Tandem Trainer Aircraft

Sizing model for a tandem trainer plane.

Mission

Variables	Value	Units	Description
$MTOW$		$[lbf]$	max take off weight
$W_{fuel-tot}$		$[lbf]$	total fuel weight
R_{min}	400.000	$[nmi]$	minimum flight range

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & ['Mission']_{[0]} \\ & ['Mission']_{[0]} \\ & MTOW = W_{start} \\ & MTOW \geq W_{fuel-tot} + W_{zfw} \\ & W_{fuel-tot} \geq W_{fuel-fs} \\ & W_{end} \geq W_{zfw} \\ & W_{structures} \geq MTOW f_{structures} \\ & \vec{R} \geq 0.2 R_{min} \end{array}$$

Aircraft

Variables	Value	Units	Description
W_{pay}	500.000	$[lbf]$	payload
$f_{structures}$	0.700		fractional structural weight
$W_{structures}$		$[lbf]$	structural weight
W_{zfw}		$[lbf]$	zero fuel weight

$$\begin{array}{ll} \text{minimize} & 1 \\ \text{subject to} & ['Mission', 'Aircraft']_{[0,0]} \\ & W_{structures} = W_{structures} \\ & f_{structures} = f_{structures} \\ & W_{zfw} \geq W_{pay} + W_{structures} \end{array}$$

FlightSegment

Variables	Value	Units	Description

$W_{fuel-fs}$	$[lbf]$	flight segment fuel weight

$$
\begin{array}{ll}
\text{minimize} & 1 \\
\text{subject to} & ['Mission']_{[0]} \\
& ['Mission','FlightSegment']_{[0,0]} \\
& ['Mission','FlightSegment']_{[0,0]} \\
& ['Mission','FlightSegment']_{[0,0]} \\
& ['Mission','FlightSegment']_{[0,0]} \\
& W_{fuel-fs} \geq W_{fuel} + W_{fuel} + W_{fuel} + W_{fuel} + W_{fuel} \\
& \begin{bmatrix} W_{end}\ [\text{lbf}] & W_{end}\ [\text{lbf}] & W_{end}\ [\text{lbf}] & W_{end}\ [\text{lbf}] \end{bmatrix} \geq \begin{bmatrix} W_{start}\ [\text{lbf}] & W_{start}\ [\text{lbf}] & W_{start}\ [\text{lbf}]
\end{array}
$$

Wing

Variables	Value	Units	Description
c_{MAC}		$[ft]$	mean aerodynamic chord
S		$[ft**2]$	planform area
AR	27.000		aspect ratio
b		$[ft]$	wing span

$$
\begin{array}{ll}
\text{minimize} & 1 \\
\text{subject to} & b^2 = ARS \\
& c_{MAC} = \frac{S}{b}
\end{array}
$$

FlightState

Variables	Value	Units	Description
ρ	0.771	$[kg/m**3]$	air density
V_{min}	10.000	$[m/s]$	minimum true airspeed
μ	0.000	$[N*s/m**2]$	dynamic viscosity
V		$[m/s]$	true airspeed
h	15000.000	$[ft]$	flight altitude
h_{ref}	15000.000	$[ft]$	reference altitude

$$
\begin{array}{ll}
\text{minimize} & 1 \\
\text{subject to} & \vec{V} \geq \vec{V_{min}} \\
& \vec{\rho} = \vec{\rho} \\
& \vec{\mu} = \vec{\mu} \\
& \vec{h} = \vec{h} \\
& \vec{h_{ref}} = \vec{h_{ref}}
\end{array}
$$

AircraftPerf

Variables	Value	Units	Description
η_{prop}	0.700		propulsive efficiency
C_D			aircraft drag coefficient
W_{start}		$[lbf]$	vector-begin weight
CDA_0	0.005		non-wing drag coefficient
W_{end}		$[lbf]$	vector-end weight

$$
\begin{array}{ll}
\text{minimize} & 1 \\
\text{subject to} & \vec{C_D} \geq \begin{bmatrix} CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d & CDA_0 + C_d \end{bmatrix} \\
& \vec{W_{start}} = \vec{W_{start}} \\
& \vec{W_{end}} = \vec{W_{end}} \\
& \vec{\eta_{prop}} = \vec{\eta_{prop}} \\
& ['Mission','FlightSegment','AircraftPerf']_{[0,0,0]} \\
& ['Mission','FlightSegment','AircraftPerf']_{[0,0,0]}
\end{array}
$$

