{P_{total}}{P_{shaft-max}} & \frac{P_{total}}{P_{shaft-max}} & \frac{P_{total}}{P_{shaft-max}} \end{bmatrix} = \Big[1.115 \frac{RPM^{1.6}}{RPM_{max}{}^{1.6}} \quad 1.115 \frac{RPM}{RPM_{m}} \\
& \begin{bmatrix} \frac{\dot{m}^{0.1}}{\dot{m}^{0.1}_{f-min}} & \frac{\dot{m}^{0.1}}{\dot{m}^{0.1}_{f-min}} & \frac{\dot{m}^{0.1}}{\dot{m}^{0.1}_{f-min}} & \frac{\dot{m}^{0.1}}{\dot{m}^{0.1}_{f-min}} & \frac{\dot{m}^{0.1}}{\dot{m}^{0.1}_{f-min}} \end{bmatrix} = \Big[1.152 \frac{RPM^{0.23}}{RPM_{max}{}^{0.23}} \quad 1.152 \frac{RPM^{0.23}}{RPM_{max}{}^{0.23}} \quad 1.152 \\
& \begin{bmatrix} \frac{P_{shaft-max}}{P_{sl-max}} & \frac{P_{shaft-max}}{P_{sl-max}} & \frac{P_{shaft-max}}{P_{sl-max}} & \frac{P_{shaft-max}}{P_{sl-max}} & \frac{P_{shaft-max}}{P_{sl-max}} \end{bmatrix} = \vec{L_{eng}} \\
& \vec{P_{shaft-max}} \geq \vec{P_{total}} \\
& \vec{P_{total}} \geq \Big[0.001341 \frac{P_{avn}}{\eta_{alternator}} + P_{shaft} \ [\text{hp}] \quad 0.001341 \frac{P_{avn}}{\eta_{alternator}} + P_{shaft} \ [\text{hp}] \quad 0.001341 \frac{P_{avn}}{\eta_{alternator}} \\
& \vec{m} \geq \dot{\vec{m}}_{f-min} \\
& R\vec{P}M \leq RP\vec{M}_{max}
\end{aligned}$$