SteadyLevelFlight

Variables	Value	Units	Description
T		$[N]$	thrust

$$
\begin{array}{ll}
\text{minimize} & 1 \\
\text{subject to} & \begin{bmatrix} W_{end}{}^{0.5} W_{start}{}^{0.5} \ [\mathrm{lbf}] & W_{end}{}^{0.5} W_{start}{}^{0.5} \ [\mathrm{lbf}] & W_{end}{}^{0.5} W_{start}{}^{0.5} \ [\mathrm{lbf}] & W_{end}{}^{0.5} W_{start}{}^{0.5} \ [\mathrm{lbf}] \end{bmatrix} \\
& \vec{T} \geq \begin{bmatrix} 0.5 S C_D V^2 \rho \ \left[\frac{\mathrm{ft}^2 \cdot \mathrm{kg}}{(\mathrm{m} \cdot \mathrm{s}^2)}\right] & 0.5 S C_D V^2 \rho \ \left[\frac{\mathrm{ft}^2 \cdot \mathrm{kg}}{(\mathrm{m} \cdot \mathrm{s}^2)}\right] & 0.5 S C_D V^2 \rho \ \left[\frac{\mathrm{ft}^2 \cdot \mathrm{kg}}{(\mathrm{m} \cdot \mathrm{s}^2)}\right] & 0.5 S C_D V^2 \rho \ \left[\frac{\mathrm{ft}^2 \cdot \mathrm{kg}}{(\mathrm{m} \cdot \mathrm{s}^2)}\right] \end{bmatrix} \\
& \vec{P_{shaft}} \geq \begin{bmatrix} \frac{TV}{\eta_{prop}} \ \left[\frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}}\right] & \frac{TV}{\eta_{prop}} \ \left[\frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}}\right] & \frac{TV}{\eta_{prop}} \ \left[\frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}}\right] & \frac{TV}{\eta_{prop}} \ \left[\frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}}\right] & \frac{TV}{\eta_{prop}} \ \left[\frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}}\right] \end{bmatrix}
\end{array}
$$

BreguetRange

Variables	Value	Units	Description
g	9.810	$[m/s**2]$	gravitational acceleration
R		$[nmi]$	range
W_{fuel}		$[lbf]$	segment-fuel weight
z_{bre}			Breguet coefficient
ρ_{JetA}	6.750	$[lb/gal]$	Jet A fuel density

minimize 1

subject to $\vec{z_{bre}} \geq \Big[0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{R\dot{m}\rho_{JetAg}}{VW_{end}{}^{0.5}W_{start}{}^{0.5}} \quad 0.01386\frac{R\dot{m}\rho_{JetAg}}{VW}$

$\Big[\frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}} \quad \frac{W_{fuel}}{W_{end}}\Big] \geq \big[0.1667z_{bre}{}^3 + 0.5z_{bre}{}^2 + z_{bre} \quad 0.1667z_{bre}{}^3 + 0.5z_{bre}{}^2 +$

$\vec{W_{start}} \geq \big[W_{end} + W_{fuel} \;[\mathrm{lbf}] \quad W_{end} + W_{fuel} \;[\mathrm{lbf}] \quad W_{end} + W_{fuel} \;[\mathrm{lbf}] \quad W_{end} + W_{fuel} \;[\mathrm{lbf}] \quad W$

NACA652Aero

Variables	Value	Units	Description
Re			Reynold's number
c_{dp}			wing profile drag coeff
C_L			lift coefficient
e	0.900		Oswald efficiency
C_d			wing drag coefficient

minimize 1

subject to $\vec{C_d} \geq \Big[0.3183\frac{C_L{}^2}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^2}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^2}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^2}{ARe} + c_{dp} \quad 0.3183\frac{C_L{}^2}{ARe} + c_d$

$\big[c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18} \quad c_{dp}{}^{18}\big] \geq \Big[1.15 \times 10^{56}\frac{C_L{}^{93}}{Re^{14}} + 1.563 \times 10^{-10}\frac{C_L{}^{0.062}}{Re^{5.2}} + 2.443 \times 10^{-49}C$

$\vec{Re} = \Big[0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu} \quad 0.3048\frac{c_{MAC}V\rho}{\mu}\Big]$

EnginePerf

Variables	Value	Units	Description
$\eta_{alternator}$	0.800		alternator efficiency
P_{shaft}		$[hp]$	Shaft power
P_{total}		$[hp]$	Total power, avionics included
RPM		$[rpm]$	Engine operating RPM
RPM_{max}	5800.000	$[rpm]$	Maximum RPM
P_{avn}	40.000	$[W]$	Avionics power
$\dot{m}$		$[l/hr]$	fuel burn rate
L_{eng}	1.000		shaft power loss factor
$P_{shaft-max}$		$[hp]$	Max shaft power at altitude
$\dot{m}_{f-min}$	7.000	$[l/hr]$	minimum fuel burn rate

$$
\begin{aligned}
\text{minimize} \quad & 1 \\
\text{subject to} \quad & \left[\frac{P_{total}}{P_{shaft-max}} \quad \frac{P_{total}}{P_{shaft-max}} \quad \frac{P_{total}}{P_{shaft-max}} \quad \frac{P_{total}}{P_{shaft-max}} \quad \frac{P_{total}}{P_{shaft-max}}\right] = \left[1.115\frac{RPM^{1.6}}{RPM_{max}{}^{1.6}} \quad 1.115\frac{RPM}{RPM_{m}} \right. \\
& \left[\frac{\dot{m}^{0.1}}{\dot{m}_{f-min}^{0.1}} \quad \frac{\dot{m}^{0.1}}{\dot{m}_{f-min}^{0.1}} \quad \frac{\dot{m}^{0.1}}{\dot{m}_{f-min}^{0.1}} \quad \frac{\dot{m}^{0.1}}{\dot{m}_{f-min}^{0.1}} \quad \frac{\dot{m}^{0.1}}{\dot{m}_{f-min}^{0.1}}\right] = \left[1.152\frac{RPM^{0.23}}{RPM_{max}{}^{0.23}} \quad 1.152\frac{RPM^{0.23}}{RPM_{max}{}^{0.23}} \quad 1.152 \right. \\
& \left[\frac{P_{shaft-max}}{P_{sl-max}} \quad \frac{P_{shaft-max}}{P_{sl-max}} \quad \frac{P_{shaft-max}}{P_{sl-max}} \quad \frac{P_{shaft-max}}{P_{sl-max}} \quad \frac{P_{shaft-max}}{P_{sl-max}}\right] = \vec{L_{eng}} \\
& \vec{P_{shaft-max}} \geq \vec{P_{total}} \\
& \vec{P_{total}} \geq \left[0.001341\frac{P_{avn}}{\eta_{alternator}} + P_{shaft} \ \text{[hp]} \quad 0.001341\frac{P_{avn}}{\eta_{alternator}} + P_{shaft} \ \text{[hp]} \quad 0.001341\frac{P_{avn}}{\eta_{alternator}} \right. \\
& \vec{\dot{m}} \geq \vec{\dot{m}_{f-min}} \\
& \vec{RPM} \leq \vec{RPM_{max}}
\end{aligned}
$$

Solution

Minimizing the max take off weight we arrive at the solution.

Free Variables	Value	Units	Description
Mission			
$MTOW$	1747	lbf	max take off weight
$W_{fuel-tot}$	24.05	lbf	total fuel weight
Mission/Aircraft			
$W_{structures}$	1223	lbf	structural weight
W_{zfw}	1723	lbf	zero fuel weight
Mission/Aircraft/Wing			
S	31.9	ft^2	planform area
b	29.35	ft	wing span
c_{MAC}	1.087	ft	mean aerodynamic chord
Mission/FlightSegment			
$W_{fuel-fs}$	24.05	lbf	flight segment fuel weight
Mission/FlightSegment/AircraftPerf			
C_D	[0.0155 0.0154 0.0153 0.0151 ...]		aircraft drag coefficient
W_{end}	[1.74e+03 1.74e+03 1.73e+03 1.73e+03 ...]	lbf	vector-end weight
W_{start}	[1.75e+03 1.74e+03 1.74e+03 1.73e+03 ...]	lbf	vector-begin weight
Mission/FlightSegment/AircraftPerf/EnginePerf			
$P_{shaft-max}$	[98.6 98.6 98.6 98.6 ...]	hp	Max shaft power at altitude
P_{shaft}	[40.7 40.7 40.7 40.7 ...]	hp	Shaft power
P_{total}	[40.8 40.8 40.8 40.8 ...]	hp	Total power, avionics included
RPM	[3.17e+03 3.17e+03 3.17e+03 3.17e+03 ...]	rpm	Engine operating RPM
$\dot{m}$	[7 7 7 7 ...]	$\frac{\mathrm{l}}{\mathrm{hr}}$	fuel burn rate