Fits

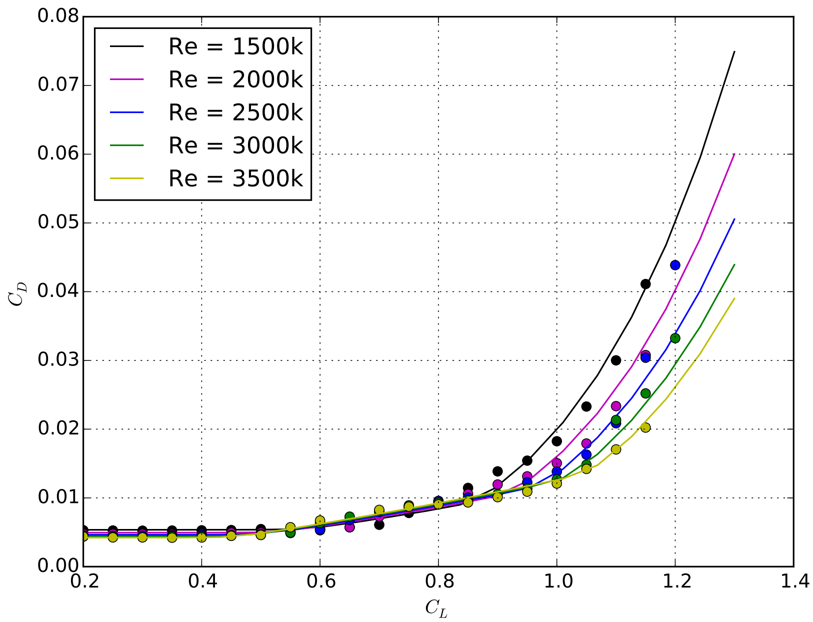
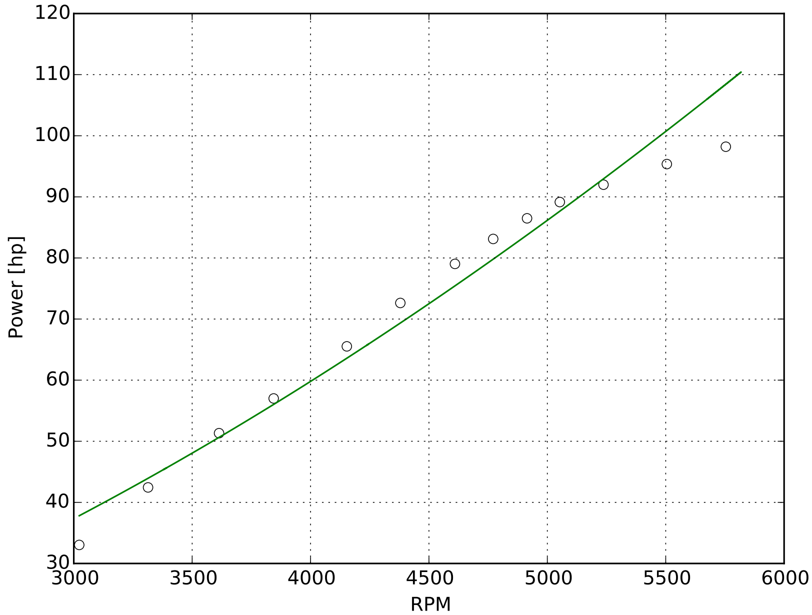
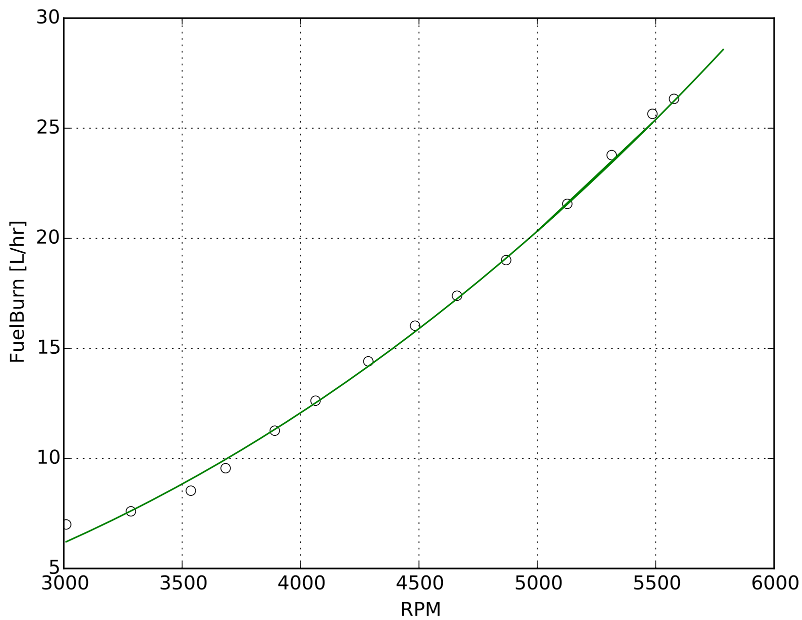


Figure 1: Fit of drag polar data from NACA 652-2412 airfoil



120
110
100
90
80
70
60
50
40
30
Power [hp]
3000
3500
4000
4500
5000
5500
6000
RPM



30
25
20
15
10
5
FuelBurn [L/hr]
3000
3500
4000
4500
5000
5500
6000
RPM

Solution

Minimizing the max take off weight we arrive at the solution.