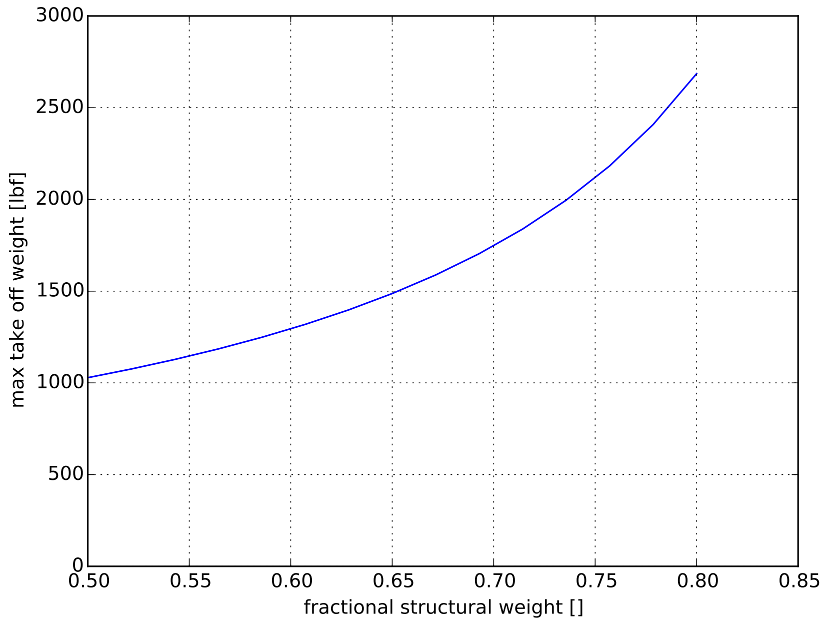
Mission/FlightSegment/AircraftPerf/NACA652Aero			
C_L	[0.602 0.597 0.592 0.587 ...]		lift coefficient
C_d	[0.0105 0.0104 0.0103 0.0101 ...]		wing drag coefficient
Re	[1.65e+06 1.66e+06 1.66e+06 1.67e+06 ...]		Reynold's number
c_{dp}	[0.00579 0.00573 0.00568 0.00563 ...]		wing profile drag coeff
Mission/FlightSegment/BreguetRange			
R	[80 80 80 80 ...]	nmi	range
W_{fuel}	[4.84 4.82 4.81 4.8 ...]	lbf	segment-fuel weight
z_{bre}	[0.00277 0.00277 0.00277 0.00277 ...]		Breguet coefficient
Mission/FlightSegment/FlightState			
V	[106 107 107 107 ...]	$\frac{\mathrm{m}}{\mathrm{s}}$	true airspeed
Mission/FlightSegment/SteadyLevelFlight			
T	[200 200 199 199 ...]	N	thrust

Constants	Value	Units	Description
Mission			
R_{min}	400	nmi	minimum flight range
Mission/Aircraft			
W_{pay}	500	lbf	payload
$f_{structures}$	0.7		fractional structural weight
Mission/Aircraft/Engine			
P_{sl-max}	98.56	hp	Max shaft power at sea level
Mission/Aircraft/Wing			
AR	27		aspect ratio
Mission/FlightSegment/AircraftPerf			
CDA_0	[0.005 0.005 0.005 0.005 ...]		non-wing drag coefficient
η_{prop}	[0.7 0.7 0.7 0.7 ...]		propulsive efficiency
Mission/FlightSegment/AircraftPerf/EnginePerf			
L_{eng}	[1 1 1 1 ...]		shaft power loss factor
P_{avn}	[40 40 40 40 ...]	W	Avionics power
RPM_{max}	[5.8e+03 5.8e+03 5.8e+03 5.8e+03 ...]	rpm	Maximum RPM
$\dot{m}_{f-min}$	[7 7 7 7 ...]	$\frac{1}{\mathrm{hr}}$	minimum fuel burn rate
$\eta_{alternator}$	[0.8 0.8 0.8 0.8 ...]		alternator efficiency
Mission/FlightSegment/AircraftPerf/NACA652Aero			
e	[0.9 0.9 0.9 0.9 ...]		Oswald efficiency

Mission/FlightSegment/BreguetRange			
ρ_{JetA}	[6.75 6.75 6.75 6.75 ...]	$\frac{\mathrm{lb}}{\mathrm{gal}}$	Jet A fuel density
g	[9.81 9.81 9.81 9.81 ...]	$\frac{\mathrm{m}}{\mathrm{s}^2}$	gravitational acceleration
Mission/FlightSegment/FlightState			
V_{min}	[10 10 10 10 ...]	$\frac{\mathrm{m}}{\mathrm{s}}$	minimum true airspeed
μ	[1.64e-05 1.64e-05 1.64e-05 1.64e-05 ...]	$\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$	dynamic viscosity
ρ	[0.771 0.771 0.771 0.771 ...]	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	air density
h	[1.5e+04 1.5e+04 1.5e+04 1.5e+04 ...]	ft	flight altitude
h_{ref}	[1.5e+04 1.5e+04 1.5e+04 1.5e+04 ...]	ft	reference altitude

Sensitivities	Value	Units	Description
Mission/FlightSegment/BreguetRange			
g	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{\mathrm{m}}{\mathrm{s}^2}$	gravitational acceleration
ρ_{JetA}	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{\mathrm{lb}}{\mathrm{gal}}$	Jet A fuel density
Mission/FlightSegment/AircraftPerf/NACA652Aero			
e	[-0.00315 -0.00301 -0.00287 -0.00274 ...]		Oswald efficiency
Mission/FlightSegment/AircraftPerf/EnginePerf			
$\dot{m}_{f-min}$	[0.00956 0.00956 0.00956 0.00956 ...]	$\frac{1}{\mathrm{hr}}$	minimum fuel burn rate
L_{eng}	[-0.0103 -0.00993 -0.00956 -0.0092 ...]		shaft power loss factor
Mission/FlightSegment/AircraftPerf			
CDA_0	[0.00331 0.00322 0.00313 0.00303 ...]		non-wing drag coefficient
η_{prop}	[-0.0103 -0.00992 -0.00955 -0.00919 ...]		propulsive efficiency
Mission/Aircraft/Wing			
AR	-0.01465		aspect ratio
Mission/Aircraft/Engine			
P_{sl-max}	-0.04788	hp	Max shaft power at sea level
Mission/Aircraft			
$f_{structures}$	2.446		fractional structural weight
W_{pay}	1	lbf	payload
Mission			
R_{min}	0.0478	nmi	minimum flight range