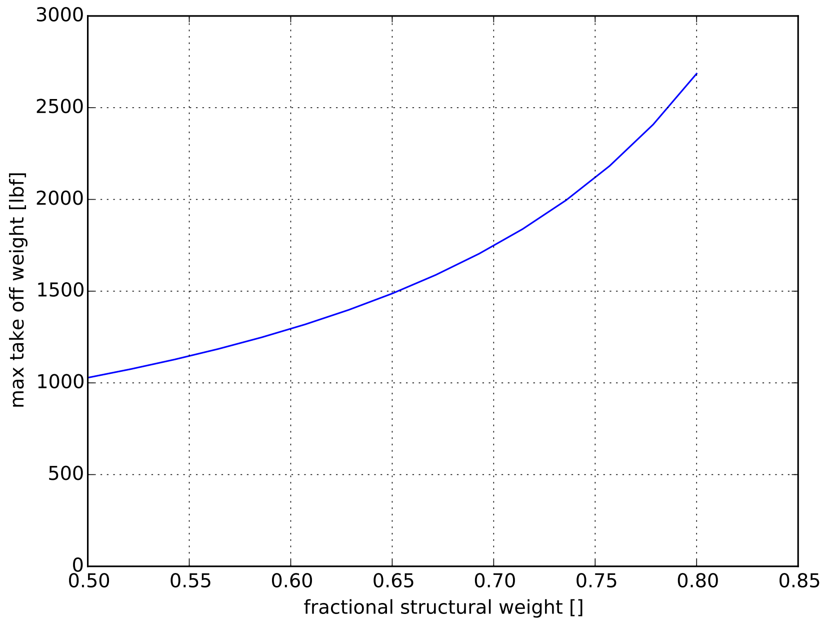
Free Variables	Value	Units	Description
Mission			
$MTOW$	1747	lbf	max take off weight
$W_{fuel-tot}$	24.05	lbf	total fuel weight
Mission/Aircraft			
$W_{structures}$	1223	lbf	structural weight
W_{zfw}	1723	lbf	zero fuel weight
Mission/Aircraft/Wing			
S	31.9	ft^2	planform area
b	29.35	ft	wing span
c_{MAC}	1.087	ft	mean aerodynamic chord
Mission/FlightSegment			
$W_{fuel-fs}$	24.05	lbf	flight segment fuel weight
Mission/FlightSegment/AircraftPerf			
C_D	[0.0155 0.0154 0.0153 0.0151 ...]		aircraft drag coefficient
W_{end}	[1.74e+03 1.74e+03 1.73e+03 1.73e+03 ...]	lbf	vector-end weight
W_{start}	[1.75e+03 1.74e+03 1.74e+03 1.73e+03 ...]	lbf	vector-begin weight
Mission/FlightSegment/AircraftPerf/EnginePerf			
$P_{shaft-max}$	[98.6 98.6 98.6 98.6 ...]	hp	Max shaft power at altitude
P_{shaft}	[40.7 40.7 40.7 40.7 ...]	hp	Shaft power
P_{total}	[40.8 40.8 40.8 40.8 ...]	hp	Total power, avionics included
RPM	[3.17e+03 3.17e+03 3.17e+03 3.17e+03 ...]	rpm	Engine operating RPM
$\dot{m}$	[7 7 7 7 ...]	$\frac{\mathrm{l}}{\mathrm{hr}}$	fuel burn rate
Mission/FlightSegment/AircraftPerf/NACA652Aero			
C_L	[0.602 0.597 0.592 0.587 ...]		lift coefficient
C_d	[0.0105 0.0104 0.0103 0.0101 ...]		wing drag coefficient
Re	[1.65e+06 1.66e+06 1.66e+06 1.67e+06 ...]		Reynold's number
c_{dp}	[0.00579 0.00573 0.00568 0.00563 ...]		wing profile drag coeff
Mission/FlightSegment/BreguetRange			
R	[80 80 80 80 ...]	nmi	range
W_{fuel}	[4.84 4.82 4.81 4.8 ...]	lbf	segment-fuel weight
z_{bre}	[0.00277 0.00277 0.00277 0.00277 ...]		Breguet coefficient
Mission/FlightSegment/FlightState			
V	[106 107 107 107 ...]	$\frac{\mathrm{m}}{\mathrm{s}}$	true airspeed

Mission/FlightSegment/SteadyLevelFlight			
T	[200 200 199 199 ...]	N	thrust

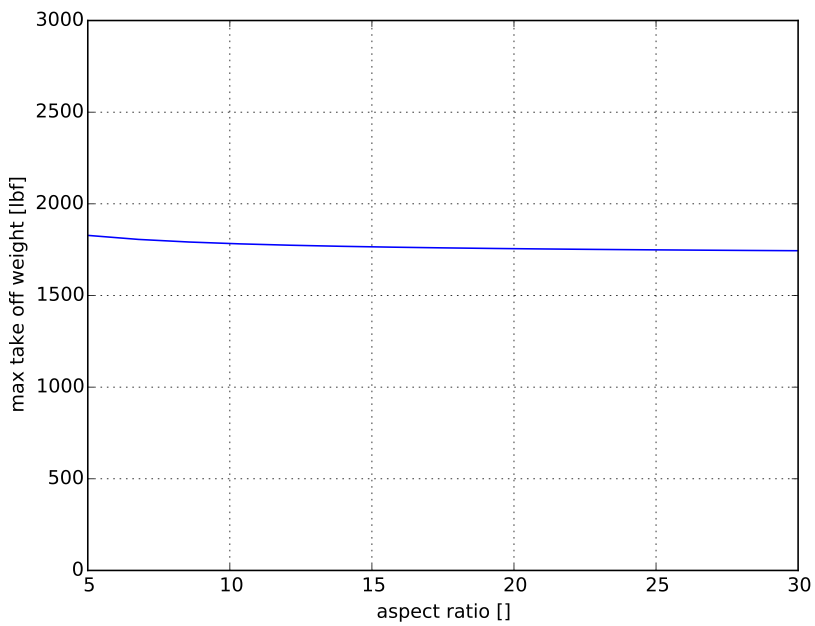
Constants	Value	Units	Description
Mission			
R_{min}	400	nmi	minimum flight range
Mission/Aircraft			
W_{pay}	500	lbf	payload
$f_{structures}$	0.7		fractional structural weight
Mission/Aircraft/Engine			
P_{sl-max}	98.56	hp	Max shaft power at sea level
Mission/Aircraft/Wing			
AR	27		aspect ratio
Mission/FlightSegment/AircraftPerf			
CDA_0	[0.005 0.005 0.005 0.005 ...]		non-wing drag coefficient
η_{prop}	[0.7 0.7 0.7 0.7 ...]		propulsive efficiency
Mission/FlightSegment/AircraftPerf/EnginePerf			
L_{eng}	[1 1 1 1 ...]		shaft power loss factor
P_{avn}	[40 40 40 40 ...]	W	Avionics power
RPM_{max}	[5.8e+03 5.8e+03 5.8e+03 5.8e+03 ...]	rpm	Maximum RPM
$\dot{m}_{f-min}$	[7 7 7 7 ...]	$\frac{\mathrm{l}}{\mathrm{hr}}$	minimum fuel burn rate
$\eta_{alternator}$	[0.8 0.8 0.8 0.8 ...]		alternator efficiency
Mission/FlightSegment/AircraftPerf/NACA652Aero			
e	[0.9 0.9 0.9 0.9 ...]		Oswald efficiency
Mission/FlightSegment/BreguetRange			
ρ_{JetA}	[6.75 6.75 6.75 6.75 ...]	$\frac{\mathrm{lb}}{\mathrm{gal}}$	Jet A fuel density
g	[9.81 9.81 9.81 9.81 ...]	$\frac{\mathrm{m}}{\mathrm{s}^2}$	gravitational acceleration
Mission/FlightSegment/FlightState			
V_{min}	[10 10 10 10 ...]	$\frac{\mathrm{m}}{\mathrm{s}}$	minimum true airspeed
μ	[1.64e-05 1.64e-05 1.64e-05 1.64e-05 ...]	$\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$	dynamic viscosity
ρ	[0.771 0.771 0.771 0.771 ...]	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	air density
h	[1.5e+04 1.5e+04 1.5e+04 1.5e+04 ...]	ft	flight altitude
h_{ref}	[1.5e+04 1.5e+04 1.5e+04 1.5e+04 ...]	ft	reference altitude

Sensitivities	Value	Units	Description
Mission/FlightSegment/BreguetRange			
g	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{\mathrm{m}}{\mathrm{s}^2}$	gravitational acceleration
ρ_{JetA}	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{\mathrm{lb}}{\mathrm{gal}}$	Jet A fuel density
Mission/FlightSegment/AircraftPerf/NACA652Aero			
e	[-0.00315 -0.00301 -0.00287 -0.00274 ...]		Oswald efficiency
Mission/FlightSegment/AircraftPerf/EnginePerf			
$\dot{m}_{f-min}$	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{\mathrm{l}}{\mathrm{hr}}$	minimum fuel burn rate
L_{eng}	[-0.0103 -0.00993 -0.00956 -0.0092 ...]		shaft power loss factor
Mission/FlightSegment/AircraftPerf			
CDA_0	[0.00331 0.00322 0.00313 0.00303 ...]		non-wing drag coefficient
η_{prop}	[-0.0103 -0.00992 -0.00955 -0.00919 ...]		propulsive efficiency
Mission/Aircraft/Wing			
AR	-0.01465		aspect ratio
Mission/Aircraft/Engine			
P_{sl-max}	-0.04788	hp	Max shaft power at sea level
Mission/Aircraft			
$f_{structures}$	2.446		fractional structural weight
W_{pay}	1	lbf	payload
Mission			
R_{min}	0.0478	nmi	minimum flight range