Sweeps


3000
2500
2000
1500
1000
500
0
max take off weight [lbf]
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0.55
0.60
0.65
0.70
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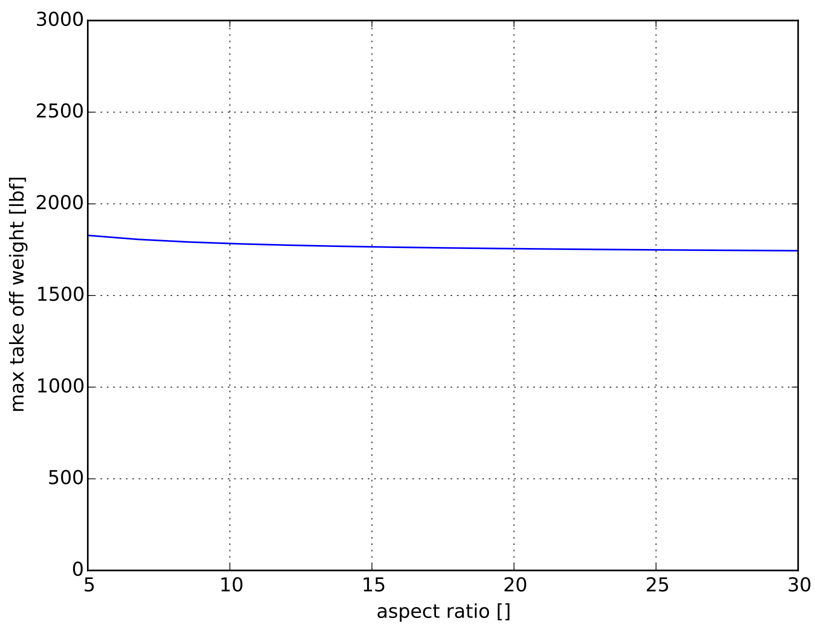
0.80
0.85
fractional structural weight []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2500
2000
1500
1000
500
0
max take off weight [lbf]
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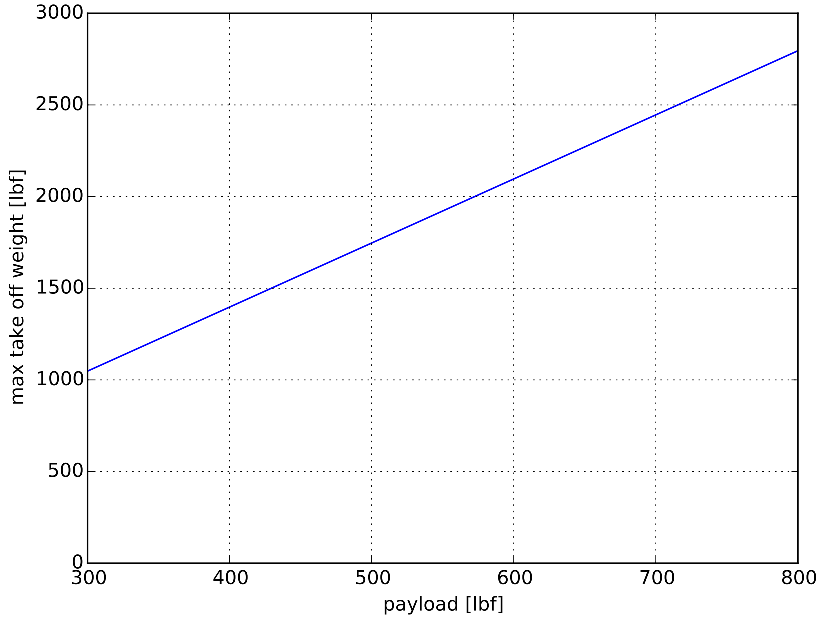
10
15
20
25
30
aspect ratio []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payload [lbf]
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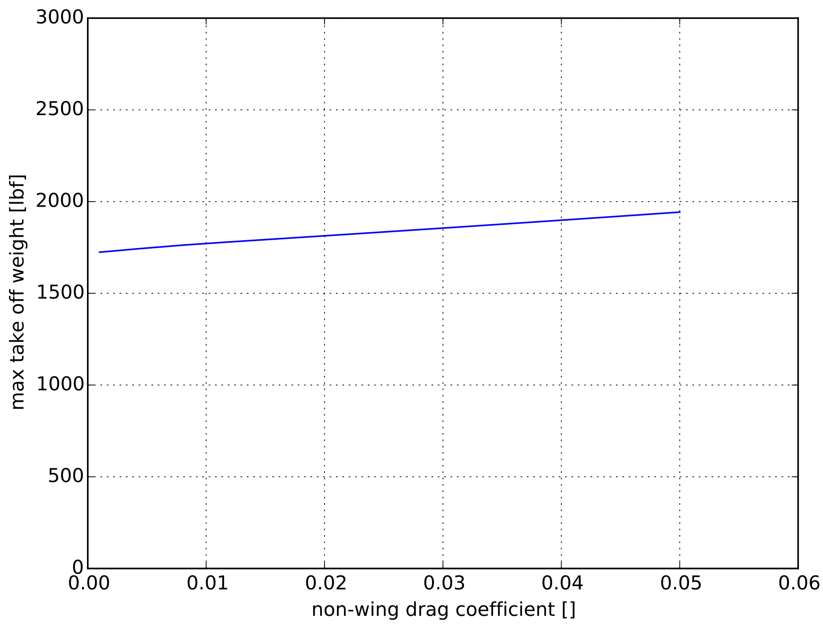
500
1000
1500
2000
2500
3000
300
400
500
600
700
800


max take off weight [lbf]
non-wing drag coefficient []
0
500
1000
1500
2000
2500
3000
0.00
0.01
0.02
0.03
0.04
0.05
0.06