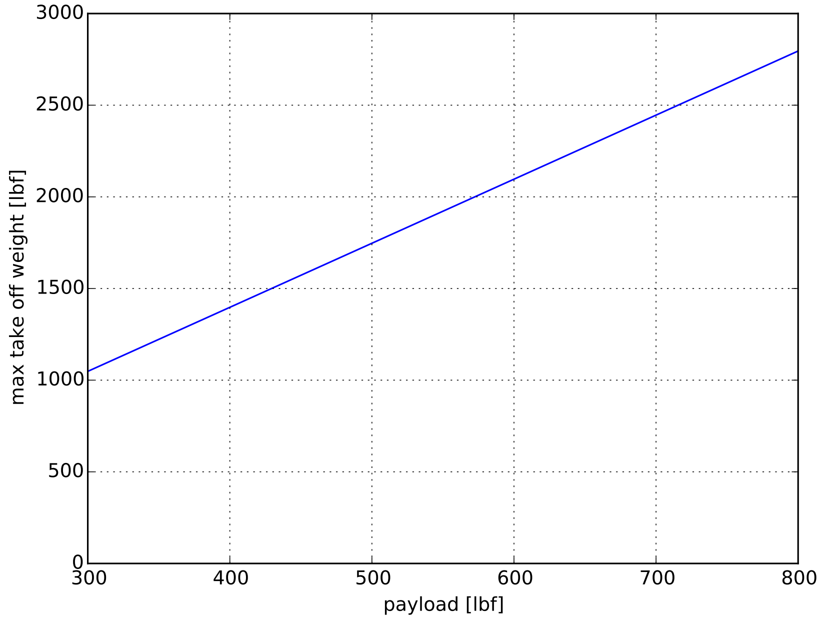
Sweeps



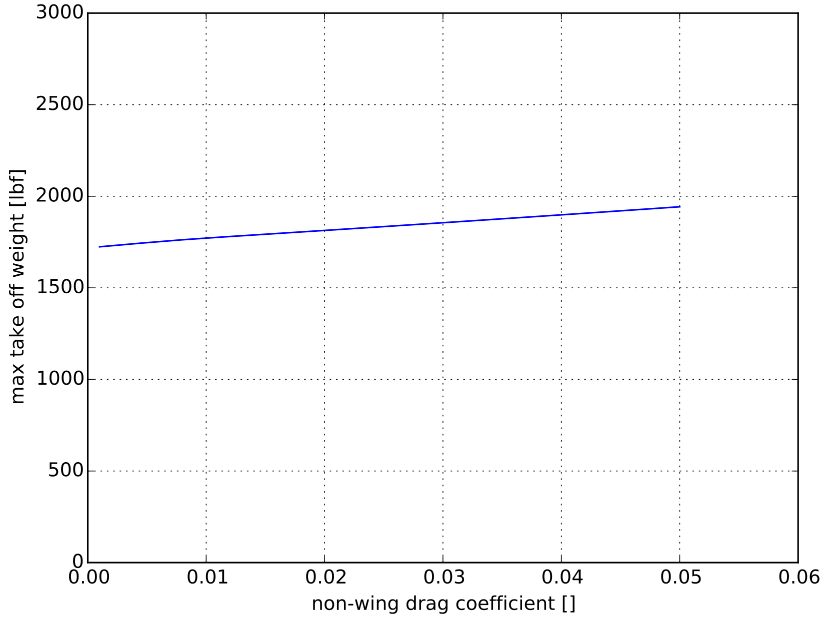
3000
2500
2000
1500
1000
500
0
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
max take off weight [lbf]
fractional structural weight []



3000
2500
2000
1500
1000
500
0
5
10
15
20
25
30
max take off weight [lbf]
aspect ratio []



max take off weight [lbf]
payload [lbf]



max take off weight [lbf]
non-wing drag coefficient []